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Uniform scatter bands to analyse the fatigue strength of welded joints

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Abstract

This paper analyses the possibilities of a rational and uniform interpretation of the fatigue strength of welded joints, as a necessary basis for the development of sound and easily applicable design procedures. The re-analyses of experimental data here considered refer to as-welded joints, in steel and aluminium alloys, fabricated by using arc-welding, under axial or bending loads, under different stress ratios and modelled with the assumption of a null notch tip radius at the crack initiation location. The first part of the paper gives a synthetic presentation of the principal unifying parameters proposed in the literature and of the fatigue scatter bands derived with their application. The second part applies to welded joints a new parameter, recently introduced as an extension of the averaged Strain Energy Density approach. This parameter, which has been called the Strain Energy Density Intensity Factor, does not depend on the radius of the chosen control area, since it is not related to the fatigue strength of a specimen without geometrical notches. For this reason, it should make the averaged strain energy density concept more easily applicable to welded structures and give more general validity to the scatter bands derived with its application.

Keywords: welded joints; fatigue; strain energy density; steel; aluminium

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1. Introduction

The fatigue initiation and short propagation life of welded structures is today ruled by several standards (in Europe Eurocodes EN3 for steel and EN9 for aluminium alloys (2005, 2011)) which give the fatigue SN curves of typical welded joints as a function of the nominal stress. Since quite often the application of the rules to real structures is not easy, several approaches have been developed to correlate the fatigue strength to the local conditions around the critical points, measured by strain gages or evaluated by FEM. The extension of the LEFM criteria to sharp open notches (often called Linear Elastic Notch Mechanics) seems to be the most appropriate approach for this problem since, for as welded joints obtained with the usual welding technologies, the radius at the toe of the weld is very small. The pioneering works of Haibach for steel (Haibach 1968) and Atzori for aluminium alloys (Atzori and Bufano; Haibach and Atzori 1974; Atzori and Indrio 1976) evidenced that, on relative scale, a uniform scatter band (constant inverse slope k and scatter index T) could be assumed to represent the fatigue behaviour of different geometries independently of the crack initiation location. The application of the concept of unified scatter bands by a Notch Stress Intensity Factor approach, developed by Atzori and his research group (Atzori and Haibach 1979; Atzori et al. 1987, 1990, 1992, 1999, 2008), evidenced the problem of comparing the fatigue strength for cracks starting from weld toes (2α = 135°) and roots (2α = 0°). Lazzarin overcame this problem in a brilliant way with the proposal of the averaged Strain Energy Density concept (Lazzarin and Zambardi 2001). The approach was then deeply developed by Lazzarin and his research group with the study of many related theoretical implications and the analysis of some typical applications (Berto and Lazzarin 2009; Radaj and Vormwald 2013).

Recently an extension of the SED approach has been proposed, that converts the averaged Strain Energy Density in an averaged Strain Energy Density Intensity Factor L (Atzori et al. 2019). For notch opening angle 2α = 0 this new parameter L does not depend on the radius R_0 chosen for the considered integration area. Although the L SEDIF seems to be a useful parameter to simplify and make more general the practical applications of the SED approach, the extension to the fatigue strength of welded joints faces some difficulties, since for open notches this parameter does depend on the chosen radius R_0.

Aim of the paper is to analyse the ways in which this problem could be overcome, to verify if the relevant results on fatigue strength of welded structures obtained by Lazzarin and his research group (with the definition of well-known unified scatter bands for welded joints subjected to tension and bending loads, for a wide range of thicknesses, both for steel and aluminium alloys) are suitable (and at which extent) to a Strain Energy Density Intensity Factor approach.

2. Fatigue strength of welded joints

After the second world war the diffusion of the welding technology evidenced the fatigue problems connected with welded structures, as a consequence both of the welding technology and of the notch effect due to the complex geometries of the welded joints. Although many experimental results on the subject were readily obtained and made available in the literature, it was only in 1968 that Haibach was able to develop a systematic approach to the interpretation of the fatigue behaviour of welded joints in steel (Haibach 1968). This was made possible as the result of a wide program of fatigue tests on welded joints characterized by different types of steel, joint geometries and stress ratios. The tests were performed monitoring the fatigue life as a function not only of the applied nominal stress but also of the local strain near the weld toe, usual starting point of the fatigue failures. On a phenomenological basis Haibach found that a unified scatter band was able to represent the fatigue lives of different series of axial tests on steel welded joints, whichever the type of steel, the welding parameters, the geometry of the joint and the absolute dimensions of the joint. When analysed in terms of nominal stress, the influence of those parameters causes a vertical translation of the scatter band. When analysed in terms of local strains, all the results merge in a unique scatter band. The unified scatter band for conventional arc-welded joints in steel was defined by Haibach as having an inverse slope k=3.75 up to $N_A=2\cdot10^6$ cycles, with a scatter of the results defined by the ratio between the stress amplitudes for a probability of survival (PS) equal to 10% and 90%, respectively, i.e. $T_\sigma = \sigma_{a,10\%}/\sigma_{a,90\%}$. At $N_A=2\cdot10^6$ cycles was evaluated as T_σ =1.5.

Several years later Atzori extended the unified scatter band approach to welded joints in aluminium alloys. On the basis of more than 6000 fatigue test results on welded joints of different types of aluminium alloys (Atzori and Bufano;

Haibach and Atzori 1974; Atzori and Indrio 1976), a unified scatter band for conventional arc-welded joints in aluminium alloys with thickness of the principal plate ranging from 4 to 12 mm was defined as having an inverse slope k=4.3 up to $N_A=2\cdot10^6$ cycles, and k=17 for higher number of cycles. The scatter of the results was evaluated at $N_A=2\cdot10^6$ cycles as $T_\sigma=1.55$. The different types of welded joints reanalysed were then classified in terms of nominal stresses, according to the existing standards on welded joints in steel, and the first code on aluminium alloys containing detailed fatigue design rules was developed on this basis (Atzori and Dattoma 1983a, b; 1985).

A theoretical basis to the phenomenological findings above described was proposed by Atzori (Atzori 1985), which suggested that, for sharp notches, the complete stress field around the tip of the notch should be taken into consideration (as in Linear Elastic Fracture Mechanics approach) and not the peak stress (as in Stress Concentration approach). After verifying that the radius at the toe of fusion welded joints was always very small (Atzori et al. 1985) it was proposed that the dimension of this radius in the crack initiation region could be assumed as equal to zero without influencing the sharp notch stress field. With this simplifying assumption the fatigue life of welded joints was evaluated as a function of the Notch Stress Intensity Factor K_1, already proposed in the literature to extend to open notches the SIF K_I used in LEFM (Atzori et al. 1989; Lazzarin and Tovo 1998). The ΔK-N scatter band was evaluated from fatigue test results of welded steel joints with various thicknesses of the principal plate, ranging from 3 to 100 mm (Lazzarin and Livieri 2001; Livieri and Lazzarin 2005).

For fatigue failures starting at the weld toe ($2\alpha=135°$) the evaluated scatter band was defined as having an inverse slope k=3.0 up to $N=5\cdot10^6$ cycles, with a scatter of the results (considering PS=2.3% and PS=97.7%) defined by $T_\sigma=1.85$. The value of the NSIF stress range ΔK at $N_D=5\cdot10^6$ cycles for PS=50% was evaluated as $\Delta K_D=$ 211 MPa $mm^{0.326}$.

