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A unilateral nonlocal tensile damage model for masonry structures

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Abstract

In the present paper, a constitutive nonlocal damage model is proposed for the non-linear incremental finite element analysis of masonry structures. The mechanical model is based on the assumptions of linear elasticity under compression and softening behaviour under tension, described by the adoption of a unique strain-driven nonlocal damage variable. Specifically, non-locality of the integral type is introduced in order to prevent spurious strain localization. It can be noted that the unilateral nature of the model is suitable to contemplate both diffused macro-cracks induced by the tensile damage process and the stiffness recovery in the transition from tension to compression, considering the anisotropy induced by the damage process as well. This is performed by realizing a decomposition of the strain tensor in its positive and negative components, and accounting for stiffness degradation only along tensile direction. The assumption of a linear elastic behaviour in compression is motivated by the fact that the main interest of the model is represented by investigating the response of masonry structures under service loads, condition in which very low compressive states are usually predominant. Consequently, the number of constitutive parameters is more limited with respect to other models that include a damage criterion also in compression. Finally, the validation of the proposed damage model is carried out with reference to a plane problem, in order to check the capability of the model to treat damage in an anisotropic way as well as the almost null dependence of the results on the discretization.

Keywords: masonry structures; anistropic damage; strain decomposition; crack-closure effect; non-locality

1. Introduction

Masonry is one of the most widespread structural material used in historical buildings of both Western and Eastern countries. The formulation of methods able to describe the mechanical behaviour of these constructions is and has been a stimulating issue in civil engineering and architecture. The complexities related to such a task derive from the

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fact that masonry presents several features that differentiate it from a homogeneous isotropic linear elastic material. In fact, it is characterized by marked heterogeneity and anisotropy, due to its particular arrangement of units and mortar joints; in addition, it has a pronounced non-symmetrical response in tension and in compression, with tensile strength that is very low when compared with the compressive one. Consequently, the scarce resistance in tension is responsible for a non-negligible non-linear behaviour even for low strain levels, with the formation of cracks.

In the present paper, the main interest is addressed to study the static stress regime of masonry structures in service load conditions, without the aim to describe properly the situation at compressive failure. Among the FEM based approaches, it is useful to recall a distinction between two main different strategies, i.e. micro- and macro-mechanical approaches: the former considers separately bricks and mortar joints while the latter regards masonry as a fictitious homogeneous continuum. For full structures, that represent the main object of this formulation, the macro-modelling appears the most suitable choice since it implies advantages related to computational effort and meshing procedure, together with an acceptable accuracy (Addessi et al. (2014)). In the field of macro-modelling approaches, several proposals exist, based on the adoption of non-linear constitutive laws, involving damage and/or plasticity. The majority of the damage models for quasi-brittle materials foresees an isotropic criterion since the damage variable introduced in the constitutive law affects in the same way all the components of the stiffness matrix. This can be found in the formulation provided by Addessi et al. (2002), where damage is described by a single scalar variable that is inclusive of both damage processes in tension and in compression. Other proposals deal with the adoption of two parameters, to take into account separately of tension and compression induced damage. This approach, followed by Toti et al. (2013) and by Contraffatto and Cuomo (2006), allows to take into account the crack-closure phenomenon typical of quasi-brittle materials, as experimentally shown by Reinhardt (1984). However, some models treat damage as a tensor. These models are classified as anisotropic and constitute a step forward since the degradation in stiffness becomes dependent on the spatial directions. Among these contributions, Berto et al. (2002)'s work can be cited, together with the model proposed by Faria et al. (1998) and successively extended by Pelà et al. (2011). In the latter case, anisotropy is induced in the damaged material thanks to a decomposition of the effective stress tensor into its positive and negative components.

