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ZETA™ METHODOLOGY AND VARIATION IN THE SYSTEMIC RISK OF DEFAULT: ACCOUNTING FOR THE EFFECTS OF TYPE II (FALSE NEGATIVE) ERRORS VARIATION ON LENDING

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Abstract

The loan manager – dealing with one single borrower at a time and being responsible for that single decision to lend – is exposed to the idiosyncratic risk of default of his customer just like the physician is exposed to the risk of a wrong diagnosis with our strep throat. At the same time – if we do not expect the strep throat diagnostic test kit to change – we would still expect that physician reading that test to become more careful – or update his prior beliefs – about his diagnoses when a flu epidemic is likely to kick in with a certain estimated probability (likelihood). However, this has not been the case with loan management – there is in fact some consensus that before 2007 a reduction in the standards of idiosyncratic risk assessment by lenders – prior to risks pooling – coupled with a worsening of the systemic risk scenario, is partly to blame for the well known 2007–2008 financial crisis, with some of the blame falling also on the incapacity of actuarial mathematical models (test kits) to update worst case scenarios or be calibrated continuously on the basis of variation in the likelihood of default of the underlying risks pool.

The authors of this paper argue that, on the other hand, a standard Bayesian transformation of the ZETA bankruptcy prediction methodology introduced by Altman in 1968–1977 allows for a continuous *a posteriori* update of conditional Type I and II errors due to variation in the systemic likelihood of default. The Bayesian transformation can be used both to condition the loan manager's *prior* decision (generally based on Basel II-compliant Internal Rating Based system or Credit Agency's Rating) and to update such decision on the basis of any *posterior* hypothesis (based on actuarial frequentist assumptions of conditional hazard rates) regarding the creditworthiness and the probability of default of an underlying pool of securities.

At the same time – under a Bayesian framework – the ZETA diagnostic test can be conditioned on the new evidence introduced by other tests to increase the total sensitivity of the *default* prediction models (IRB ratings, TTC ratings, logit, probit, neural) to update the commercial bank's lending decisions.

A ground-state, static meta-analysis of Altman's et al. ZETA original article (1977) reveals that the odds of the commercial bank detecting a *default* after the ZETA score has been introduced (post-test) is 13.2 times more effective than the a priori prediction. Under the same assumptions, the odds of the commercial bank detecting a *survival* after (post-test) the ZETA score has been introduced is 12.2 times more effective than the *a priori*. Integration of the ZETA model with other default prediction models reaches a credibility interval of CI ≥ 95% when the updated likelihood of default is equal to 60%. As expected, the Efficiency Comparison Test ECZETA= 0.00243 is invariant under the Bayesian transformation.

Key words: Altman's Z Score, Risk of Default, Bankruptcy prediction, Bayesian Transformation

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Introduction

In general modeling correlation and dependence (Li, 2000; Embrechts et al., 2002; Embrechts et al., 2008; Embrechts, 2009) of pooled idiosyncratic risks should perform the task of increasing the efficiency of financial markets by transferring some of banks' credit risks and allow them to free up capital (Donnelly and Embrechts, 2010). However, there is some consensus (Brunnermeier, 2009;

Crouhy, 2008; Salmon, 2009) that before 2007 a reduction in the standards of idiosyncratic credit risk assessment by lenders prior to risk transfer (Donnelly 2010) – coupled with a worsening of the systemic risk scenario (Stiglitz, 2008) – has led to the well known 2007–2008 financial crisis, with part of the blame falling on the incapacity of the mathematical models (Derman, 2013; Lohr, 2009; Donnelly, 2010) to update the worst case scenarios on the basis of variation in correlation in the underlying risks pool and the systemic probabilities of default.