For fatigue failures starting at the weld root ($2\alpha=0°$) the evaluated scatter band was defined as having an inverse slope k=3.2 up to $N_D=5\cdot10^6$ cycles, with a scatter of the results for PS=2.3% and 97.7% defined by $T_\sigma=2.1$. The value of the NSIF stress range ΔK at $N_D=5\cdot10^6$ cycles for PS= 50% was evaluated as $\Delta K_D=180$ MPa $mm^{0.5}$.

For welded joints made of aluminium alloys with fatigue failures starting from the weld toe ($2\alpha=135°$), the evaluated scatter band was defined as having an inverse slope k=4.0 up to $N_D=5\cdot10^6$ cycles, with a scatter of the results for PS=2.3% and 97.7% defined by $T_\sigma=1.78$. The value of the NSIF stress range ΔK at $N_D=5\cdot10^6$ cycles for PS=50% was evaluated as $\Delta K_D=99$ MPa $mm^{0.326}$.

To correlate the numerical values from FE analyses with the experimental strain gages results, the ΔK-N scatter band for steel was transformed into a $\Delta\varepsilon_{loc}$–N scatter band (Atzori and Meneghetti 2001). The unified scatter band for conventional arc-welded joints was defined as having an inverse slope k=3.0 up to $N_A=2\cdot10^6$ cycles, with a scatter of the results for PS=10% and PS=90% defined by $T_\sigma=1.4$. The value of the strain amplitude at $N_A=2\cdot10^6$ cycles for PS=50% was evaluated (for nominal load ratio equal to 0) as $\varepsilon_A=1416$ με at a distance x=0.01mm and $\varepsilon_A=220$-280 με at a distance x=2.5mm (depending mainly on the real extension of the singular stress field, that is on the absolute dimensions of the joint). The analysis evidenced the good agreement between numerical and experimental evaluations of the local stress values for the interpretation of the fatigue strength of welded joints and also the very strong dependence of the local strength values to the chosen distance of evaluation.

Several authors proposed similar techniques for sharp notches. In particular Tanaka (Tanaka 1983), Atzori and Tovo (Atzori and Tovo 1994) and Taylor (Taylor 1999), Radaj (Radaj 1990) and Meneghetti and Lazzarin (Meneghetti and Lazzarin 2007).

The Point Method, independently proposed by Tanaka, Tovo and Taylor, but developed mainly by Taylor (Taylor 2007), was applied by Susmel (Al Zamzami and Susmel 2017) to the re-analysis of a large number of fatigue test results on aluminium welded joints. The assumed critical distance, able to correlate the fatigue strength of the analysed geometries to that of a ground butt weld, was $x_{PM}=0.25$ mm. A scatter band of the re-analysed fatigue data is not given in the original paper (Al Zamzami and Susmel 2017), but the results evaluated in terms of the stress range $\Delta\sigma_{PM}$ for the analysed geometries are plotted against the assumed design curve (the Eurocode 9 design curve for aluminium ground butt welds). This curve has an inverse slope k=4.5 up to N= 5 10^6 cycles and 6.5 for higher number of cycles. Consequently the value of the Point Method stress range $\Delta\sigma_{PM}$ at $N_A=2\cdot10^6$ cycles for PS=97.7% assumes the value $\Delta\sigma_{PM,A}=55$ MPa.

The fictitious notch-rounding concept was developed mainly by Raday (Radaj and Sonsino 1998) and is proposed as a design approach by the IIW Recommendations (Hobbacher 2016) (FAT 225 for steel and FAT 71 for aluminium alloys, with k=3 for both materials). Pedersen applied the criterion to the re-analysis of 767 fatigue test results on steel-welded joints (Pedersen et al. 2010). The assumed reference radius was ρ_r=1mm, which gives a peak stress at the tip of the radiused notch corresponding to the stress evaluated at a distance of about 0.125 mm from the tip of the same notch but with a null radius (Atzori et al. 2003). The evaluated scatter band was defined as having an inverse slope k=3.0 up to N=10^7 cycles, with a scatter of the results for PS=2.3% and 97.7% defined by T_σ=2.3. The value of the maximum stress range $\Delta\sigma$ at $N_A=2\cdot10^6$ cycles for PS=50% was evaluated as $\Delta\sigma_A$=305 MPa.

The Peak Stress Method, developed mainly by Meneghetti, was applied to welded joints in steel and in aluminium alloys (Meneghetti 2008). The size of the 4-node quadrilateral plane elements (PLANE 182 of Ansys element library) in the critical zone was d=1mm (that, for the adopted mesh and element type, corresponds to a distance of about 0.16 mm (Atzori et al. 2018)). For steel welded joints, the evaluated scatter band was defined as having an inverse slope k=3.0 up to N=$5\cdot10^6$ cycles, with a scatter of the results for PS=2.3% and 97.7% defined by T_σ=1.90. The value of the peak stress range at $N_D=5\cdot10^6$ cycles for PS=50% was evaluated as $\Delta\sigma_{peak,D}$=149 MPa. For aluminium alloys the evaluated scatter band was defined as having an inverse slope k=3.80 up to $N_D=5\cdot10^6$ cycles, with a scatter of the results for PS=2.3% and 97.7% defined by T_σ=1.81. The value of the peak stress range at $N_D=5\cdot10^6$ cycles for PS=50% was evaluated as $\Delta\sigma_{peak,D}$=70 MPa.

The fatigue strength of welded structures was analysed by several authors also on a strain energy basis. We refer here to the works of Lazzarin, Livieri and Tovo. An extension to sharp open notches of the path independent line integral J was applied in (Lazzarin et al. 2002) to the analysis of welded joints in steel and in aluminium alloys. When applied to open notches, this integral, which was called J_V, is dependent on the location of the two extremity points of the chosen path. For this reason, the ΔJ_V-N scatter bands were evaluated assuming a distance r=1 mm of these points from the notch tip. The scatter band for steel joints was defined as having an inverse slope k=1.47 up to $N_D=5\cdot10^6$ cycles, with a scatter of the results for PS=2.3% and 97.7% defined by $T_J=(T_\sigma)^2$ =3.75. The value of the line integral range ΔJ_V at $N_D=5\cdot10^6$ cycles for PS=50% was evaluated as $\Delta J_{V,D}$ =0.0883 MPa mm. The scatter band for aluminium alloys joints was defined as having an inverse slope k=2.02 up to $N_D=5\cdot10^6$ cycles, with a scatter of the results for PS=2.3% and 97.7% defined by $T_J=(T_\sigma)^2$ =3.19. The value of the line integral range ΔJ_V at $N=5\cdot10^6$ cycles for PS=50% was evaluated as $\Delta J_{V,D}$ =0.0598 MPa mm.

A few years later several new fatigue test results were added and the scatter band was improved and presented with a new parameter, with physical dimensions that are independent on the notch opening angle, the "equivalent SIF" defined as $K_V=(J_V\,E')^{0.5}$ (Livieri and Tovo 2009). Since the fatigue strength at $N_D=5\cdot10^6$ cycles were related to the fatigue strength of butt ground welds, the distance r of the two extremity points of the chosen integration path from the tip of the notch resulted different for the two materials: r=1 mm for steel and 0.4 for aluminium alloys. The scatter band for steel joints was defined as having an inverse slope k=2.97 up to $N_D=5\cdot10^6$ cycles, with a scatter of the results for PS=2.3% and 97.7% defined by T_σ=1.97. The value of the equivalent SIF range ΔK_V at $N_D=5\cdot10^6$ cycles for PS=50% was evaluated as $\Delta K_{V,D}$ =145 MPa $mm^{0.5}$. The scatter band for aluminium alloys joints was defined as having an inverse slope k=4.04 up to $N_D=5\cdot10^6$ cycles, with a scatter of the results for PS=2.3% and 97.7% defined by T_σ=1.78. The value of the equivalent SIF range ΔK_V at $N=5\cdot10^6$ cycles for PS=50% was evaluated as $\Delta K_{V,D}$ =53 MPa $mm^{0.5}$.