The continuum mechanical model proposed in the present paper takes inspiration from this formulation but, differently from Faria et al. (1998) and Pelà et al. (2011), a split of the strain tensor is performed and only the tensile damage process related to growth of diffused cracks in mode I is taken into account. Consequently, only one damage variable is adopted while the material is considered infinitely linear-elastic in compression. Such a methodology allows catching the softening response of the material in tension, the unilateral effects in case of load reversal as well as the damage induced anisotropy, even if starting from the inaccurate hypothesis of isotropic undamaged material. The assumption of neglecting the compressive strength is in line with the goal to study masonry structures under service loads and allows decreasing the number of constitutive parameters with respect to other formulations. In addition, damage mechanics is coupled with a nonlocal integral approach, based on the model originally proposed by Pijaudier-Cabot and Bažant (1987). First, the reasons of non-locality are numerical, since it works as a regularization technique able to reduce the dependence of the results on the spatial discretization with respect to the local case. In addition, as recalled by Bažant and Jirásek (2002), non-locality has a physical relevance because crack growth strongly depends on the energy released in the surrounding and not only on the conditions at a point.

The paper is organized as follows: in section 2, the mechanical model is explained and its potentialities are investigated with reference to a 1D cyclic loading case. In section 3, after some details about the implementation of the model in a FORTRAN code, it is applied to the problem of a masonry panel subjected to pure shear.

2. Mechanical model

2.1. Constitutive relationships

In tensorial notation, the considered stress-strain law is:

$$\boldsymbol{\sigma} = (1-d)\boldsymbol{C} : \boldsymbol{\varepsilon}^{+} + \boldsymbol{C} : \boldsymbol{\varepsilon}^{-} \tag{1}$$

where $\boldsymbol{\sigma}$ is the stress tensor, d is the damage variable ranging from 0 (no damage) to 1 (full damage), $\boldsymbol{C}$ is the fourth-order elastic tensor and $\boldsymbol{\varepsilon}^+$ and $\boldsymbol{\varepsilon}^-$ are respectively the positive and negative parts of the strain tensor $\boldsymbol{\varepsilon}$, linked by the relation $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^+ + \boldsymbol{\varepsilon}^-$. The definition of $\boldsymbol{\varepsilon}^+$ and $\boldsymbol{\varepsilon}^-$ is based on the transformation that relates the components of a generic second order tensor in the principal reference system with the ones in a Cartesian reference system:

$$\boldsymbol{\varepsilon}^+ = \sum_{i=1}^{3} < \varepsilon_i > \boldsymbol{n}_i \otimes \boldsymbol{n}_i \qquad \boldsymbol{\varepsilon}^- = \sum_{i=1}^{3} - < -\varepsilon_i > \boldsymbol{n}_i \otimes \boldsymbol{n}_i \tag{2}$$

The positive-part operator, represented by the Macaulay brackets, is applied to the *i-th* principal strain value ε_i (for a scalar variable y: if $y \leq 0$, $<y> = 0$; otherwise $<y> = y$). In the expressions (2), $\boldsymbol{n}_i$ is the unit vector, in Cartesian components, that identifies the principal direction associated to the principal strain value ε_i. Recalling the fundamental property concerning eigenvalues and eigenvectors of the symmetric second order tensor $\boldsymbol{\varepsilon}$, $\boldsymbol{\varepsilon}\, \boldsymbol{n}_i = \varepsilon_i\, \boldsymbol{n}_i$, it is possible to find the relation between $\boldsymbol{\varepsilon}^+$ and $\boldsymbol{\varepsilon}$, as well as the relation between $\boldsymbol{\varepsilon}^-$ and $\boldsymbol{\varepsilon}$, ruled by the fourth-order tensor $\boldsymbol{R}$:

$$\boldsymbol{\varepsilon}^+ = \boldsymbol{R} : \boldsymbol{\varepsilon} \qquad \boldsymbol{\varepsilon}^- = (\boldsymbol{I} - \boldsymbol{R}) : \boldsymbol{\varepsilon} \qquad \boldsymbol{R} = \sum_{i=1}^{3} H(\varepsilon_i)\, \boldsymbol{n}_i \otimes \boldsymbol{n}_i \otimes \boldsymbol{n}_i \otimes \boldsymbol{n}_i \tag{3}$$

In the expression providing $\boldsymbol{R}$, the Heaviside function, dependent on the sign of the principal value ε_i, is present (for a scalar variable y: if $y \leq 0$, $H(y) = 0$; otherwise $H(y) = 1$). Let $\boldsymbol{D} = d\boldsymbol{R}$. The substitution of relations (3) in the constitutive law (1) leads to a more compact formulation of the stress-strain law:

$$\boldsymbol{\sigma} = \boldsymbol{C} : (\boldsymbol{I} - \boldsymbol{D}) : \boldsymbol{\varepsilon} \tag{4}$$

This is due to the fact that the decomposition of the strain tensor into its positive and negative counterparts is incorporated in the fourth-order damage tensor $\boldsymbol{D}$, that is related to the variable d and to the positive strain principal directions by the expression $\boldsymbol{D} = d\boldsymbol{R}$.

