

Investigation on Planck scale physics by the AURIGA gravitational bar detector

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Abstract

We have recently shown that the very low mechanical energy achieved and measured in the main vibration mode of gravitational wave bar detectors can set an upper limit to possible modifications of the Heisenberg uncertainty principle that are expected as an effect of gravity. Here we give more details on the data analysis procedure that allows one to deduce the energy of the bar mode (i.e., the meaningful parameter for our purpose). Furthermore, we extend the analysis of our results, discussing their implication for physical models that face quantum



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gravity from different points of view, e.g., proposing modified commutation relations or exploring spacetime discreteness.

Keywords: Planck scale, generalized uncertainty principle, gravitational bar detector

1. Introduction

General relativity and quantum physics are expected to merge at the Planck scale, defined by distances of the order of $L_p = \sqrt{\hbar G/c^3} = 1.6 \times 10^{-35}$ m and/or extremely high energies of the order of $\sim E_p = c\hbar/L_p = 1.2 \times 10^{19}$ GeV. At this scale, gravity is supposed to have a strength close to the electro-weak and strong forces, and quantum gravitational effects can no longer be neglected. Since the study of particles collisions around the Planck energy is well beyond the possibilities of current and foreseeable accelerators, high-energy astronomical events (e.g., γ -ray bursts) have been considered as the privileged natural system to unveil quantum gravitational effects [1–4]. This common view has been enriched in recent years thanks to a number of studies proposing that signatures of the Planck-scale physics could also manifest at low energies. It is indeed widely accepted that, when gravity is taken into account, deviations from standard quantum mechanics are expected.

A paradigmatic example, provided by Wigner in 1957 [5, 6], illustrates the consequences produced by the combination of quantum mechanics and some basic features of gravity in the possibility of determining the position of an object. His starting idea is the measurement of space-like distances by using the time of flight of light beams, thus exploiting clocks (i.e., possible quantum objects) instead of macroscopic standard rods. A typical measurement scheme consists in positioning a clock in the first reference frame with an accuracy Δx_{clock} ; sending a light pulse from the clock to the second reference frame, where it is reflected back; and detecting the arrival time on the clock $T = 2D/c$, from which we deduce the distance D . Due to Heisenberg's principle, the velocity of the clock whose mass is m , is known with a minimal uncertainty $\Delta v = \frac{\hbar}{2m\Delta x_{clock}}$; therefore, during the time T , it moves a step whose length has an uncertainty $T\Delta v$. As a consequence, we have a spread ΔD in the measured distance between the two reference frames, given by:

$$(\Delta D)^2 = (\Delta x_{clock})^2 + \left(\frac{\hbar D}{mc\Delta x_{clock}} \right)^2 \quad (1)$$

and actually a minimal uncertainty $\Delta D_{min} = \sqrt{\frac{2\hbar D}{mc}}$. More refined speculations can be developed on this conceptual scheme. For instance, the same procedure of sending and detecting a light pulse imposes quantum-mechanical constraints, and the original work by Salecker and Wigner discusses in detail the properties of a possible quantum clock. However, we remark that the obtained relation (1) is quite close to the expression of the standard quantum limit in consecutive position measurements, a concept later developed in the framework of the studies of gravitational wave detectors [7, 8]. Any Wigner-like measurement scheme can likely be traced back to a series of position measurements. If we now consider the complete quantum-mechanical uncertainty ΔD_{min} , we notice that it can be arbitrarily reduced by increasing the mass m . Therefore, in quantum mechanics there are no limits to the accuracy of distance

measurements. However, taking into account gravity, m is limited by the requirement that the clock should not turn into a black hole, expressed by $m < \frac{Dc^2}{G}$. This implies a lower bound on the uncertainty in D , given by $\Delta D_{\min} \geq \sqrt{\frac{2\hbar G}{c^3}} = \sqrt{2}L_p$.

Although these simple considerations cannot be considered as a rigorous demonstration, the existence of a minimal measurable length of the order of the Planck length is a common feature of different approaches to quantum gravity [9, 10], such as string theory [11, 12] and loop quantum gravity, as well as of doubly special relativity [13] and gedanken experiments in black hole physics [14–16].

As just mentioned, in the framework of the Heisenberg relation, the minimal position uncertainty could be made arbitrarily small toward zero at the cost of our knowledge about the momentum. However, the above arguments show that when gravity is taken into account, a minimal observable length appears naturally. This consideration has motivated the introduction of generalized Heisenberg uncertainty principles (GUPs), in which the existence of a minimum length scale is encoded:

$$\Delta x \Delta p \geq \frac{\hbar}{2} \left(1 + \beta_0 \left(\frac{L_p \Delta p}{\hbar} \right)^2 \right), \quad (2)$$

where β_0 is a positive dimensionless parameter. Equation (2) implies the existence of a nonzero minimal uncertainty in position measurements $\Delta x_{\min} = \sqrt{\beta_0} L_p$, which defines a new fundamental length scale where some new physics should come into play [17]. Although the GUP (2) was originally proposed in the context of string theory [11, 12], different considerations combining quantum mechanics and general relativity suggest that this formula might have a more general validity in quantum gravity, and it is not necessarily related to strings [14–16]. For instance, considering again the Wigner measurement scheme, we remark that, according to equation (1), its accuracy is limited by the total uncertainty in the clock position:

$$\Delta x \approx \Delta x_Q + \frac{\hbar D}{mc \Delta x_Q} \geq \Delta x_Q + \frac{L_p^2}{\Delta x_Q}, \quad (3)$$

where we have used the relation $m \leq Dc^2/G$. Then, Δx consists of the sum of two contributions, Δx_Q and $\Delta x_G = L_p^2/\Delta x_Q$. The first one is the measurement precision while the second one, proportional to the gravitational constant G , is due to the combination of the Heisenberg uncertainty principle and gravitational effects. Using $\Delta x_Q \sim \frac{\hbar}{2\Delta p}$, equation (3) reduces to equation (2) with a deformation parameter $\beta_0 = 4$. In this framework, the quantum uncertainty in the position of a particle implies an uncertainty in its momentum. Therefore, due to the mass-energy equivalence, it also implies an uncertainty in the geometry which in turn introduces an additional uncertainty in the position of the particle.

To a GUP it is possible to associate a modified canonical commutator between position and momentum [18], which implies changes in the energy spectrum and in the time evolution of quantum systems and suggests that space is quantized in units of a fundamental length [19]. As a consequence, an alternative way to check quantum gravitational effects would be to perform high-sensitivity measurements of the uncertainty relation in order to reveal any possible deviation from predictions of standard quantum mechanics [17, 20].

To this end, two optical experiments have been recently proposed for observing Planck-scale effects on the center-of-mass coordinates of some macroscopic bodies [21, 22]. [21] aims

at measuring quantum gravitational modifications of the canonical commutator for the variables associated to the center of mass of a macroscopic oscillator. With a different approach, [22] proposes to study the motion of a macroscopic dielectric block crossed by a single photon as a probe to test the Wheeler's concept of quantum foam [23], i.e., the non-smooth texture of spacetime on length scales of the Planck order.

As recently remarked [24], the heuristic arguments used to introduce GUPs, as well as those concerning the spacetime quantization, do not apply to these proposals since they do not involve Planckian energies concentrated in a Planck-length-sized region. More specifically, it is the high energy used to probe small distances that significantly disturbs the spacetime structure and thus increases the position quantum uncertainty. On the other hand, one can consider the quantum spacetime fluctuations responsible for $\Delta x_{\min} \geq L_p$ to be related to the creation and annihilation of particles (which may in principle have arbitrarily high energies) and virtual black holes [25]. Given an underlying discrete spacetime, fundamental questions are how ordinary matter propagates in such a background and whether signatures of discreteness can appear on macroscopic scales. Some indications might be provided, e.g. by causal set theory, a Lorentz invariant approach to quantum gravity [26]. In this framework, propagating particles are subject to small, stochastic fluctuations due to the uncertainty in spacetime structure at the Planck scale [27]. Although the definition of a quantum measurement in discrete spacetime is still an open problem, we remark that the emergence of extended uncertainty relations for discrete coordinate and momentum operators, which can be formulated in the form of a GUP, have been investigated in finite-dimensional discrete phase spaces [28]. In this spirit, the nonzero minimum uncertainty in position measurements could be the manifestation of an inherent spacetime feature, i.e., the existence of a fundamental scale at which the very concept of distance becomes physically meaningless. As such, it would also affect the low-energy motion of the center of mass of a macroscopic body. This is the key assumption of proposals based on precise metrological systems, such as those described in [21, 22], and on correlated Michelson interferometers [29].

The aim of this paper is to review and extend the results of our analysis described in [30], where we have exploited the sub-millikelvin cooling of the normal modes of the ton-scale gravitational wave detector AURIGA to place an upper limit for possible Planck-scale modifications on the ground-state energy of an oscillator. We discuss some possible interpretations of our results, including possible consequences on deformed commutators, and an upper limit on the length scale at which quantum fluctuations of the spacetime geometry should come into play.

2. GUP and ground state energy of harmonic oscillators

The ground state energy of a quantum harmonic oscillator is a direct consequence of the Heisenberg uncertainty relation. If the uncertainty principle is modified by quantum gravity, then the zero-point energy would also be changed. We then consider the generalized uncertainty relation (2), re-written in the form:

$$\Delta x \Delta p \geq \frac{\hbar}{2} \left(1 + \beta_0 \left(\frac{\Delta p}{M_p c} \right)^2 \right), \quad (4)$$

where M_p is the Planck mass $M_p = E_p/c^2 \simeq 22 \mu\text{g}$. The Hamiltonian operator for a harmonic oscillator with mass m and angular frequency ω_0 is:

$$H = \frac{m\omega_0^2}{2} x^2 + \frac{1}{2m} p^2, \quad (5)$$

and its minimal energy is found for $\langle x \rangle = \langle p \rangle = 0$ using the equality in equation (4). Extracting Δx from this last equality, inserting it into (5), and minimizing with respect to Δp , we find the minimal energy:

$$E_{\min} = \frac{\hbar\omega_0}{2} \left[\sqrt{1 + \frac{\beta^2}{4}} + \frac{\beta}{2} \right], \quad (6)$$

where $\beta = \beta_0 \frac{\hbar m \omega_0}{M_p^2 c^2}$ is a dimensionless deformation parameter that includes the properties of the specific mechanical oscillator.