Lazzarin realized a substantial improvement in the generality of possible applications with the proposal of a strain energy density approach (Lazzarin and Zambardi 2001; Berto and Lazzarin 2009; Radaj and Vormwald 2013). On the basis of the Neuber's structural volume needed to cause a fatigue failure, the parameter to be considered according to this approach is the averaged strain energy density W in a circular control area of radius R_0. This parameter has the same dimensions for closed and open notches and is applicable also to multi-axial loadings (Lazzarin et al. 2008b). Since it was applied with reference to the fatigue strength of butt ground welds (Livieri and Lazzarin 2005), the radius of the control area is different for steel (R_0=0.28 mm) and for aluminium alloys (R_0=0.12 mm). The scatter band for steel joints was defined as having an inverse slope k=1.5 up to $N_D=5\cdot10^6$ cycles, with a scatter of the results for PS=2.3% and 97.7% defined by $T_W=(T_\sigma)^2$=3.3. The value of the SED range ΔW at $N_D=5\cdot10^6$ cycles for PS=50% was evaluated as ΔW_D =0.105 MJ/m^3. The scatter band for aluminium alloys joints was defined as having an inverse slope k=2.0 up to $N_D=5\cdot10^6$ cycles, with a scatter of the results for PS=2.3% and 97.7% defined by $T_W= (T_\sigma)^2$ =3.2. The value of the SED range ΔW at $N_D=5\cdot10^6$ cycles for PS=50% was evaluated as ΔW_D =0.103 MJ/m^3.

From the above synthesized results, it appears that the parameters k and T_σ, although apparently different between stress and energy-based approaches, are quite independent on the chosen approach, if the value of energy is reduced to stress. As far as the inverse slope is concerned, the variation of k for steel is between 2.92 and 3.75, with a most frequent value of k=3.0 (as the one assumed in IIW Recommendations and in Eurocode 3 (2005; Hobbacher 2016));

for aluminium alloys the variation is between 3.80 and 4.3, with a most frequent value of k=4.0 (while IIW (Hobbacher 2016) assumes k=3 and Eurocode 9 (2011) k=3.4-4.3-7.0 depending on the joint geometry). As far as the T_σ ratio is concerned, when reduced to PS=10% and 90%, the variation for steel is between 1.40 and 1.9, with a most frequent value of 1.5, while for aluminium alloys the variation is between 1.47 and 1.55, with a most frequent value of 1.5. The PS=50% reference values at $N_A=2\cdot10^6$ cycles or at $N_D=5\cdot10^6$ cycles show a very large variation and are not easy to be synthesized, since the majority of them depend on a chosen characteristic length, function of the applied criteria but also of the absolute dimensions of the joint. In any case, due to the existing correlation between the different parameters, the fatigue scatter bands corresponding to each of them are very similar and the PS=50% reference values are correlated. In principle, as far as it is allowed to assume a null notch tip radius, each one of the discussed approaches could be used, since each of them has its peculiar advantages and disadvantages, but the discussion of this subject is beyond the aims of this paper. Aim of this paper is instead to verify the applicability to welded joints of a new parameter that links together the advantages of SIF and SED approaches.

3. The averaged Strain Energy Density Intensity Factor (SEDIF)

Lazzarin and co-workers (Lazzarin and Zambardi 2001; Livieri and Lazzarin 2005) assumed the strain energy density (SED) averaged over a structural volume surrounding the crack initiation location as a fatigue strength criterion. They assumed a structural volume having circular shape with radius R_0 and provided the closed-form expression of the averaged SED parameter as a function of the relevant NSIFs. Dealing with notched components under pure mode I loading, the averaged SED parameter can be written in closed-form expression as a function of the mode I NSIF K_1:

$$\Delta\bar{W} = \frac{e_1(2\alpha,\nu)}{E}\left(\frac{\Delta K_1}{R_0^{\,1-\lambda_1}}\right)^2 \tag{1}$$

In previous expression E is the Young's modulus of the material; λ_1 is the degree of singularity of the local stress field and is function of the notch opening angle 2α, e_1 is a parameter depending on the notch opening angle 2α and on the Poisson's ratio ν; finally ΔK_1 is the range (maximum value minus minimum value) of the NSIF-parameter. Previous expression (1) is valid when the stress fields within the control volume having size R_0 are governed solely by the NSIFs, i.e. the contribution of higher-order non-singular terms is negligible. The control radius R_0 can be calibrated by equaling the averaged SED in two experimental conditions, namely (i) the high-cycle fatigue strength, typically at N_D cycles, of un-notched specimens and (ii) the high-cycle fatigue strength, at the same number of cycles, of notched specimens characterized by an opening angle $2\alpha \geq 0°$, as reported in Eq. (2).

$$\Delta\bar{W}_{un-notched} = \Delta\bar{W}_{notched} \rightarrow \frac{\Delta\sigma_D^2}{2E} = \frac{e_1(2\alpha,\nu)}{E}\left(\frac{\Delta K_{1D}}{R_0^{\,1-\lambda_1}}\right)^2 \rightarrow R_0 = \sqrt{2\cdot e_1(2\alpha,\nu)}\left(\frac{\Delta K_{1D}}{\Delta\sigma_D}\right)^{\frac{1}{1-\lambda_1}} \tag{2}$$

Recently, an extension of the SED approach has been proposed, which converts the averaged Strain Energy Density in an averaged Strain Energy Density Intensity Factor L (Atzori et al. 2019). The SEDIF parameter L has been defined as follows:

$$L = \Delta\bar{W}\cdot R_0 \tag{3}$$

For a notch opening angle $2\alpha = 0$ (i.e. crack case), which is of interest for welded joints exhibiting fatigue failure at the weld root side, the parameter L does not depend on the radius R_0 chosen for the considered integration area, indeed:

$$L(2\alpha=0) = \Delta\bar{W}(2\alpha=0)\cdot R_0 = \frac{e_1(2\alpha=0,\nu)}{E}\frac{(\Delta K_1)^2}{R_0}\cdot R_0 = \frac{e_1(2\alpha=0,\nu)}{E}(\Delta K_1)^2 \tag{4}$$

Although the SEDIF seems to be a useful parameter to simplify and make more general the practical applications of the SED approach, the extension to the case of open notches, which is of interest for welded joints exhibiting fatigue failure at the weld toe side (where 2α is typically 135°), faces some difficulties, since for $2\alpha > 0$ the L parameter does depend on the chosen radius R_0, as demonstrated below:

$$L(2\alpha > 0) = \Delta\bar{W}(2\alpha > 0)\cdot R_0 = \frac{e_1(2\alpha > 0, \nu)}{E}\left(\frac{\Delta K_1}{R_0^{1-\lambda_1}}\right)^2 \cdot R_0 \tag{5}$$