As it can be observed in (4), the present constitutive model is able to take into account the anisotropy induced by the damage process even though only one scalar parameter is adopted to simulate the progressive degradation of the material. Before the appearance of damage, the material is isotropic while when the damage variable grows, the reduction in stiffness is in general directional and ruled by tensor $\boldsymbol{R}$. The damage-induced anisotropy, described by the constitutive tensor $\boldsymbol{C}^* = \boldsymbol{C} : (\boldsymbol{I}-\boldsymbol{D})$, is specifically orthotropic and the principal directions of strains constitute the principal reference system of the material. The relation between the components C_{ijkl}^* in the reference system of the material and the ones C_{pqrs}^* in the Cartesian reference system is needed for the computation of the maximum and minimum rigidities of the damaged material and is made explicit in the following expression:

$$C_{ijkl}^* = C_{pqrs}^* \boldsymbol{n}_i(p)\, \boldsymbol{n}_j(q)\, \boldsymbol{n}_k(r)\, \boldsymbol{n}_l(s) \tag{5}$$

where the summations on indices p,q,r,s are tacit and $\boldsymbol{n}_i(p)$ represents the Cartesian component p of the unit vector $\boldsymbol{n}_i$. According to the present formulation, maximum axial rigidity is found along the direction of maximum shortening while minimum axial rigidity along the one of maximum elongation. This is true in all cases characterized by principal strains with discordant sign. Otherwise, the activation of damage preserves isotropy: in condition of full tensile regime, $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^+$ and an isotropic damage model is recovered; in complete compression, $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^-$ and a linear elastic isotropic constitutive law is maintained.

2.2. Strain-driven damage formulation coupled with non-locality

The damage variable introduced in the constitutive law (1) is responsible to describe the cracking phenomena occurring in brittle materials under tensile regime. In the present model, the quantity chosen to affect the damage evolution is the positive part of the strain tensor. The choice of a strain-driven damage formulation with respect to a stress-based one can be justified considering the specific situation of compressed elements with free lateral expansion (for example masonry columns). In this case, since transversal stresses are null, the longitudinal cracks produced by the different elastic properties of bricks and mortar joints can be represented through a macro-modelling approach only considering the damage induced by transversal strains.

The definition of the equivalent strain variable ε_{eq}, adopted as indicator of the strain state in a generic point, is derived from Lemaitre and Mazars (1982) and is a norm of the positive principal strains:

$$\varepsilon_{eq} = \sqrt{\sum_{i=1}^{3} < \varepsilon_i >^2} \tag{6}$$

This definition allows to take into account the absence of a symmetrical behaviour between tension and compression. Differently other formulations providing the equivalent variable as an energy norm of the strain tensor are not specifically capable to deal with it.

The non-locality is introduced in the model with reference to the equivalent strain quantity: the local variable $\varepsilon_{eq}(\boldsymbol{x})$ at a generic point $\boldsymbol{x}$ is replaced by its nonlocal counterpart $\varepsilon_{NL}(\boldsymbol{x})$, obtained by a weighted average over a representative volume of the material, whose size is identified by means of the internal length l_c.

$$\varepsilon_{NL}(\boldsymbol{x}) = \frac{\int_V \psi_0(\boldsymbol{x},\xi)\varepsilon_{eq}(\xi)d\xi}{\int_V \psi_0(\boldsymbol{x},\xi)d\xi} \qquad \psi_0(\boldsymbol{x},\xi) = \exp\left(-\left(\frac{2(\boldsymbol{x}-\xi)}{l_c}\right)^2\right) \tag{7}$$

$\psi_0(\boldsymbol{x}, \xi)$ is the nonlocal weight function, here chosen, in line with the majority of integral nonlocal models, as the Gauss distribution function. The internal length l_c has to be approximately chosen as the ratio between the fracture energy G [N/mm] and the specific dissipated energy g [N/mm^2], as suggested by Bažant and Pijaudier-Cabot (1989).