The modified ground state energy is larger with respect to the standard quantum mechanical value $\hbar\omega_0/2$. Therefore, an experiment to measure a low energy level E_{exp} for a mechanical oscillator puts a straightforward upper limit to the corresponding E_{\min} :

$$E_{\min} < E_{\text{exp}}. \quad (7)$$

Equation (7) expresses the key idea of our analysis.

Comparing equation (6) with the measured E_{exp} , according to equation (7), we derive for β the limit:

$$\beta < \frac{2E_{\text{exp}}}{\hbar\omega_0} - \frac{\hbar\omega_0}{2E_{\text{exp}}}. \quad (8)$$

The energy of a ‘standard’ quantum oscillator in a thermal state is usually given in the form:

$$E = \hbar\omega_0 (1/2 + n_T), \quad (9)$$

where n_T is the average occupation number. In quantum gravity theory, the complete energy spectrum is likely to be modified according to the specific assumptions of the theory (an example is described in [31]). However, for high temperatures (well above the ground state) the ‘standard’ theory is likely to remain a good approximation and, in any case, equation (9) can always be used for defining n_T . equation (8) can thus be written as $\beta < 4n_T(1 + n_T)/(1 + 2n_T)$. In the limit $n_T \gg 1$, valid for the experiment on the AURIGA detector analyzed in [30] and this work, the inequality can be simplified to $\beta < 2n_T$. In the opposite case of a mechanical oscillator cooled down quite close to its ‘standard’ ground state (i.e., with energy close to $\hbar\omega_0/2$), the experimental limit to β is given by the accuracy ΔE in the measurement of the energy in the form $\beta < 4\Delta E/\hbar\omega_0$.

If we call β_{\max} the experimental upper limit to β , the corresponding limit to the deformation parameter β_0 can be written in the meaningful form:

$$\beta_0 < \beta_{\max} \frac{M_p}{m} \frac{M_p c^2}{\hbar \omega_0}. \quad (10)$$

The two factors multiplying β_{\max} suggest that more stringent limits are obtained from oscillators with large mass (compared to M_p) and large ‘standard’ ground state energy (compared to E_p).

It is useful noticing that the product $m\omega_0$ appears in the expression of the ground state wavepacket size, i.e., of the zero-point fluctuations $x_{zpf}^2 = \hbar/2m\omega_0$. Using this characteristic parameter of the oscillator, the limit on β_0 can be written as:

$$\beta_0 < \beta_{\max} \frac{x_{zpf}^2}{2L_p^2}. \quad (11)$$

In summary, the best oscillator for testing quantum gravity effects is the one with the smallest zero-point fluctuations. Of course, it must be cooled to the lowest occupation number or, when $n_T < 1$, its energy should be measured with the best accuracy. This point of view suggests further considerations that will be developed in section 5.

3. Huge and cold mechanical oscillators: the bar detectors of gravitational waves

Experimental systems particularly suitable for exploiting the relations (10) and (11) are the cryogenic Weber bars, originally conceived and still working as detectors for gravitational waves [32, 33]. They consist of large metallic bars weighing several tons and having a main longitudinal mechanical mode oscillating around $\omega_0/2\pi \simeq 1$ kHz. For our purpose, their favorable characteristics are their very large mass (around $\sim 10^{13}$ times the Planck mass), the very small zero-point fluctuations associated to their main modes (where, e.g., with respect to micro-oscillators, the relatively low frequency is compensated by the large effective mass M_{eff}), and the low level of thermal energy that is reached experimentally (also due to their high mechanical quality factor).

Some Weber bars have been operated at an ultra-cryogenic temperature [34], and we will show in section 5 that this regime is particularly interesting. For the moment, as in [30], we focus on the AURIGA detector that, in its present configuration, is working at the background temperature of 4.2 K. The peculiarity of AURIGA is that its main longitudinal mode has been further cooled down to the mK [44], exploiting a *cold damping* technique.

Modal cooling techniques have been recently exploited to bring mechanical modes of micro-oscillators in their quantum ground state, and these experiments have proven that quantum behavior of macroscopic coordinates can be obtained in this way [35–38]. Therefore cold damping is a valid technique for our purpose of reaching and measuring the lowest oscillator modal energy, even if such modal cooling cannot be exploited to increase the sensitivity of the oscillating system as a detector of external excitation [39].

3.1. Signal extraction and calibration

The problem of the readout of the bar motion, including its calibration, is well studied in the literature. However, the aim is typically to extract the signal of an impulsive force acting on the bar (i.e., a possible effect of a gravitational wave). In our case, we instead have to deduce the bar modal energy from the measured spectra at the output of the readout chain. Here we briefly discuss this problem, which had not been detailed in our previous publication [30].

