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On the Wind Turbine Wake Mathematical Modelling

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Abstract

The present paper deals with a study on the wind turbine wake mathematical modelling as well as experimental validation by means of wind tunnel experiments. In particular, different wind turbine wake's equations were implemented and results compared with experimental data.
Therefore, an experimental setup was implemented in the wind tunnel test section with a small-scale wind turbine, while velocity deficit was measured. A design of experiment based on three parameters variation was defined: wind velocity, turbine rotational speed and distance from the wind turbine rotor. In the same experimental conditions simulations were carried out by means of three 1D equations. In particular, Jensen, Larsen and Frandsen equations were studied.
Comparing theoretical and experimental results, it is evident that Larsen mathematical model is in good agreement with experimental data, while Jensen and Frandsen mathematical models are able to identify only mean and peak velocity deficit, respectively.

Keywords: Wind Turbine; Wake; Mathematical Modelling; Wind Tunnel; Optimal WT placement.

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1. Introduction

In the last decades, wind energy was the main renewable source worldwide and in the next future it will play a dominant role in the world energy scenario [1]. Generally, wind turbines operate in wind farms. Therefore, turbines interaction occurs (wake effects). A wind turbine wake is characterized by both a mean air velocity deficit and a strong increase in turbulence level. This phenomenon modifies the inflow wind conditions for wind farm turbines if to the ambient wind field that apply for stand-alone wind turbines [2]. For these reasons the study of wind turbine wake and its effects on downstream is crucial to evaluate energy production.

Thus, it is of high importance the study of wind turbine wake and its effects on downstream wind turbine performance to evaluate energy production.

Mathematical modeling of the wake is an important tool in energy harvesting in wind farms to take into account wind turbine interaction. The present paper deals with a study on wind turbine wake aerodynamic characterization by means of theoretical and experimental analysis. Three different simple wake mathematical models were implemented and the results compared with experimental data.

2. Wake Mathematical Models

In the present paragraph a brief description of the three wake mathematical models implemented and compared with experimental data, is presented. In particular, Jensen/Katic, Larsen and Frandsen models are treated in the present paper.

2.1. Jensen/Katic Wake Mathematical Model

The wake mathematical model developed by Jensen [3] and modified by Katic [4] is based on the Momentum balance in the wake. In the present paper, the Katic version of the model was implemented and used.

The model treats the wake as a turbulent wake or a negative jet, neglecting the near field (the near wake) behind wind turbine rotors where periodic and deterministic swirling vortices strongly contribute to wake structure. Usually, it is based on the description of a single wake as a function of initial velocity deficit, wake decay coefficient and distance from the rotor. The velocity profile within the wake is ideal, maintaining constant the velocity inside the wake section [3]. On the contrary, the behavior of wind turbine (stall-turbine, pitch-regulated turbine, etc.) could be modelled using the Katic model.

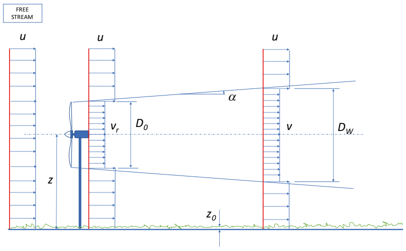


Fig. 1. Jensen Model Wake schema.

According to Fig. 1, the Momentum balance of the wake could be expressed as in Eq. (1). Once v is calculated taking into account Eq. (2) and Eq. (4), Eq. (6) is solved.

$$\pi\, D_0^2\, v_r + \pi\,(D_w^2 - D_0^2)\, u = \pi\, D_w^2\, v \tag{1}$$

where v_r is the velocity just behind the rotor, v is the velocity inside the wake at a distance x from the rotor, u is the velocity of the free stream, while D_0 and D_w are respectively the rotor diameter and the wake diameter at distance x.

$$v_r = (1 - 2\,a)\,u \tag{2}$$

where a is the axial induction factor of the turbine. This factor is also defined as the initial velocity deficit caused by the rotor (see Eq. (3)).