The damage d is a function of the internal state variable r that is initially equal to the damage threshold r_0 and during the damage process matches the maximum value of ε_{NL} reached in the loading history; in this way, the irreversibility of damage is taken into account. The damage threshold r_0 is assumed equal to the strain at the elastic limit in the uniaxial case; in fact, $r_0 = f_t / E$ where f_t is the tensile strength of the material and E is the Young's modulus. The well-known Kuhn-Tucker conditions govern the updating of the damage variable:

$$s = \varepsilon_{NL} - r \le 0 \qquad \dot{r} \ge 0 \qquad \dot{r}s = 0 \tag{8}$$

where $s = 0$ represents the damage limit surface (Fig. 1a), whose size increases in case of loading ($\dot{r} > 0$, $s = 0$) and remains unaltered in case of unloading or in the undamaged case ($s < 0$, $\dot{r} = 0$). The here proposed damage evolution law has an exponential trend, as shown in Fig. 1b together with the tensile softening 1D curve resulting from this assumption (Fig. 1c); its expression is:

$$d = 1 - \frac{\varepsilon_0}{r}\exp\left(-\left(\frac{r-\varepsilon_0}{\varepsilon_f - \varepsilon_0}\right)\right) \tag{9}$$

where ε_0 coincides with r_0 while ε_f is related to the energy g dissipated per unit volume by a completely damaged material in uniaxial tension, assuming a triangular trend for stress-strain curve $\varepsilon_f = (2g / f_t)$.

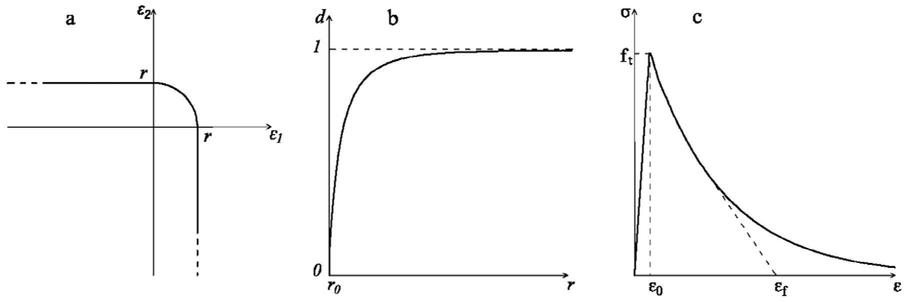

Fig. 1 Damage surface (a), damage trend with respect to variable *r* (b) and 1D stress-strain curve with softening (c).

2.3. Uniaxial stress-strain law

In order to highlight which are the features taken into account by the present model, the results of a uniaxial cyclic load history are considered. The material is cyclically subjected to loading in tension (cycle I), unloading in compression (cycle II) and reloading in tension (cycle III). The response of the material is shown in Fig. 2.

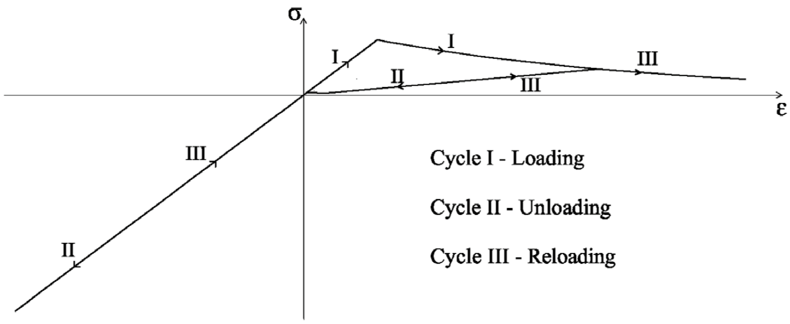

Fig. 2 Uniaxial cyclic response.