The authors of this paper argue that, on the other hand, the Bayesian transformation (Ross, 2002; Gelman et al., 2004) of the ZETA™ scoring methodology introduced by Altman in 1968-1977 allows for a continuous *a posteriori* update of conditional Type I and II errors due to variation in the systemic likelihood of default (prevalence of disease in epidemiology (– Peacock, 2011). The Bayesian transformation can be used *both* to condition some *prior* decision – generally based on Basel II-compliant Internal Rating Based system or Credit Agency's Rating – and to update any *posterior* hypothesis (based on actuarial frequentist assumptions of conditional hazard rates) regarding the creditworthiness and the probability of default of an underlying pool of securities.

At the same time – under a Bayesian framework –the ZETA™ diagnostic test can be integrated with other tests (Ross Sheldon, 2002; Gelman et al., 2004) to increase the sensitivity of other default prediction models (IRB ratings, TTC ratings, logit, probit, neural) to condition the commercial bank's lending decisions.

Background

Bankruptcy prediction has always been a central theme for banks, which must decide whether or not to approve credit requests on the basis of some estimate of the likelihood of default of the borrower at the time of the request and for a reasonable future period. Since the 1930s scholars started to develop models using statistics and economic-financial indicators (e.g. Smith, 1930; FitzPatrick, 1931; FitzPatrick, 1932; Ramser and Foster, 1931; Smith and Winakor, 1935; Wall, 1936). From the 1960s (Tamari, 1966; Beaver, 1966; Altman, 1968, Deakin, 1972, 1977; Joy et al., 1975) to the 1980s discriminant analysis models became the principal approach[1].

Among them, probably the most popular is Altman's Z-score model, formulated in 1968 (Altman, 1968) composed of a discriminant function based on five variables weighted by coefficients [2]. The model has been revised several times by its author (Altman, 1983; 2002; etc.) who has constantly updated the parameters and adapted the indices for different populations of companies other than American manufacturers quoted on the Stock Market. The Z'-Score (Altman, 1983) is a variation for private companies. The Z" Score (Altman, Hartzell and Peck, 1995; Altman & Hotchkiss, 2006, p. 314) was introduced for the non-manufacturing sector or companies operating in developing countries (the original study investigated a sample of Mexican companies) and was recently tested for Italy (Altman, Danovi, Falini 2013). Another adaptation was the introduction of the Z-Metrics System (Altman, Rijken et. al., 2010) that refines the original model, including both market equity levels and volatility, as well as macro-economic variables.

Research Questions

In medical statistics the Bayesian approach to prior hypotheses testing and updating when new evidence has been introduced is uncontroversial, and represents the contemporary epidemiological standard for clinical and pharmacological research (Peackock, 2011). In particular, Bayesian

1. Univariate and multivariate statistical analysis are still in use, (Edmister, 1972; Blum, 1974; Elam, 1975; Libby, 1975; Taffler, 1976, 1982; Altman et. al., 1977, 1993; Wilcox, 1976; Argenti, 1976; Appetiti, 1984; Lawrence and Bear, 1986; Aziz, Emanuel and Lawson, 1988; Baldwin and Glezen, 1992; Flagg, Giroux and Wiggins, 1991; Bijnen and Wijn, 1994; Kern and Rudolph, 2001; Altman, Rijken, et. al., 2010 Kruchynenko, 2012). For a comprehensive overview see Bellovary, Giacomino e Akers (2007) that summarizes 165 works from the 1930s to 2007.

2. The first application of the model involved a group of 66 American manufacturing companies (33 healthy and 33 bankrupt), listed on the Stock Exchange and showed that companies with a Z Score of less than 1.81 were highly risky and likely to go bankrupt; companies with a score more than 2.99 were healthy and scores between 1.81 and 2.99 were in a grey area with uncertain results). The model was extremely accurate since the percentage of correct predictions was about 95% and it received many positive reactions and only a few criticisms.

transformations are widely used when there is a rapid variation in the prevalence and incidence in the underlying disease pool during an epidemic or a pandemic (Ashby, 2006; Bland, 1998) and the physician needs to update the probability of Type I and II diagnostic errors.