$$2\,a = 1 - \frac{v_r}{u} \tag{3}$$

$$D_w = D_0 + 2\,\alpha\,x \tag{4}$$

where α is the decay coefficient (see Eq. (5)).

$$\alpha = \frac{0.5}{ln\left(\frac{z}{z_0}\right)} \tag{5}$$

where z is the turbine hub height and z_0 is the terrain roughness.

$$v = u\left[1 - \frac{2\,a}{\left(1 + 2\,k\,\frac{x}{D_0}\right)^2}\right] \tag{6}$$

2.2. Larsen Wake Mathematical Model

The wake mathematical model developed by Larsen [5 – 8] is based on the consideration that a wind turbine wake is a perturbation on a mean flow. According to Larsen [5], the apparent mean flow, developing downstream a wind turbine, is due to both conventional shear and wake contributions, which expand in space and attenuate with downstream distance from the turbine.

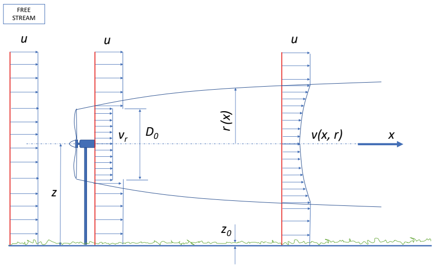


Fig. 2. Larsen Model Wake schema.

The wake extension and the wake deficit in the mean wind direction could be evaluated using Larsen simplified wake mathematical model [8]. According to this model, the wake radius is expressed as a function of the downstream distance, while the velocity deficit as a function of distance and radial coordinate (see Eq. (7) and Eq. (8) with reference to Fig. 2).

$$R_w(x) = \left(\frac{105\,C_1^2}{2\,\pi}\right)^{\frac{1}{5}} \left(C_T\,\Omega\,(x + x_0)\right)^{\frac{1}{3}} \tag{7}$$

$$\Delta U(x,r) = \Delta U_1(x,r) + \Delta U_2(x,r) \tag{8}$$

where x and r are respectively the axial and radial coordinates, ΔU_1 and ΔU_2 are the first and second order contributions to the velocity deficit ΔU, Ω is the turbine rotor area, C_T is the turbine trust coefficient. In Eq. (7), C_1 and x_0 come out form Eq. (9) and Eq. (10).

$$C_1 = \left(\frac{k\,D_0}{2}\right)^{\frac{5}{2}} \left(\frac{105}{2\,\pi}\right)^{-\frac{1}{2}} (C_T\,\Omega\,x_0)^{-\frac{5}{6}} \tag{9}$$

$$x_0 = \frac{9.6\,D_0}{\left(\frac{2\,R_{9.6}}{k\,D_0}\right)^3 - 1} \tag{10}$$

where k is determined by Eq. (11). The imposed boundary conditions refer to the magnitude of the turbulent mixing, expressed in terms of the mixing length, and the definition of the origin of the along wind coordinate, respectively [5].

$$k = \sqrt{(m+1)/2} \tag{11}$$

$$m = \frac{1}{\sqrt{1 - C_T}} \tag{12}$$

The wake radius at a downstream distance of 9.6 times rotor diameter ($9.6D_0$) could be determined using semi-empirical Eq. (13) [5].

$$R_{9.6} = a_1\, e^{(a_2\, C_T^2 + a_3\, C_T + a_4)}\, (b_1\, I_a + 1)\, D_0 \tag{13}$$

where I_a is the ambient turbulence intensity, a_1, a_2, a_3, a_4 and b_1 are coefficients that were empirically determined as reported in Table 1 [5].