The following considerations can be done:

- the model is able to capture the non-linearity of the stress-strain curve under tension. The secant stiffness governing the first part of unloading cycle II, as well as the reloading in tension of cycle III, is lower than the initial one and reduces for increasing damage. In addition, the tensile peak stress is also affected by damage since during reloading in tension (cycles III) the maximum stress reached is lower than the initial peak one ;
- the behavior is infinitely linear elastic under compressive regime;
- a modification in stiffness is visible in the transition from tension to compression, or viceversa. Specifically, the transition from a damaged stiffness to the initial elastic one (visible in II) is representative of the crack-closure effect, typical of quasi-brittle materials in general.

These features result in accordance with experimental data collected by Lourenco (2004) for the characterization of the behavior of masonry joints in tension, compression and tension-compression.

Although the assumption of infinite linearity in compression does not coincide with reality, it is however reasonable since crushing behavior has in general a minor relevance in the response of masonry structures, where compressive stresses in the service phase are low with respect to the ultimate strength. Such hypotheses have been already adopted by Heyman (1966) for applying limit analysis to masonry structures and in some no tension material models, for instance by Cuomo and Ventura (2000). An advantage of neglecting softening in compression is that the nonlocal damage model is characterized by a minimum number of input variables, which are the elastic properties of masonry (E and ν), the tensile strength f_t and the inelastic parameters ε_f and l_c, both related to the specific dissipated energy g.

3. Example of application

The unilateral nonlocal damage model described in section 2 has been introduced in a finite element code written in FORTRAN for the general 3D case. The non-linear analysis is carried out according to a Newton-Raphson method, i.e. in an incremental-iterative way, adopting simultaneously convergence checks on both displacement increments and residual forces. For what concerns the implementation of damage, in order to foster the procedure, the code evaluates the quantities defined in (6)-(7)-(8)-(9) only in correspondence of the centroid of each finite element. Moreover, the non-locality is assigned defining for each element i the neighbouring set of interacting elements, on the basis of a purely geometric criterion: all elements whose centroid has a distance lower or equal to l_c from the centroid of element i belongs to this set.

3.1. Shear Panel

The problem of a masonry wall subjected to pure shear loading is solved with the proposed tensile damage model. The shear wall analysed has dimensions 1000 mm×1000 mm×1 mm and an horizontal displacement u equal to 1.2 mm is imposed at all points belonging to the top side (Fig. 3a). The constitutive parameters adopted, representive of the macroscopic behaviour of masonry, are the following: $E = 2000$ MPa, $\nu = 0.1$, $f_t = 0.1$ MPa and $g = 0.0003$ MPa. Different values of the internal length l_c are considered, including the local case, in order to evaluate how this parameter affects the solution. Three different meshes, all composed of 4-node elements, are taken into account: 15×15, 30×30 and 50×50 elements. The response of the structural member is characterized by a diagonal damage band, induced by tension, simulating the crack expected in pure shear condition. Once the tensile strength is achieved and damage grows, the resistant mechanism becomes coincident with the one of an inclined column. The map of the minimum principal stresses at the end of the loading history confirms this observation (Fig. 3b).

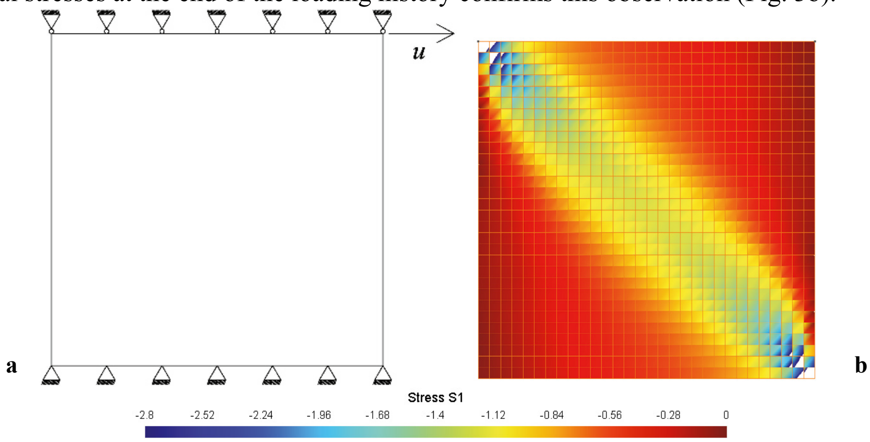

Fig. 3 Static scheme of the shear panel (a) and map of minimum principal stresses (b) in MPa, for mesh 30×30 and $l_c = 200$ mm.