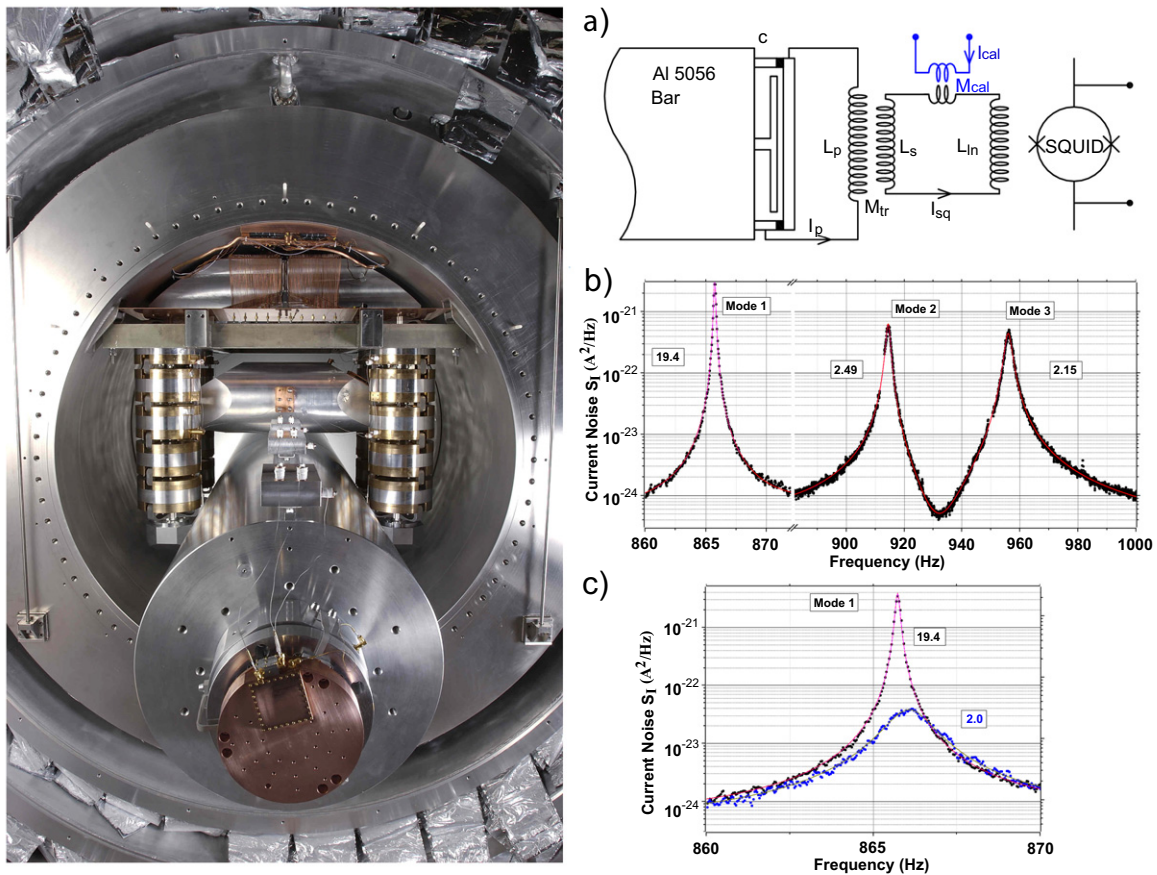


Figure 1. Left: photo of the AURIGA bar detector in its vacuum tank. The suspension system, composed of a central copper wire hanging from a horizontal beam and suspending with four multi-stage mechanical isolators, is clearly visible. On the bar face: the resonant mechanical amplifier and (in the copper box) the readout electronics. (a) Block diagram of the electro-mechanical system. The main line is drawn in black, while the auxiliary line used for the detector calibration is drawn in blue. A more realistic and complete scheme can be found in [55]. (b) Power spectrum of the output current in the regime of moderate damping. The resonances corresponding to the three modes are visible, with labels showing the respective effective temperatures. (c) Enlarged view of the first mode, with moderate (black dots with red fitting curve) and strong (blue dots with green fitting curve) damping.

The AURIGA readout is composed of a resonant mechanical amplifier. Its motion is read by a capacitive transducer [40] coupled through a superconducting matching transformer to a double stage SQUID amplifier [41, 42]. The overall system can be seen as a set of three coupled oscillators (the bar first longitudinal mode, the mechanical amplifier, and the electrical resonant circuit formed by the transducer capacitance and by the inductance of the primary coil of the matching transformer) forming therefore three normal modes [43]. The first problem is determining the effective temperature (i.e., the energy) associated to each mode. To this purpose, we will now describe a model-independent procedure. Figure 1 shows a block diagram of the whole system. Few auxiliary lines are integrated in the readout, and one of them is used

for the detector calibration. Through this line, the thermal energy contents of each mode are estimated in two steps.

First, a known current $I_{cal}(\omega)$ is injected into the calibration port. The induced electromotive force $V_{cal}(\omega) = i\omega M_{cal} I_{cal}(\omega)$ produces a current $I_{sq}(\omega)$ in the readout circuit, which is detected by the SQUID amplifier. The overall readout impedance at the SQUID input $Z_{read}(\omega)$ is thus estimated as:

$$Z_{read}(\omega) = \frac{V_{cal}(\omega)}{I_{sq}(\omega)}. \quad (12)$$

$Z_{read}(\omega)$ contains all the information about the detector electromechanical parameters and topology, and it is experimentally estimated without any *a priori* assumption on them. Around each resonance, $Z_{read}^{-1}(\omega)$ may be described by an equivalent *RLC* resonator, according to:

$$\frac{1}{Z_{read}(\omega \approx \omega_j)} \approx \frac{i\omega}{L_j} \frac{1}{\omega_j^2 - \omega^2 + i\omega_j\omega/Q_j}, \quad (13)$$

where ω_j , Q_j , and L_j are, respectively, the resonant pulsation, the quality factor, and the effective inductance of the mode j . They are estimated by fitting the experimental data with the function (13). Note that Q_j includes all the regenerative effects due to the dynamic SQUID input impedance [45] and the effects of the cold damping loop.