The hypothesis of the authors of transferring the epidemiological experience to bankruptcy classification rests on the two following assumptions:

1. The credit manager will utilize the ZETA methodology to condition some *a priori* expectation (probability distribution) based on some Basel II-compliant Internal Rating Based system or Credit Agency's Rating regarding the probability of default of the corporation he is evaluating. However, the conditional probability that the credit manager will grant or withhold credit on the basis of the further evidence provided by the ZETA analysis is not the same as the underlying true probability of bankruptcy of the corporation (Ross Sheldon, 2002; Gelman et al., 2004), and neither it is the same as the probability of the corporation getting a negative ZETA if they are effectively going to go bankrupt (Ross Sheldon, 2002; Gelman et al., 2004).
2. Variation in the systemic risk of default P(D) of the credit pool should affect the credit manager's belief (Ross Sheldon, 2002; Gelman *et al.*, 2004) in the likelihood of default of the corporation he is evaluating, even if the ZETA methodology discriminant coefficients are based on idiosyncratic risks and are not updated continuously.

We performed a ground-state, static meta-analysis of Altman's ZETA original article (1977) – to which we refer in full – and transformed the variables under a standard Bayesian transformation.

In this sense, these are the research questions:

1. We would expect the commercial bank's conditional estimates of default P(D) given that Z is negative (loan rejected) $P(D|Z \leq 2.675)$ to be different from the discriminant's analysis probabilities of being negative given actual default $P(Z \leq 2.675|D)$;
2. The pre-test effectiveness (odds ratio) $\frac{P(D)}{P(D^c)}$ of detecting a default increases by the post-test ratio $\frac{P(Z|D)}{P(Z|D^c)}$.
3. Reveal a commercial bank's lower tolerance for conditional Type I Errors and a higher tolerance for conditional Type II Errors. i.e. $P(D|Z^c) \leq P(D^c|Z)$.
4. Verify that the Bayesian framework permits expansion and integration of the contribution $P(D_j|Z)$ that the ZETA Analysis gives to any prior hypotheses D_j (TTC ratings, neural networks, logit, probit) about the probability of default of the borrower.
5. Verify that ZETA Analysis can be continuously updated on the basis of data relative to the systemic hazard rate of the underlying risks pool P(D).
6. Confirm the postulate that the ZETA test causes, in epidemiological jargon and on a macroeconomic level, a selection of the fittest corporations (patients) amongst the fittest (*cream skimming*) (Levaggi, 2003) by the commercial bank.
7. confirm that the pre-test and post-test efficiency comparison $EC_{ZETA} = 0.00243$ is invariant under the Bayesian transformation.

All datasets, variables and parameters utilized in the Meta-Analysis are from Altman EI, Haldeman R.G., Narayanan P. (1977), "ZETA™ Analysis", Journal of Banking and Finance; 1: 29–54. The version utilized can be accessed and freely downloaded from E. Altman's website at the New York University Stern School of Business at http://people.stern.nyu.edu/ealtman/papers.html (last accessed April 2013).

The Bayesian notation utilized is coherent with Berkley University's textbook: Ross Sheldon (2002), *Probability*, 6th Ed., Prentice Hall (NJ), USA; 3: 64–121.

The epidemiological approach to Bayesian statistics is coherent with Oxford University's textbook: Peacock, JL, Peacock, P.J. (2011), Medical Statistics, Oxford University Press, UK; 7: 234–235 and 14: 478–497.

Methods

We partitioned Altman et al. ZETA™ Analysis (1977) sample space S=P(D)+P(Z)+P(Dc∩Zc)=1 on the basis of mixed probabilities P(Z∩D) = 0.018, P(D∩Zc) = 0.002, P(Dc∩Z) = 0.069 and P(Dc∩Zc) = 0.911 and we conditioned the a priori probabilities of default P(D) = 0.02 on the a posteriori evidence of the ZETA tests calculated by Altman (1977).