4. SEDIF: a unifying approach to the fatigue strength assessment of welded joints and sharp open notches

The averaged SED calculated at the weld root ($2\alpha = 0$) and at the weld toe ($2\alpha = 135°$) sides of welded joints subjected to mode I loadings, by assuming that $R_{0,root}$ and $R_{0,toe}$ are in principle different, results in the following expressions (Livieri and Lazzarin 2005):

$$\Delta\bar{W}_{root} = \frac{e_1(2\alpha = 0, \nu)}{E}\left(\frac{\Delta K_{1root}}{R_{0,root}^{1-\lambda_1}}\right)^2 = \frac{e_1(2\alpha = 0, \nu)}{E}\left(\frac{\Delta K_{1root}}{R_{0,root}^{1-0.5}}\right)^2 = \frac{e_1(2\alpha = 0, \nu)}{E}\frac{(\Delta K_{1root})^2}{R_{0,root}} \tag{6}$$

$$\Delta\bar{W}_{toe} = \frac{e_1(2\alpha = 135°, \nu)}{E}\left(\frac{\Delta K_{1toe}}{R_{0,toe}^{1-\lambda_1}}\right)^2 = \frac{e_1(2\alpha = 135°, \nu)}{E}\left(\frac{\Delta K_{1toe}}{R_{0,toe}^{1-0.674}}\right)^2 = \frac{e_1(2\alpha = 135°, \nu)}{E}\frac{(\Delta K_{1toe})^2}{R_{0,toe}^{0.652}} \tag{7}$$

Dealing with welded joints made of structural steels, R_0 has been calibrated in (Livieri and Lazzarin 2005) by applying Eq. (2) and it resulted in 0.28 mm and in 0.36 mm for joints with failures from the weld toe and root sides, respectively. It has been suggested in (Livieri and Lazzarin 2005) to adopt a constant value, i.e. 0.28 mm in the safe direction. Concerning welded joints made of aluminium alloys, R_0 has been calibrated again in (Livieri and Lazzarin 2005) and it resulted in 0.12 mm for joints with failures from the weld toe as well as from the root sides.

The averaged SED approach according to Eqs. (6) and (7) has been adopted to synthesize experimental fatigue results generated from welded joints subjected to axial or bending loadings in the as-welded conditions and with a nominal load ratio R close to 0, made of structural steels (see Fig. 1a) or aluminium alloys (see Fig. 1b) and exhibiting fatigue crack initiation either at the weld root ($2\alpha = 0$) or at the weld toe ($2\alpha = 135°$) sides (Livieri and Lazzarin 2005; Lazzarin et al. 2008b). More in detail, Fig. 1a includes about 600 fatigue data generated by T-welded as well as cruciform fillet-welded joints having a main plate thickness in the range between 3 and 100 mm, and the joints were made of structural steels characterized by a yield stress ranging from 235 to 690 MPa, while Fig. 1b include about 120 fatigue data generated by T- or cruciform fillet-welded joints with main plate thickness ranging from 3 to 25 mm and manufactured by aluminium alloy sheets belonging to the 5000 and 6000 series with a yield stress varying between 215 and 315 MPa.

It is worth noting that for both steel and aluminium welded joints, $R_{0,root}$ has been calibrated by equalling the averaged SED of un-notched, i.e. ground butt welded joints, and notched specimens, i.e. welded joints with failures at weld root side, both evaluated at N_D cycles. In the same way, $R_{0,toe}$ has been calibrated by equalling the averaged SED of un-notched, i.e. ground butt welded joints, and notched specimens, i.e. welded joints with failures at weld toe side, both evaluated at N_D cycles. As a consequence, the averaged SED values relevant to welded joints with failures at weld root and at weld toe sides, respectively, must be equal each other at N_D cycles and therefore a relationship between the control radii $R_{0,root}$ and $R_{0,toe}$ can be derived as follows:

- steel welded joints under mode I loading are characterised by $\Delta K_{1D,root}$ = 180 MPa·mm$^{0.5}$ and $\Delta K_{1D,toe}$ = 211 MPa·mm^{326} (Livieri and Lazzarin 2005), therefore by imposing $\Delta\bar{W}_{root} = \Delta\bar{W}_{toe}$ it results:

$$\frac{R_{0,root}}{R_{0,toe}^{0.652}} = \frac{e_1(2\alpha = 0, \nu = 0.3)}{e_1(2\alpha = 135°, \nu = 0.3)}\left(\frac{\Delta K_{1D,root}}{\Delta K_{1D,toe}}\right)^2 = \frac{0.133}{0.118}\left(\frac{180\text{MPa}\cdot\text{mm}^{0.5}}{211\text{MPa}\cdot\text{mm}^{0.326}}\right)^2 = 0.82\frac{\text{mm}}{\text{mm}^{0.652}} \rightarrow R_{0,toe} = \left(\frac{R_{0,root}}{0.82}\right)^{\frac{1}{0.652}} \tag{8a}$$

- welded joints made of aluminium alloys under mode I loading are characterised by $\Delta K_{1D,root}$ = 71 MPa·mm$^{0.5}$ and $\Delta K_{1D,toe}$ = 99 MPa·mm^{326} (Livieri and Lazzarin 2005; Lazzarin et al. 2006), therefore by imposing $\Delta\bar{W}_{root} = \Delta\bar{W}_{toe}$ it results:

$$\frac{R_{0,root}}{R_{0,toe}^{0.652}}=\frac{e_1(2\alpha=0,\nu=0.33)}{e_1(2\alpha=135°,\nu=0.33)}\left(\frac{\Delta K_{1D,root}}{\Delta K_{1D,toe}}\right)^2=\frac{0.125}{0.113}\left(\frac{71MPa\cdot mm^{0.5}}{99MPa\cdot mm^{0.326}}\right)^2=0.48\frac{mm}{mm^{0.652}}\rightarrow R_{0,toe}=\left(\frac{R_{0,root}}{0.48}\right)^{\frac{1}{0.652}} \tag{8b}$$

Accordingly, once calibrated the control radius at the root side, i.e. $R_{0,root,}$ then the control radius at the toe side, i.e. $R_{0,toe}$, can be derived by applying Eq. (8). Some examples of the values of the control radii, which satisfy the conditions provided by Eq. (8), are reported in Tables 1 and 2. It is worth noting that Tables 1 and 2 include values of $R_{0,root}$ and $R_{0,toe}$ on the order of several millimetres, while typical values calibrated by Lazzarin and co-workers (Lazzarin and Zambardi 2001; Livieri and Lazzarin 2005) are on the order of tenths of a millimetre. High values of R_0 are valid only by assuming that the stress fields within the control volume having size R_0 are governed solely by the NSIFs, i.e. the contribution of higher-order non-singular terms is negligible.

Table 1. Values of the control radii $R_{0,root}$ and $R_{0,toe}$, which satisfy the conditions provided by Eq. (8a) and relevant values of the averaged SED and SEDIF parameter referred to PS=50% and $N_D=5\cdot10^6$ cycles for steel welded joints.