The second calibration step consists in measuring the SQUID input current power spectral density $S_{I_{sq}I_{sq}}(\omega)$. According to the fluctuation dissipation theorem, and using equation (13) around each mode, it should be equal to:

$$S_{I_{sq}I_{sq}}(\omega \approx \omega_k) = \frac{4k_B T_j}{L_j} \Re \left\{ \frac{i\omega}{\omega_j^2 - \omega^2 + i\omega_j\omega/Q_j} \right\}, \quad (14)$$

where k_B is the Boltzmann constant and T_j the ' j '-mode equivalent temperature, which in the absence of cold damping and neglecting the back action contribution should be equal to the thermodynamic temperature T_{Th} . In the presence of cold damping, $T_j = T_{Th} \cdot Q_j/Q_j^{int}$, where Q^{int} is the intrinsic mode quality factor. The relation (14) allows us to estimate the mode equivalent temperature and then ultimately the thermal energy E_j stored on each mode as:

$$E_j = \frac{1}{2} L_j I_{sq-j}^2 = \frac{1}{2} k_B T_j, \quad (15)$$

where the last equality follows from:

$$I_{sq-j}^2 = \frac{1}{2\pi} \int_{-\infty}^{+\infty} S_{I_{sq}I_{sq}}(\omega \approx \omega_j) d\omega. \quad (16)$$

As a check of the calibration procedure, we also calculated the bar expected temperature ($T_j \cdot Q_j^{int}/Q_j$) that within the experimental errors (5 ÷ 10%) turns out to be equal to the temperature measured using thermometers. All of the three modes have been cooled down by cold damping to roughly the same temperature [44], with an overall thermal energy of about $3.3 \cdot 10^{-26}$ J.

For the purpose of this work, we have now to establish the fraction of this total modal energy that is actually stored in the bar resonator. This can be performed using a lumped model for the detector. With reference to figure (1), the equations describing the motion of the system are:

Table 1. The readout parameters of the AURIGA detector.

Symbol	Meaning	Value
m_a	Bar equivalent mass	1150 Kg
m_t	Transducer equivalent mass	5.92 Kg
$\omega_a/2\pi$	Bar resonant frequency	898.85 Hz
$\omega_t/2\pi$	Transducer resonant frequency	900.40 Hz
L_p	Transformer primary inductance	3.78 H
L_s	Transformer secondary inductance	3.48 μ H
M_{Tr}	Transformer mutual inductance	4.51 mH
L_{in}	SQUID input inductance	1.46 μ H
C	Transducer capacitance	7.57 nF
E_{bias}	Transducer bias field	7.5×10^6 V/m

$$\begin{cases} m_a \ddot{x}_a + m_a \omega_a^2 x_a + m_t \omega_t^2 (x_a - x_t) = F_{Th-1} + E_{bias} Q_p \\ m_t \ddot{x}_t - m_t \omega_t^2 (x_a - x_t) = -E_{bias} Q_p + F_{Th-2} \\ L_{eff} \ddot{Q}_p + Q_p/C + M_{tr} \ddot{Q}_{sq} = E_{bias} (x_a - x_t) + F_{Th-3} \end{cases}, \quad (17)$$

where we have for the moment neglected the dissipative terms. F_{Th-i} is the Langevin thermal noise generator associated to the i th oscillator $L_{eff} = L_p (1 - M_{Tr}^2 / (L_p (L_s + L_{in})))$. The different system parameters are described and quantified in table 1. Hereafter, the bar end face displacement x_a , the mechanical transducer displacement x_t , and the primary coil charge Q_p will be defined with the symbol x_i , with i ranging from 1 to 3 respectively. The SQUID input current I_{sq} is related to the primary coil current I_p as:

$$I_{sq} = - \frac{M_{Tr}}{(L_{in} + L_s)} I_p. \quad (18)$$

The Hamiltonian associated to the system (17) can be diagonalized, giving the set of coordinates of the normal modes X_j , linked to the original x_i by the linear relation:

$$x_i = \sum_{j=1}^3 a_{ij} X_j, \quad (19)$$

where the coefficients a_{ij} are completely determined by the known system parameters. In the new coordinate system the equations of the motion read:

$$\ddot{X}_j + \omega_j^2 X_j = \mathcal{F}_j, \quad (20)$$

where \mathcal{F}_j (with spectral density $S_{\mathcal{F}_j \mathcal{F}_j}$) is a linear combination of the stochastic forces F_{Th-i} , and $\omega_j/2\pi$ is the mode frequency. After reintroducing phenomenologically the modal quality factor Q_j , the power spectral density of the original oscillator coordinates is written as:

$$S_{x_i x_i} = \sum_{j=1}^3 a_{ij}^2 \frac{S_{\mathcal{F}_j \mathcal{F}_j}}{(\omega_j^2 - \omega^2)^2 + \omega^2 \omega_j^2 / Q_j^2}. \quad (21)$$