We utilized Bayes' Formula to condition the probability of default P(D|•) and of survival P(Dc|•) on P(•|Z) given the a priori probability of default P(D) = 0.02, of survival [P(Dc) = 1 - P(D)] = 0.98, the a posteriori experimental accuracy of the ZETA test P(Z|D) = 0.924 and P(Zc|Dc) = -.930, and the experimental Type II P(Z|Dc) = 0.070 and Type I P(Zc|D) = 0.076 Errors.

$$P(D|Z)=\frac{P(Z|D)P(D)}{P(Z|D)P(D)+P(Z|D^c)\left[1-P(D)\right]}$$

and

$$P(D^c|Z^c)=\frac{P(Z^c|D^c)\left[1-P(D)\right]}{P(Z^c|D^c)\left[1-P(D)\right]+P(Z^c|D)P(D)}.$$

We then applied the results to the test utilized by Altman in his original paper (Altman *et al.* 1977, page 43) to compare the efficiency of the ZETA™ methodology with alternative strategies, which will be invariant after the Bayesian transformation if:

$$q_1\left(M_{12}/N_1\right)C_I+q_2\left(M_{21}/N_2\right)C_{II}\equiv P\left(Z^c\right)P\left(D|Z^c\right)C_I+P\left(Z\right)P\left(D^c|Z\right)C_{II}$$

and therefore:

$$EC_{ZETA}\equiv EC_{ZETA-BAYES}=.00243$$

where CI (Type I Error) is the cost of an accepted loan that defaults, CII is the cost of a rejected loan that would have resulted in a successful payoff; M12,M21 are the observed Type I and II errors (misses) respectively; and N1,N2 are the number of observations in the bankrupt (N1) and non-bankrupt (N2) groups (Altman et al., 1977).

Findings

All findings are summarized in Exhibit A.

The conditional probability of the commercial bank detecting a default when the corporation will actually default after the ZETA test has been introduced is P(D|Z) = 21.2% (circa 2 out of 10), with a Type II Error $P(D^c|Z)$ = 78.8% (circa 8 out of 10).

The conditional probability of the commercial bank detecting a healthy corporation when the corporation will actually not go bankrupt (survive) after the ZETA test has been introduced is $P(D^c|Z^c)$ = 99.8% (circa 10 out of 10), with a Type I Error $P(D|Z^c)$ = 0.2% (circa 0 out of 10).

The odds of the commercial bank detecting a default after (post-test) the ZETA score has been introduced $\frac{P(D|Z)}{P(D^c|Z)}=.269$ is $\frac{P(Z|D)}{P(Z|D^c)}=13.2$ times more effective than the *a priori* $\frac{P(D)}{P(D^c)}=.02$

The odds of the commercial bank detecting a default after (post-test) the ZETA score has been introduced $\frac{P(D|Z)}{P(D^c|Z)}=599.6$ is $\frac{P(Z|D)}{P(Z|D^c)}=12.2$ times more effective than the *a priori* $\frac{P(D)}{P(D^c)}=49.0$

The Efficiency Comparison Test EC_{ZETA}=.00243 is invariant after the Bayesian transformation as was expected, in fact:

$$P\left(Z^cD\right)C_I+P\left(ZD^c\right)C_{II}\equiv P\left(DZ^c\right)C_I+P\left(D^cZ\right)C_{II}=.00243$$

Exhibit A: Bayesian reduced sample space partitioning of Altman's ZETA Analysis (1977)

P(D)	**Prior hypothesis - Probability of default P(D)**	**0.020**	**2.0%**
$P(D^c)$	Prior hypothesis - Probability of survival [1 - P(D)]	0.980	98.0%
P(Z)	Probability of negative Z≤2.675	0.087	8.7%