Table 1. Larsen model coefficients

	a_1	a_2	a_3	a_4	b_1
Coefficient Values	0.43544986	0.797853685	-0.124807893	0.136821858	15.6298

The first order contribution to the wake deficit could be expressed using Eq. (14).

$$\Delta U_1(x,r) = -\frac{u}{9}\,(C_T\,\Omega\,(x + x_0)^{-2})^{\frac{1}{3}} \left(r^{\frac{3}{2}} \left(3\,C_1\,C_T\,\Omega\,(x + x_0)\right)^{-\frac{1}{2}} - \left(\frac{35}{2\,\pi}\right)^{\frac{3}{10}} (3\,C_1^2)^{-\frac{1}{5}} \right)^2 \tag{14}$$

where u is the undisturbed upstream wind velocity.

The second order to contribution to the wake deficit is generally neglected in the case of the stationary wake deficit prediction. Therefore, in the present study it was not considered.

2.3. Frandsen Wake Mathematical Model

The wake mathematical model developed by Frandsen [9] is based on the Momentum Conservation Law of the flow through and around the wind turbine rotor. Frandsen considered a cylindrical control volume with constant cross-sectional area equal to the wake area and horizontal axis parallel to the mean wind vector.

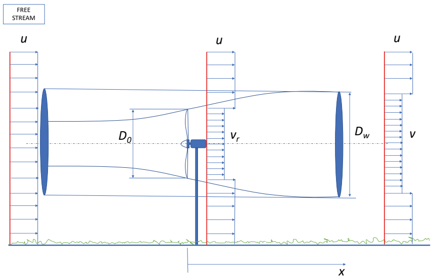


Fig. 3. Larsen Model Wake schema.

According to Fig. 3, the expansion of the wake cross-sectional area, in terms of wake diameter, at a distance x downwind the wind turbine rotor could be expressed using Eq. (15).

$$D_w(x) = D_0\left(\beta^{\frac{k}{2}} + \alpha\, s\right)^{\frac{1}{k}} \tag{15}$$

where D_0 is the wind turbine rotor diameter, α is the decay constant of the wake, s is the relative distance from the rotor (see Eq. (6)) and β is the wake expansion coefficient (see Eq. (17)). α and k are constants determined experimentally and generally they assume as values respectively 0.075 and 2 [9].

$$s = \frac{x}{D_0} \tag{16}$$

$$\beta = \frac{1 + \sqrt{1 - C_T}}{2\sqrt{1 - C_T}} \tag{17}$$

where C_T is the wind turbine thrust coefficient. According to Frandsen [9], the initial wake diameter should be evaluated using Eq. (18).

$$D_{initial} = \sqrt{\beta} D_0 \tag{18}$$

According to Larsen [9], and with reference to Fig. 3, the air velocity into the wake could be calculated by means of Eq. (19).

$$v = \frac{u}{2}\left(1 \pm \sqrt{1 - 2\,\frac{\Omega_0}{\Omega_w} C_T}\right) \tag{19}$$

where Ω_0 and Ω_w are the swept area by the rotor and the area of the wake at the distance x, respectively.

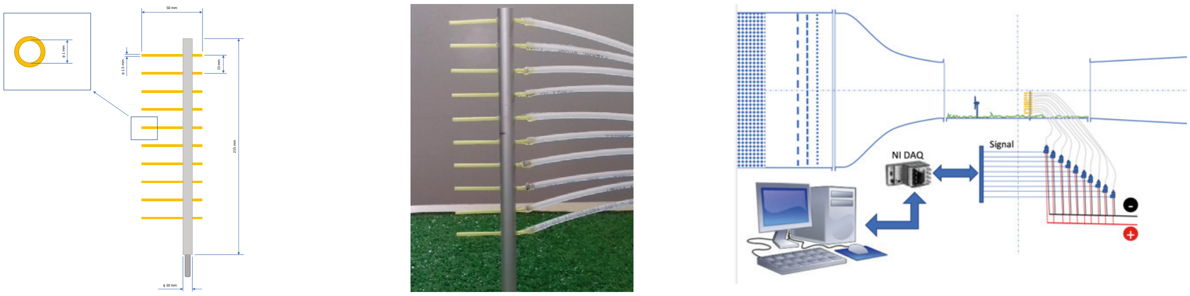
3. Experimental setup

In order to verify the accuracy of the wake mathematical models, a specific experimental setup was implemented using a closed-loop wind tunnel (see Table 2 for wind tunnel characteristics) [10, 11]. In order to measure velocity deficit in the turbine wake, a pressure rake was built and used. In Fig. 4 pressure rake schema (Fig. 4a) and photograph in the wind tunnel (Fig. 4b) are shown.