In particular, the x_3 coordinate can be related through equation (18) to the measured SQUID input current noise and therefore, as we have previously discussed, the three modal

contributions $A_j = \int_{S_{x_3x_3}} (\omega \approx \omega_j) d\omega / 2\pi$ to the variance are directly measured. Finally, the variance of the bar coordinate x_1 is calculated using the known matrix a_{ij} according to:

$$\langle x_1^2 \rangle = \sum_{j=1}^3 \frac{a_{1j}^2}{a_{3j}^2} A_j. \quad (22)$$

The lowest achieved AURIGA mean bar displacement noise, in the strong cold damping regime, corresponds to an energy of about $(1.0 \pm 0.3) \times 10^{-26}$ J. Here the error comes from the detector parameter uncertainty, and a deeper analysis brought us to slightly correct the value of $(1.3 \pm 0.1) \times 10^{-26}$ J previously reported in [30].

3.2. The AURIGA data and the GUP

A crucial issue for comparing the experimental data of AURIGA with the theory described in the previous section is the adaptation of the discussion, which deals with a point-like oscillating mass, to a normal mode of an elastic body. A first problem is that the discussion concerns the oscillations of the center-of-mass of the particle, but the center-of-mass of the bar is fixed (or, put in better form, we are in the Lorentz frame of the center-of-mass of the bar). A second crucial issue is the determination of the appropriate value of the mass to be used in relations (10) and (11). The observed modal motion can be described by the equation of a damped harmonic oscillator of effective mass M_{eff} . The effective mass contains the overlap integral between the modal profile and the readout system and, therefore, it can be quite different from the total oscillator mass. The lowest value of M_{eff} is obtained when the readout is concentrated in a region of maximum oscillation amplitude (anti-node), while it is infinite for a readout on a node. In our case the readout measures the axial displacement of a bar face, and thus we calculate $M_{\text{eff}} = M/2$, where M is the bar physical mass (see the supplementary information in [30]). Such a discretionary definition of the effective mass does not yield any problem, even in a quantum mechanical treatment. To each mode one can associate a quantum harmonic oscillator with frequency ω_n and mass M_{eff} , and the validity of this description has been implicitly confirmed by the previously mentioned recent experiments on macroscopic quantum-mechanical oscillators. On the other hand, in the framework of GUPs, the Planck-scale deviations from standard quantum mechanics depend on the probe mass. Therefore, we need to identify the ‘real mass’ involved in the motion, getting rid of the dependence from the shape of the readout.

Both problems can be tackled starting from the consideration that the motion of the fundamental bar mode is symmetrical with respect to the plane (x,y) , (perpendicular to the bar axis) that bisects the bar. The bar can be considered as rigidly constrained to the symmetry plane, and the modal motion implies an oscillation of the center-of-mass positions— z_{cm} s— of each half-bar. The reduced mass of this couple of centers of mass is $M_{\text{red}} = M/2 = 1.1 \times 10^3$ kg. Incidentally, the values of the effective and reduced mass are coincident; however, generally speaking, the reduced mass is the correct parameter to be used in equations (10) and (11).

The energy associated to the oscillation of the centers-of-mass is about 80% of the total modal energy. Therefore, the measured E_{exp} remains an upper limit also for the minimal energy E_{min} associated to the oscillation of the z_{cm} s; i.e., considered for extracting an upper limit to quantum gravity effects according to the discussion of the previous section.

Replacing in equations (8–10) the AURIGA parameters, we finally find $\beta < 4.4 \times 10^4$ and $\beta_0 < 3 \times 10^{33}$. Our upper limit for β_0 is still far from forbidding new physics at the Planck scale. On the other hand, several models involving compactified large extra dimensions predict the emergence of intermediate fundamental scales between the electro-weak and the Planck scale [46]. This is a strong motivation for proposing experiments able to explore these new intermediate scales.

4. Deformed canonical commutators

As we have seen, the emergence of a nonzero minimum position uncertainty and of generalized uncertainty relations is a common feature in quantum gravity models. It is therefore natural to ask whether there is an algebraic structure from which the GUP follows, just like the Heisenberg uncertainty principle follows from the algebra $[x, p] = i\hbar$. Starting from equation (2), the most direct modification of canonical commuting relations that can be obtained is:

$$[x, p] = i\hbar \left(1 + \beta_0 (L_p^2 p^2) \right). \quad (23)$$

The limit to the β_0 parameter in the modified commutator of equation (23), extracted from AURIGA data, is obviously the same that we had previously derived while discussing the GUP. The experiments proposed in [21], based on the modified quantum dynamics that follows from the modified commutation relations, could strongly improve this upper limit. However, we remark that the meaning of the GUP that is directly tested by AURIGA is somehow more general than that of specific modified commutation relations.

A relevant question is whether this is the most general deformed algebra reproducing equation (2) or, from a wider point of view, implying the existence of a minimal observable length. Under a certain hypothesis, Maggiore has derived in [47, 48] modified commutation relations leading to a GUP, which reduces to (2) in a suitable limit. Under a different set of assumptions it is possible to obtain even more general commutation relations, e.g., as in [52], which also lead to equation (2). However, they are not unique, and experiments could help to distinguish between the various theoretical approaches.