$R_{0,root}$ [mm]	$R_{0,toe}$ [mm]	$\Delta W_{root,D}$ [MJ/m^3]	$\Delta W_{toe,D}$ [MJ/m^3]	$L=\Delta W_{root,D}\cdot R_{0,root}=\Delta W_{toe,D}\cdot R_{0,root}$ [kJ/m^2]
0.1	0.040	0.2092	0.2092	0.0209
0.3	0.21	0.0697	0.0697	0.0209
0.5	0.47	0.0418	0.0418	0.0209
1	1.36	0.0209	0.0209	0.0209
3	7.31	0.0070	0.0070	0.0209
5	16.0	0.0042	0.0042	0.0209

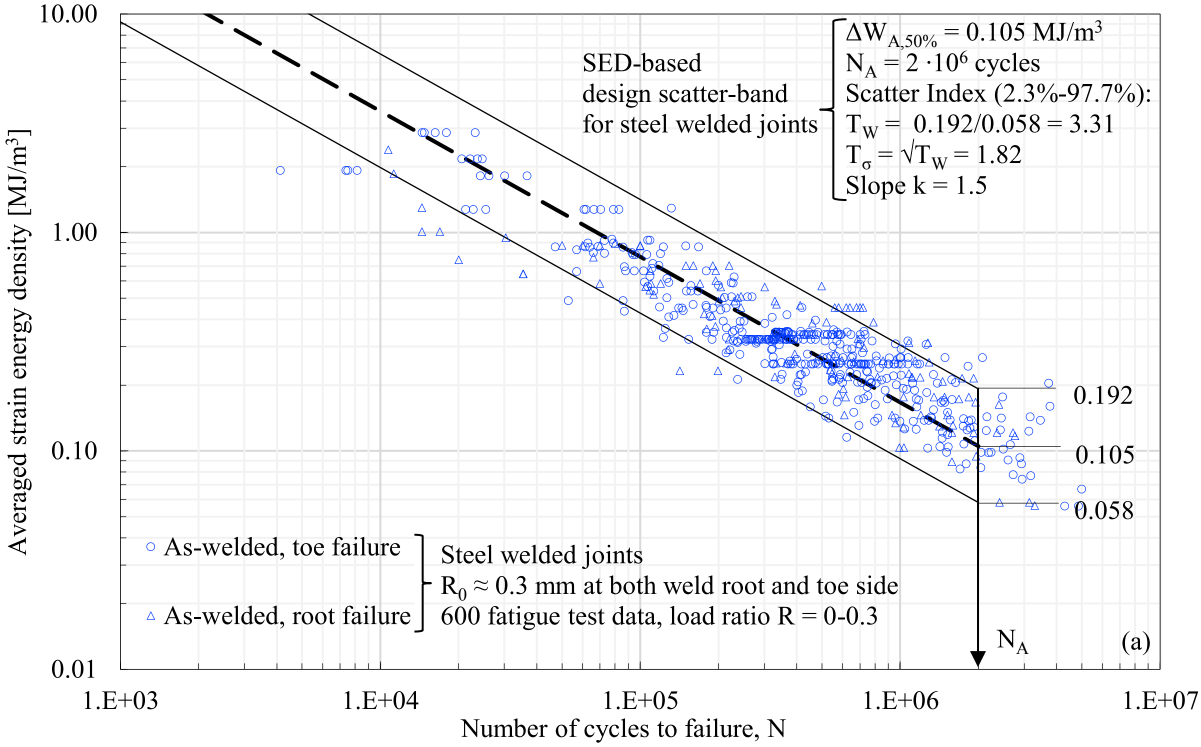
Table 2. Values of the control radii $R_{0,root}$ and $R_{0,toe}$, which satisfy the conditions provided by Eq. (8b) and relevant values of the averaged SED and SEDIF parameter referred to PS=50% and $N_D=5\cdot10^6$ cycles for welded joints made of aluminium alloys.

$R_{0,root}$ [mm]	$R_{0,toe}$ [mm]	$\Delta W_{root,D}$ [MJ/m^3]	$\Delta W_{toe,D}$ [MJ/m^3]	$L=\Delta W_{root,D}\cdot R_{0,root}=\Delta W_{toe,D}\cdot R_{0,root}$ [kJ/m^2]
0.12	0.12	0.0750	0.0750	0.0090
0.3	0.48	0.0300	0.0300	0.0090
0.5	1.05	0.0180	0.0180	0.0090
1	3.05	0.0090	0.0090	0.0090
3	16.44	0.0030	0.0030	0.0090
5	35.99	0.0018	0.0018	0.0090

Afterwards, it is proposed here to define the SEDIF parameter L as the product of the averaged SED and the control radius calibrated for a notch having zero-opening-angle ($2\alpha = 0$), i.e. $R_{0,root.}$ Taking advantage of previous relationships between $R_{0,root}$ and $R_{0,toe}$, it results:

$$L_{root}=L(2\alpha=0)=\Delta\bar{W}_{root}\cdot R_{0,root}=\frac{e_1(2\alpha=0,\nu)}{E}(\Delta K_{1root})^2 \tag{9}$$

$$L_{toe}=L(2\alpha=135°)=\Delta\bar{W}_{toe}\cdot R_{0,root}=\frac{e_1(2\alpha=135°,\nu)}{E}(\Delta K_{1toe})^2\frac{R_{0,root}}{R_{0,toe}^{0.652}}\quad \text{where}\quad \frac{R_{0,root}}{R_{0,toe}^{0.652}}=\text{const.}\quad \text{according to Eq. (8)} \tag{10}$$