Ten dynamic pressure ports were used connected with ten correspondent pressure sensors (Trafag AG ECT8473). Pressure signals were acquired by means of National Instrument DAQ system as shown in Fig. 4c. In the same figure an experimental setup schema is also shown. As far as the studied turbine it is concerned, in Fig. 5 a wind turbine schema (Fig. 5a), a photograph of the studied turbine (Fig. 5b) and a wind turbine rotor drawing (Fig. 5c) are reported.

Table 2. Wind tunnel main characteristics

Characteristic	Value
Wind tunnel type	Closed-loop
Test section dimensions	500 x 500 x 1300 mm
Maximum wind velocity	30 m/s
Turbulence level	0.4 %



(a) (b) (c)

Fig. 4. Pressure rake: (a) schema; (b) photograph in wind tunnel; (c) experimental setup.

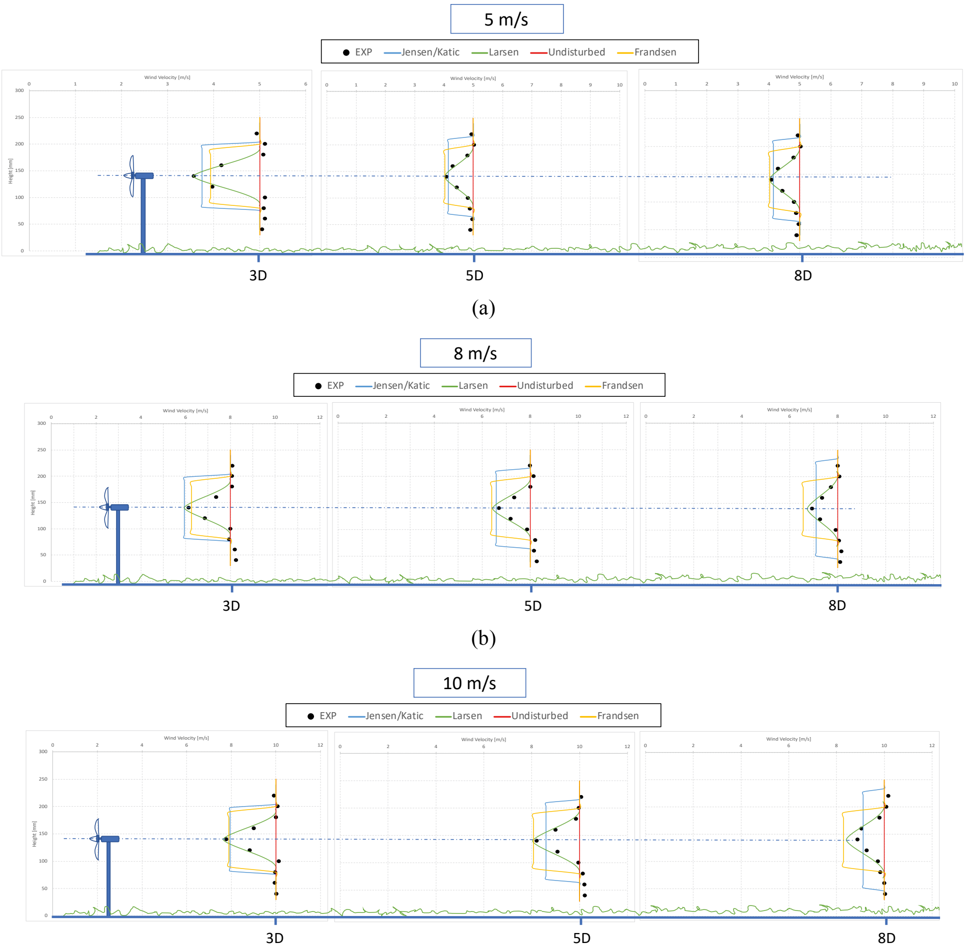
(a) (b) (c)