We then consider a different kind of modified commutator [47]:

$$[x, p] = i\hbar \sqrt{1 + 2\mu_0 \frac{(p/c)^2 + m^2}{M_p^2}}, \quad (24)$$

where m is the test particle rest mass and μ_0 is a dimensionless parameter. If the modification to the commutator is supposed to be relevant at the Planck scale, μ_0 must be of the order of the unity.

If equation (24) is applied to a macroscopic coordinate x and its associated momentum p , characterizing a normal mode of a non-relativistic macroscopic object with mass $M \gg p/c$, it can be written as:

$$[x, p] = i\tilde{\hbar} \quad (25)$$

with $\tilde{\hbar} \simeq \hbar \sqrt{1 + 2\mu_0 M^2/M_p^2}$. We can therefore keep the standard quantum equations for the harmonic oscillator, with an effective Planck constant $\tilde{\hbar}$. The ground-state energy of the oscillator becomes:

$$E_{\min} = \frac{1}{2} \tilde{\hbar} \omega_0. \quad (26)$$

We now come back to the first longitudinal mode of AURIGA with cold damping and consider the oscillation of the center-of-mass positions of the two half-bars. Using the mode resonance frequency, the reduced mass M_{red} , and the measured modal energy $E_{\text{exp}} = 1.3 \times 10^{-26}$ J, we obtain from equations (7) and (26) $\tilde{\hbar} < 4.7 \times 10^{-30}$ J s, and from the definition of $\tilde{\hbar}$ we finally find $\mu_0 < 4 \times 10^{-13}$.

This constraint is very strong and apparently disproves the validity of the modified commutator described in equation (24) when applied to the coordinates of a normal mode, even at the Planck scale. The emphasis on the *normal mode* is here particularly important: our result can alternatively reject the model or just indicate that an experiment on a macroscopic object, such as those recently proposed in [21], cannot really test it. However, such an interpretation needs to be motivated by a self-consistent development of the theory. The generalization from the position/momentum of a single particle to that defining a collective motion is indeed well described in standard quantum mechanics. The canonical commutator $[x, p] = i\hbar$ remains valid for any Lagrangian coordinate x and its conjugate momentum p , including the coordinates describing the center of mass or a normal vibrational mode of a macroscopic object. We remark that recent experiments have cooled down mechanical modes of micro-oscillators to their quantum ground state [35–37], proving that a mechanical normal mode can be described by quantum-mechanical observables [35, 38]. On the other hand, if a modified commutator is considered, a self-consistent description in terms of macroscopic coordinates is not straightforward. The direct application of a modified commutator to a macroscopic body and the rough extrapolation to the classical world based on the correspondence principle would imply a set of paradoxes. Some recent works have tackled the problem, proposing solutions that imply a strong suppression of the expected effect of Planck-scale physics when probed by multi-particle objects [49, 50]. However, none of these approaches is fully satisfactory. Such hypotheses indeed entail the fact that macroscopic bodies have quantum-spacetime properties different from those of their fundamental constituents, whose nature (atoms, quarks, etc.) remains undetermined. A different solution is based on coherence properties of quantum systems and proposes that deformed commutators hold just for coherent collections of particles (e.g. Bose–Einstein condensates or a pure state of a macroscopic oscillator) and not for incoherent states, such as daily life macroscopic objects [51]. Given this situation, any kind of experimental analysis remains meaningful in particular if performed on macroscopic variables that display quantum properties.

The results of highly accurate low-energy experiments have been analyzed with the purpose of extracting upper limits to possible quantum gravity effects. From spectroscopic measurements in the hydrogen atom, whose energy levels are predicted very precisely, the deduced upper limits are $\beta_0 < 10^{36}$ (using the Lamb shift [17]) and $\beta_0 < 4 \times 10^{34}$ (from the 1S–2S level energy difference [50]). Our AURIGA results improve the latter limit by one order of magnitude. Curiously, we are presently at the level of the electro-weak scale (defined by $\sim 10^{17} L_p$, or $\beta_0 \sim 10^{34}$). The upper limit to β_0 from the hydrogen atom is calculated starting from the modified commutator of equation (23), using a perturbative approach. It is not clear if the same limits can be also deduced by starting directly from the GUP (maybe with variational methods), as performed in [30] for AURIGA, thus obtaining a result independent from a specific modified algebra.

5. Limits on spacetime discreteness

A satisfactory treatment of macroscopic bodies in the framework of generalized commutators is a challenging, not yet solved, theoretical problem. On the other hand, experiments on quantum macroscopic systems, such as those recently realized [35, 38], allow us to reach unprecedented sensitivity in metrology applications and enable fundamental tests of quantum physics. In this context, a relevant question is whether experiments on these systems could test at least very general, model-independent features of quantum gravity. One of these features is the Wheeler proposal of quantum foam; i.e., the idea that spacetime at the smallest scales should manifest quantum fluctuations of geometry.

In this framework, Bekenstein proposed in [22] an experiment for observing Planck-scale effects on the center-of-mass position of a macroscopic body, namely a glass block. The starting hypothesis is that the block cannot move by a distance lower or comparable to L_p : roughly speaking, either it moves by more than L_p , or its motion does not happen at all (and, of course, it cannot be measured). This lack of translation would not be directly measured but inferred, exploiting momentum conservation from the behavior of a single optical photon which crosses the block and has the role of inducing its motion.