The model is able to catch the directional degradation in stiffness, making almost null the stiffness along the tensile direction and maintaining unchanged with respect to the initial elastic one the stiffness along the compressive direction. This can be noted respectively in Fig. 4a and Fig. 4b when the damage map distribution is plotted together with the directions of minimum and maximum axial rigidity (lines whose length and colour depend on the stiffness value). The values of minimum and maximum axial rigidity have been computed according to expression (5), considering the constitutive components with the same indices. The higher value of damage, visible in Fig. 4a in correspondence of the two corners, is a spurious effect of non-locality: as observed by Krayani et al. (2009), damage is attracted by boundaries since here the interacting averaging domain is reduced. From Fig. 3 and Fig. 4, it is visible how the anisotropic treatment of damage allows to describe one fundamental aspect of masonry structures, stressed by Di Pasquale (1992), i.e. the fact that the resisting structure does not coincide with the construction and on the contrary depends on loads.

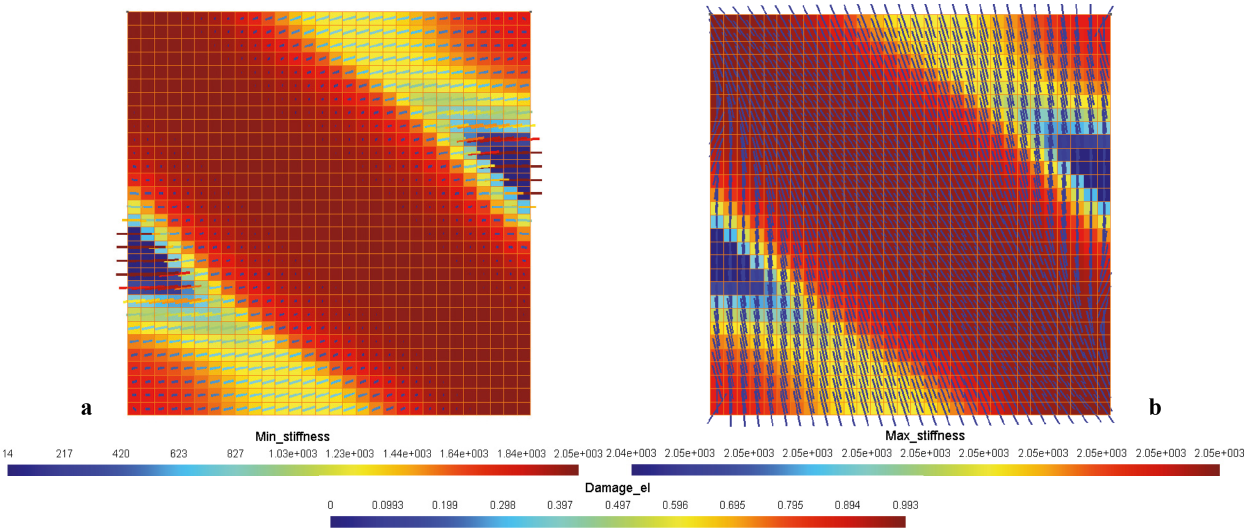

Fig. 4 Map of damage with directions of minimum (a) and maximum (b) axial rigidity in N/mm^2 for mesh 30×30 and l_c = 200 mm.

The overall structural responses related to different meshes and different values of the nonlocal parameter l_c are collected in Fig. 5, in terms of resultant horizontal force F as a function of the horizontal prescribed displacement u.