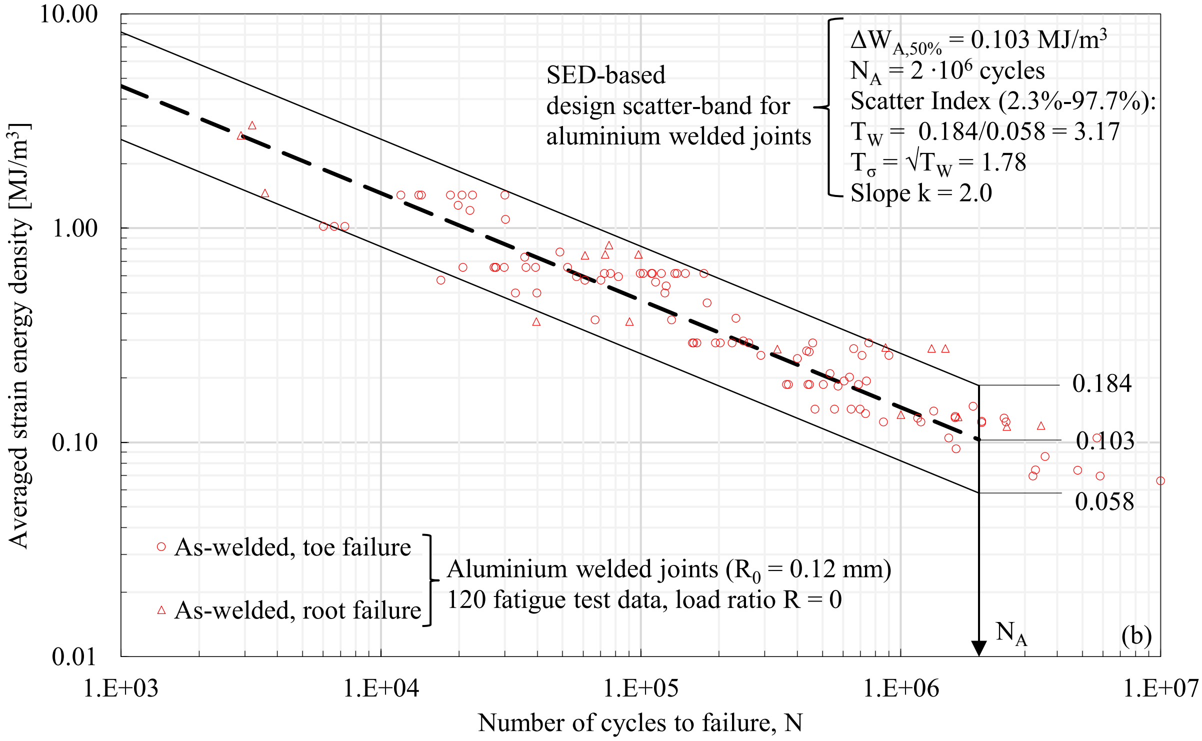
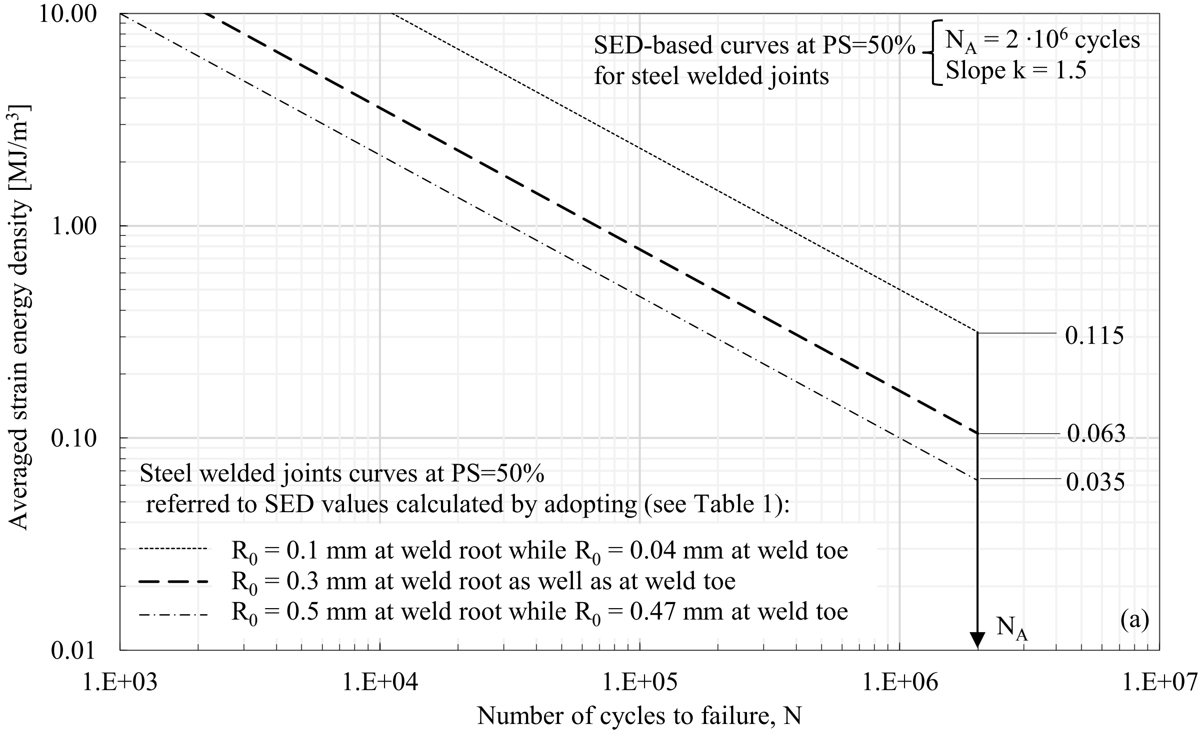


Figure 1: Fatigue test results expressed in terms of averaged SED and relevant to welded joints tested in the as-welded conditions under axial or bending loading and made by (a) structural steels and (b) aluminium alloys.

It can be observed that the averaged SED values calculated at the weld root and toe sides depend on the control radii $R_{0,root}$ and $R_{0,toe}$ according to Eqs. (6) and (7), while the SEDIF parameter L does not depend on the specific values of the control radii, but only on their ratio according to Eqs. (9) and (10), provided that condition (8) is verified. This can be observed also by comparing Fig. 2a with Fig. 2b, for the case of steel welded joints. This means that L parameter could be evaluated from Eq. (9) and (10) by adopting any value of the control radii $R_{0,root}$ and $R_{0,toe}$ provided that they verify conditions (8a) and (8b) and stress fields within the relevant control volumes are governed solely by the NSIFs. It is worth noting that also Lazzarin and co-workers (Lazzarin et al. 2008a; Fischer et al. 2013) suggested to calculate the SED averaged over a control volume of radius R_0 by adopting any value of the control radius $R_{0,FEM}$ in the FE calculation and, then, to correct the SED value resulting from the FE post-processing by a proper ratio of the radii $R_{0,FEM}$ and R_0.

It is worth noting that to apply the averaged SED approach to welded joints exhibiting fatigue failures either at the weld root or weld toe sides according to Eqs. (6) and (7), two different control radii $R_{0,root}$ and $R_{0,toe}$ must be calibrated on the basis of the fatigue results generated by (i) ground butt welded joints, (ii) welded joints with failures at weld root side and (iii) welded joints with failures at weld toe side. On the other hand, to apply the approach based on the L parameter according to Eqs. (9) and (10), only the ratio between the two control radii $R_{0,root}$ and $R_{0,toe}$ must be calibrated from Eq. (8), so that fatigue results relevant to un-notched specimens are not necessary.

Another advantage of using the L parameter as compared to the averaged SED is highlighted in Fig. 3, where it can be observed that the experimental fatigue results relevant to welded joints made of structural steels and aluminium alloys, respectively, which have been summarised in terms of averaged SED in two different scatter-bands in Fig. 1 (see also a comparison in Fig. 3a), instead, can be summarised in an uniform scatter-band in terms of SEDIF parameter, provided that fatigue data are normalized on the basis of the Young's modulus of the materials. Similar results can be observed in Fig. 4, where experimental fatigue data taken from (Kihara and Yoshii 1991) and relevant to different V-notched flat bar specimens with notch opening angle equal to $2\alpha = 90°$, $120°$, $135°$ and $150°$ and made of two different steels, i.e. mild steel (SS41) and high-strength steel (HT60), have been summarised in two different scatter-bands in terms of averaged SED, while they fall inside a uniform scatter-band when expressed in terms of SEDIF parameter, calculated by assuming for each material a constant control radius R_0 for all considered notch opening angles. It is worth noting that Kihara and Yoshii (Kihara and Yoshii 1991) obtained a similar synthesis of experimental results for the two different steels by adopting an 'equivalent stress intensity factor'.