Such a proposal requires Planck-length accuracy in the control of the center-of-mass position of the macroscopic probe; this can hardly be reached in an experiment that is not specifically conceived for this purpose. On the other hand, resonant gravitational waves detectors are already designed to detect extremely small displacements and therefore exhibit very low background length fluctuations. The AURIGA bar oscillator has not yet been cooled down to its quantum ground state, and experiments based on micro- and nanomechanical oscillators have reached much lower energy levels. However, due to its large mass and relatively low temperature, the cold longitudinal mode of the AURIGA bar is probably the most *well localized* oscillator ever observed. Indeed, for a given energy, the root mean square value of an oscillator velocity scales as $1/\sqrt{M}$ and that of its position as $1/(\omega_0 \sqrt{M})$. More specifically, the root mean square position is given by:

$$x_{rms} = \sqrt{(k_B T / m \omega_0^2)} = \sqrt{(E_{exp} / m \omega_0^2)}. \quad (27)$$

In the framework of the mentioned view of spacetime discreteness, the measured x_{rms} could be directly compared to the Planck length L_p . From the AURIGA parameters given before, we obtain $x_{rms} \sim 6 \times 10^{-19}$ m. The possibility to measure such a low level of vibration [22] (also implying that there are not extra displacement noise fluctuations originated by spacetime discreteness [53]) indicates that the threshold below which spacetime can no more be considered as smooth is lower than 6×10^{-19} m. Indeed, as previously discussed, a deformation parameter larger than unity defines a new length scale $\mathcal{L} = \sqrt{\beta_0} L_p$, where quantum gravitational effects should come into play. By comparing equation (27) with equations (8) and (10), we see that, in the limit of high β_{max} (i.e., when the oscillator is not very close to its ground state), the experimental upper limit on \mathcal{L} corresponds to $\sim x_{rms}$. (In general, as shown by equation (11), we have $\mathcal{L} < \sqrt{\beta_{max}/2} x_{zpf}$.) However, we are not simply repeating in a different form the previous discussion: the considerations contained in this section are related to a different and maybe more general point of view. In particular, they are not restricted to a specific form of GUP and do not rely on its validity when applied to macroscopic objects.

Such a derivation of the limit to spacetime discreteness, starting from the observed modal motion of AURIGA, is probably oversimplified. Indeed, in the models discussed in the previous sections the addressed physical system is the bar main mode, view as a (quantum) oscillator, and in particular its position is considered the meaningful observable. On the other hand, we are now dealing with the complete displacement of the half-bar center of mass. Such displacement contains contributions from all the bar modes that (except the first longitudinal mode) are still at the equilibrium thermodynamic temperature of 4.2 K. In this framework it is useful to consider the ultracryogenic Weber bars that, with their lower temperature, provide a meaningful length scale even without the strong cooling of the main mode. For instance, AURIGA was cooled down to ~ 0.1 K [54], as well as the similar bar detector NAUTILUS [34]. In this configuration, the rms displacement of the center of mass—i.e., the region of space explored during its motion—is $x_{rms} = 6 \times 10^{-18}$ m, corresponding to $4 \times 10^{17} L_p$. Also in this case, the huge mass and low temperature of the gravitational wave bars both contribute to their extremely low residual vibration and make them particularly suitable for testing fine space structures.

6. Conclusions

In this paper, we have analyzed the residual motion of the first longitudinal mode of the bar detector AURIGA, and in particular of its first longitudinal mode cooled down to the mK, in the search of low-energy signatures of quantum-gravitational effects. More specifically, we place an upper limit for possible Planck-scale modifications on the ground-state energy of a harmonic oscillator, and we set bounds to the scale at which quantum fluctuations of the spacetime geometry might come into play. The experiment is motivated by the fact that quantum-gravitational effects could manifest also at low energies in the form of deviation from the predictions of a standard theory, manifested as modifications of quantum mechanics and/or the appearance of a discrete spacetime structure. However, the possibility to unveil Planck-scale effects through low-energy measurements is the subject of an intense debate. In the absence of a self-consistent theory, there are different assumptions concerning the mechanisms at the basis of spacetime quantization which have deep implications on our experiment or similar proposals [21, 22, 29]. A heuristic physical interpretation of GUPs is that any attempt to determine the position of a particle with Planckian accuracy requires the use of a photon with Planckian energy, which would significantly deform the spacetime geometry, thus introducing an additional (gravitationally induced) uncertainty on the particle position. In this picture, the spacetime fuzziness appears as a direct consequence of the measurement process; i.e., it is dynamically related to the photon used to probe it. Therefore, only high-energy experiments, concentrating an energy of the order of E_p within a Planck-length-sized region, would be able to observe quantum gravitational effects. On the other hand, if discreteness emerges as an intrinsic property of the spacetime geometry when quantum mechanics is taken into account, Planck-scale effects would also affect the low-energy motion of a macroscopic body and could be tested by any dedicated experiment with the required sensitivity. Future experiments on macroscopic oscillators operating in their fundamental quantum state, whose center-of-mass motion can be controlled with Planckian accuracy, could provide even more significant limits and explore also a different hypothesis, such as the phenomenon of quantum-gravity induced decoherence.

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