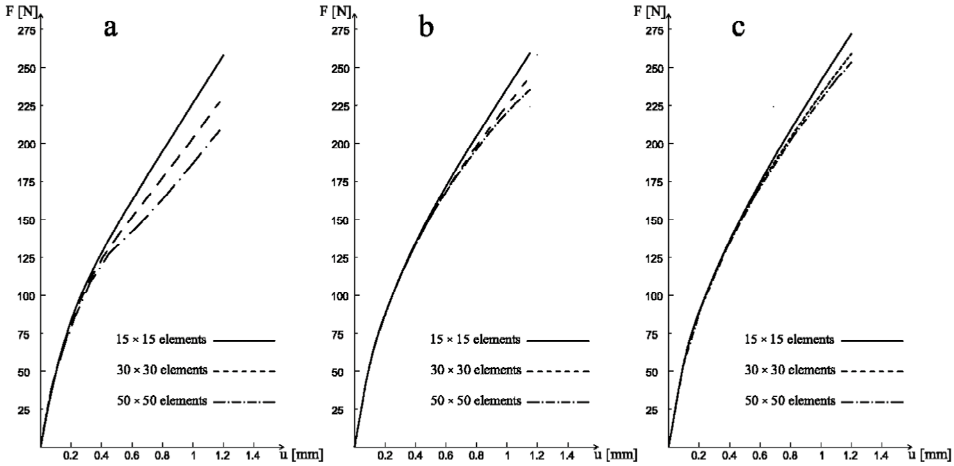

Fig. 5 Horizontal force versus horizontal displacement: different meshes compared in the local case (a), l_c = 300 mm (b) and l_c = 400 mm (c).

All the curves present an initial elastic behaviour, followed by a clear increase of deformability due to the appearance of damage. The softening in the global response is absent since the masonry crushing is not reproduced by means of a damage criterion in compression; therefore, the code finds equilibrated solutions for increasing loads simply carrying them by means of the compressed inclined column's mechanism. Fig. 5 shows how the non-locality (Fig. 5b, Fig. 5c) affects positively the solution, limiting the dependence of the results on the discretization with respect to the local case (Fig. 5a): specifically, the higher is the value of the internal length, the lower is the mesh dependence, as visible from the comparison between Fig. 5b and Fig. 5c and as shown in Table 1.

Table 1. Gaps (%) in term of peak loads between different meshes by varying of nonlocal parameter l_c

Compared meshes	*local*	l_c = *200 mm*	l_c = *300 mm*	l_c = *400 mm*
15×15 → 30×30 elements	11,3 %	8,7%	7,0%	5,0%
30×30 → 50×50 elements	8,4%	7.3%	3,1%	2,2%

In conclusion, l_c assumes the role of a constitutive parameter, being physically related to the fracture energy of the material. In accordance with Bažant and Pijaudier-Cabot (1989), $2l_c$ represents the width of the effective process zone and coincides with the damage band width, as visible in Fig. 4. For increasing l_c, this band assumes a larger size, damage is less localized and consequently the response is characterized by an increased stiffness (Fig. 5b and Fig. 5c).

4. Conclusions

In the present paper, a nonlocal damage model for the FE analysis of masonry structures has been proposed. The basic hypotheses of the formulation deal with a softening behaviour in tensile regime and linear elasticity in compression. The absence of a damage criterion in compression, assumption reliable when the goal is to investigate the static behaviour of masonry structures under service loads, allows reducing the number of input parameters with respect to other damage models. A decomposition of the strain tensor into its positive and negative components, as similarly performed by Faria (1998) and Pelà (2011), has been carried out in order to describe the non-symmetrical response of the material in tension and compression. The advantages of such a procedure have been highlighted in the paper: first, the recovery of stiffness moving from tension to compression, can be taken into account; in addition, in line with the actual behaviour of cracked structures, the decomposition makes anisotropic the damaged material.

The non-linear model, introduced in a FORTAN code, has been applied to the case of a shear panel; the main results obtained have been a realistic damage propagation and a low dependence of the solution on the mesh, thanks to the adoption of non-locality. Another positive aspect highlighted with this example is the possibility of identifying the directions of orthotropy; this appears an useful tool for the analysis of the results, since it allows understanding visually the resistant mechanism inside the structural member, once the cracking phenomenon has started.

Further research could be devoted to introduce anisotropic constitutive laws even in the undamaged elastic phase. In addition, the potentialities of the model could be validated with reference to experimental data, extrapolated from non-destructive tests, for both increasing and cyclic load histories.

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