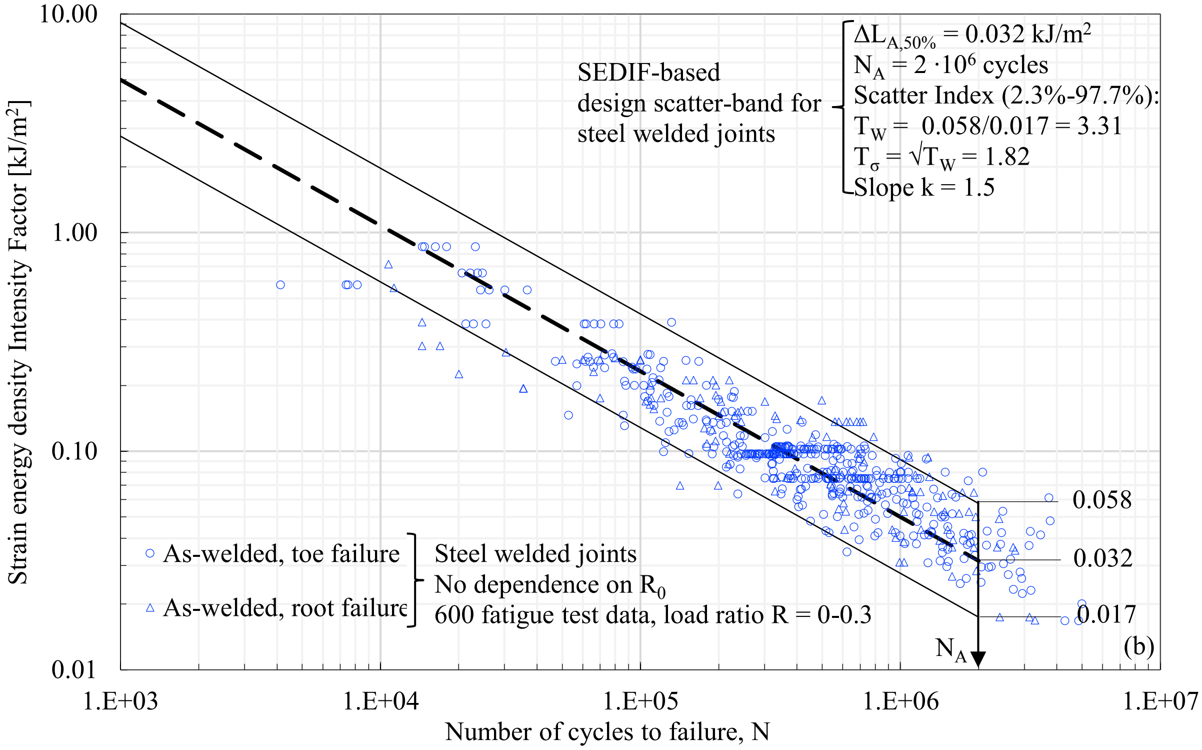
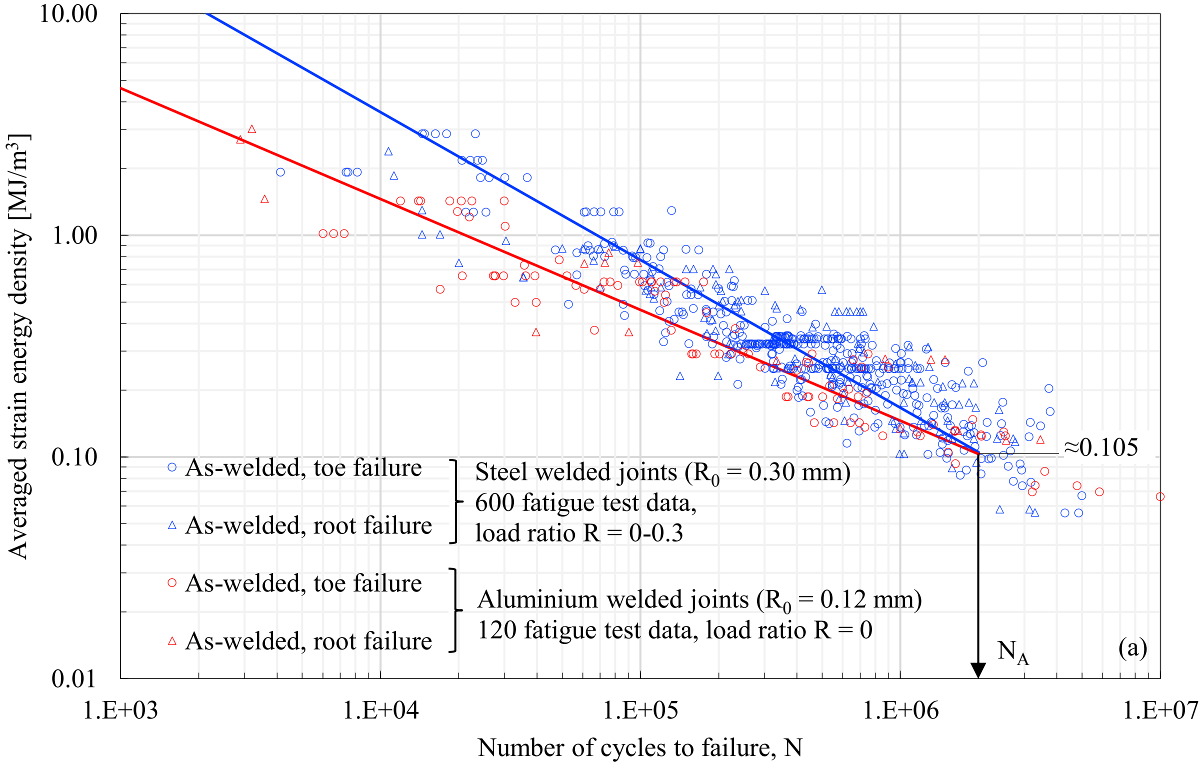


Figure 2: Fatigue test results relevant to steel welded joints tested in the as-welded conditions under axial or bending loading and re-analysed in terms of (a) averaged SED by adopting different values of the control radii $R_{0,root}$ and $R_{0,toe}$ (see Table 1) and (b) SEDIF parameter, independently of the control radius R_0. In figure (a) only the curves at PS=50% have been reported for the sake of clarity.