Fig. 5. Wind turbine: (a) schema and dimensions; (b) photograph; (c) rotor

4. Results and discussion

In order to test and verify the mathematical models, a specific experimental campaign was carried out. Wind velocity (5, 8 and 10 m/s), turbine rotational speed (1000, 2000 and 3000 r/min) as well as distance from turbine rotor (relative to the rotor diameter $3D_0$, $5D_0$ and $8D_0$) were considered as parameters in the present study. For each single combination of the parameters 10 repetitions were carried out.

Fig. 6 shows a velocity deficit as a function of wind velocity and relative distance from the turbine rotor at 1000 r/min at 5 m/s (Fig. 6a), 8 m/s (Fig. 6b) and 10 m/s (Fig. 6c). For the sake of simplicity only the results obtained at 1000 r/min are reported. On the basis of the presented results, it is possible to notice that increasing the relative distance, the velocity deficit decreases, while the wake radius increases according to Eq. (4) (Jensen/Katic model), Eq. (7) (Larsen model) and Eq. (15) (Frandsen model). This behavior is evident at all studied wind velocities.



(a)

(b)

(c)

Fig. 6. Velocity deficit as a function of downstream distance at 1000 r/min: (a) 5 m/s; (b) 8 m/s; (c) 10 m/s;

Varying undisturbed wind speed, the velocity deficit within the wake depends on the relative distance and very marginally on the undisturbed wind velocity. According to the obtained results, increasing the turbine rotational speed a higher velocity deficit was registered. This behavior was registered at all studied undisturbed wind velocities and at all relative distances.

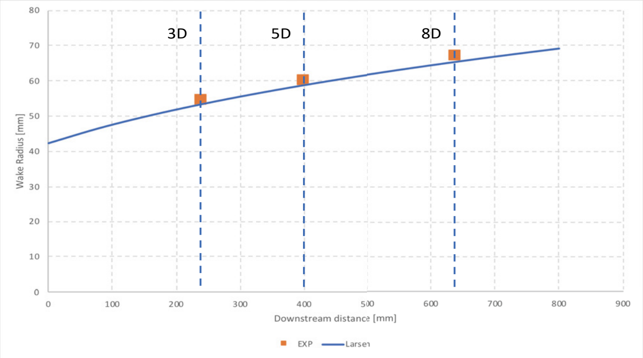


Fig. 7. Wake radius as a function of distance from the turbine rotor using Larsen Wake Model (theoretical and experimental reults).

5. Conclusions

The present work deals with a study on wind turbine wake aerodynamic characterization. This study was conducted by means of theoretical and experimental analysis. In particular, three different simple models were implemented and compared with experimental results. A specific experimental setup was implemented using a wind tunnel to measure velocity deficit within the wake by means of a pressure rake. Different undisturbed wind velocities (5, 8 and 10 m/s), turbine rotational speeds (1000, 2000 and 3000 r/min) and relative distances from turbine rotor ($3D_0$, $5D_0$ and $8D_0$) were studied. On the basis of the presented results, it is possible to state that: a) the higher the relative distance, the smaller the velocity deficit; b) the higher the turbine rotational speed, the higher the velocity deficit at all distances and undisturbed wind velocities; c) the wake radius increases with the relative distance and decreases inversely proportional with the wind velocity. Comparing the three wake mathematical models, it is possible to conclude that Larsen wake model describes better the wake profile at all wind velocity and distance from the rotor.

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