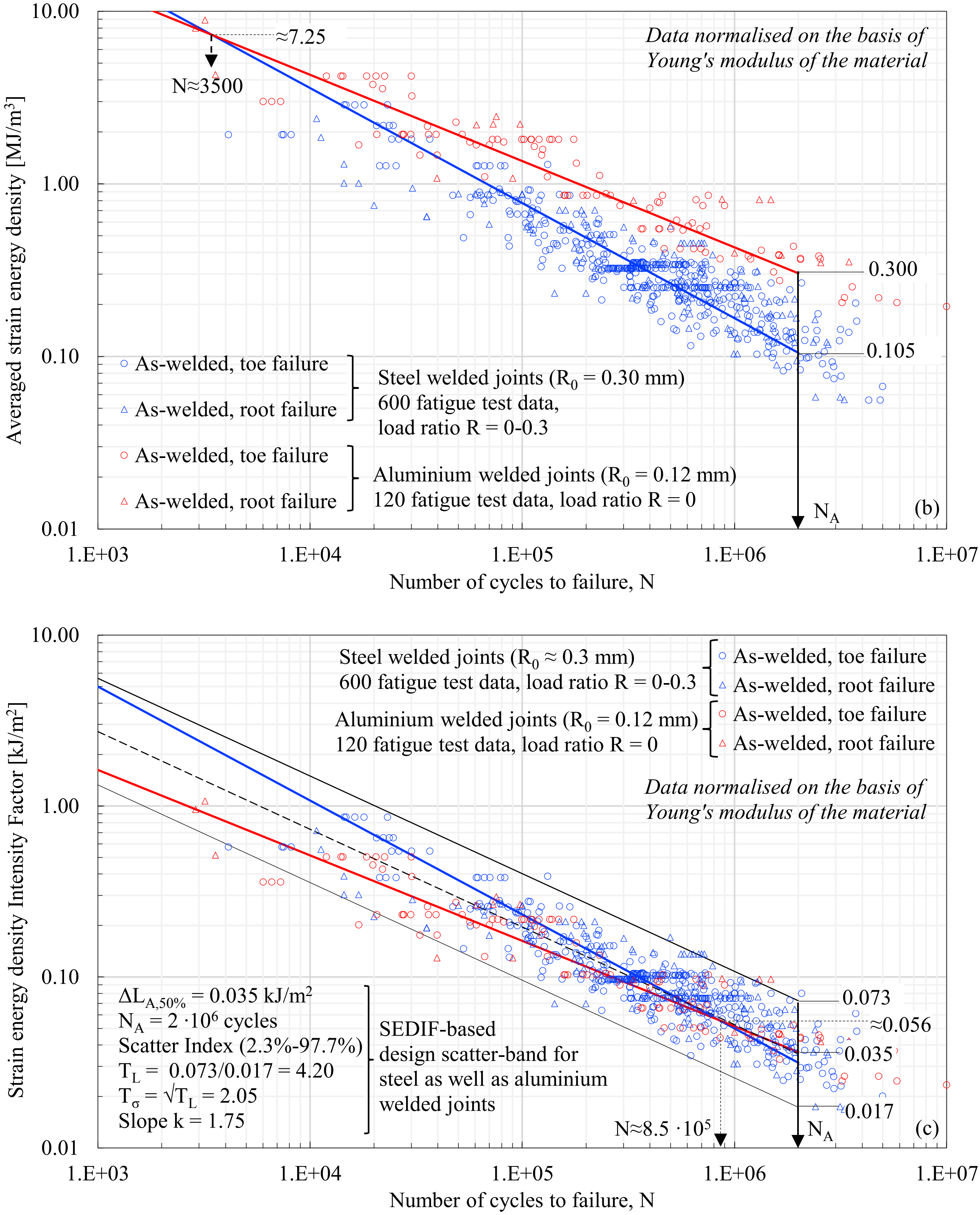


Figure 3: (a) Fatigue test results relevant to welded joints made of steels and aluminium alloys and tested in the as-welded conditions under axial or bending loading and re-analysed in terms of (a) and (b) averaged SED and (c) SEDIF parameter. Fatigue data in figures (b) and (c) have been normalized on the basis of Young's modulus of the material (E = 206000 MPa for steels and 70000 MPa for aluminium alloys).

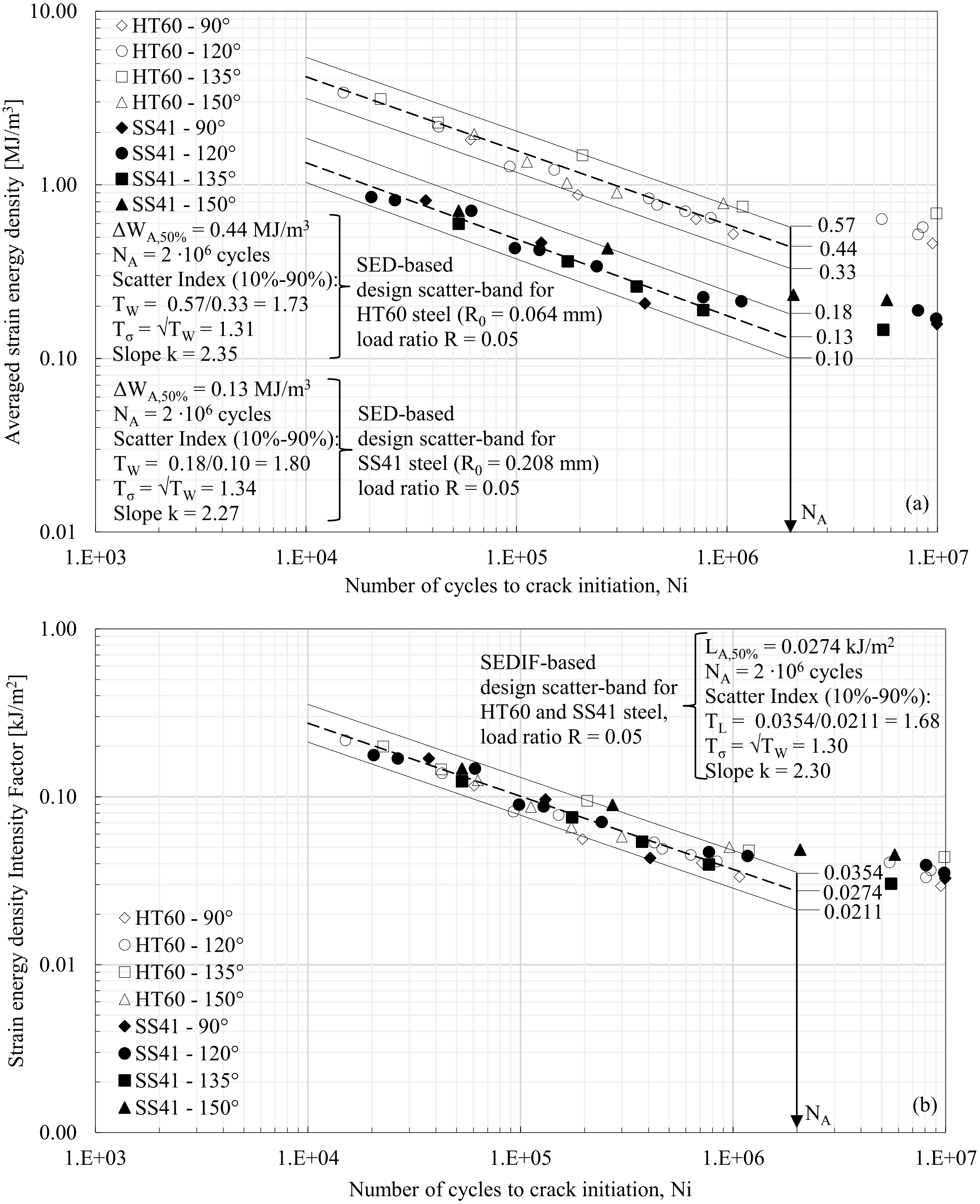


Figure 4: Fatigue test results (Kihara and Yoshii 1991) expressed in terms of (a) averaged SED and (b) SEDIF parameters over cycles to crack initiation Ni (ai = 0.2 mm) for different V-notched flat bar specimens (2α = 90°, 120°, 135° and 150°) of mild steel (SS41) and of high-strength steel (HT60) subjected to tensile loading. Nominal stress ratio R=0.05 (see also (Lazzarin and Zambardi 2001)).

5. Conclusions

A new parameter, which has been recently introduced as an extension of the averaged Strain Energy Density (SED) approach and called the Strain Energy Density Intensity Factor (SEDIF), has been applied in the present contribution to conventional arc welded joints exhibiting fatigue failures either at the weld root or at the weld toe. The re-analyses of experimental fatigue data refer to as-welded joints, made of either structural steels or aluminium alloys, under axial or bending loads, under different load ratios and modelled with the assumption of a null notch tip radius at the crack initiation location. The advantages of the SEDIF parameter as compared to the averaged SED are as follows: (i) to calibrate the method, the high cycle fatigue strength relevant to weld toe failure and weld root failure is needed; (ii) the SEDIF parameter, if multiplied by the Young modulus, allows to summarise fatigue results generated from specimens made of different materials, global geometries and notch opening angles.

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