$P(Z^c)$	Probability of positive $Z > 2.675$	0.913	91.3%
$P(Z\|D)$	Probability of negative $Z \leq 2.675$ *given* default	0.924	92.4%
$P(Z^c\|D^c)$	Probability of positive $Z > 2.675$ *given* survival	0.930	93.0%
$P(Z^c\|D)$	Type I - Probability of positive $Z > 2.675$ *given* default	0.076	7.6%
$P(Z\|D^c)$	Type II - Probability of negative $Z \leq 2.675$ *given* survival	0.070	7.0%
$P(D\|Z)$	**Probability of default *given* that $Z \leq 2.675$ is negative**	**0.212**	**21.2%**
$P(D^c\|Z^c)$	**Probability of survival *given* that $Z > 2.675$ is positive**	**0.998**	**99.8%**
$P(D\|Z^c)$	**Type I - Probability of default *given* $Z > 2.675$ is positive**	**0.002**	**0.2%**
$P(D^c\|Z)$	**Type II - Probability of survival *given* $Z \leq 2.675$ is negative**	**0.788**	**78.8%**
$P(ZD)$	negative $Z \leq 2.675$ *and* default	0.018	1.8%
$P(DZ)$	negative $Z \leq 2.675$ *and* default	0.018	1.8%
$P(ZD^c)$	negative $Z \leq 2.675$ *and* survival	0.069	6.9%
$P(D^cZ)$	negative $Z \leq 2.675$ *and* survival	0.069	6.9%
$P(Z^cD^c)$	positive $Z > 2.675$ *and* survival	0.911	91.1%
$P(D^cZ^c)$	positive $Z > 2.675$ *and* survival	0.911	91.1%
$P(DZ^c)$	positive $Z > 2.675$ *and* default	0.002	0.2%
$P(Z^cD)$	positive $Z > 2.675$ *and* default	0.002	0.2%
S	Space of events (Summation)	1.000	100.0%
$P(D) / P(D^c)$	Odds ratio of default	0.020	
$PZ\|D) / P(Z\|D^c)$	Likelihood	13.2	
$P(D\|Z) / P(D^c\|Z)$	New odds ratio of default after the Z-test has been introduced	0.269	
$P(D^c) / P(D)$	Odds ratio of survival	49.000	
$P(Z^c\|D^c) / P(Z^c\|D)$	Likelihood	12.2	
$P(D^c\|Z^c) / P(D\|Z^c)$	New odds ratio of survival after the Z-test has been introduced	599.6	

Source: Authors' elaborations on Altman's ZETA Analysis (1977)

Medicine students and practitioners are usually very surprised by the fact that any reliable diagnostic test, with an accuracy of 92.4%, Type I errors of 7.6% and Type II Errors of 7.0%, is capable of diagnosing a patient affected by a disease when the patient actually has that disease only 2 out of 10 times (Ross, 2002).

They are normally subject to the instinctive – but sometimes severely misleading - a *priori* belief in the perfect correlation between the outcome of the diagnosis (Z) and the disease itself (D), and in the *a posteriori* belief that the probability of diagnosing the disease D given a positive test Z is a linear function of utilizing a test Z which is positive given that the disease is present (Peacock, 2011).

The reason is that Bayes' rule states that the probability of the event default D is a weighted average of the conditional probability of $P(D|\bullet)$ given that $Z \equiv ZETA \leq 2.675$ has occurred and the conditional probability of D given that Z has *not* occurred ($Zc \equiv ZETA > 2.675$), each conditional probability is given as much weight as the event P(D) or P(Dc) on which it is conditioned to occur (Bayes, 1763).

In addition, the Bayesian transformation of Type II errors from $P(Z|Dc) = 7\%$ to $P(Dc|Z)=78.8\%$ is so steep at low P(D)s (2% in the 1977 ZETA Model) that, at such low P(D)s, increases in Type II Errors of the ZETA test P(Z|Dc) (>7%) (Exhibit B) implies a growing number of creditworthy corporations (> 8 out of 10) whose requests for loans are unexplainably rejected.

ZETA Analysis, after the Bayesian transformation consciously or unconsciously performed by the commercial bank's credit manager selects, from a macroeconomic point of view and according to epidemiological jargon, the fittest amongst the fittest (*cream skimming*).

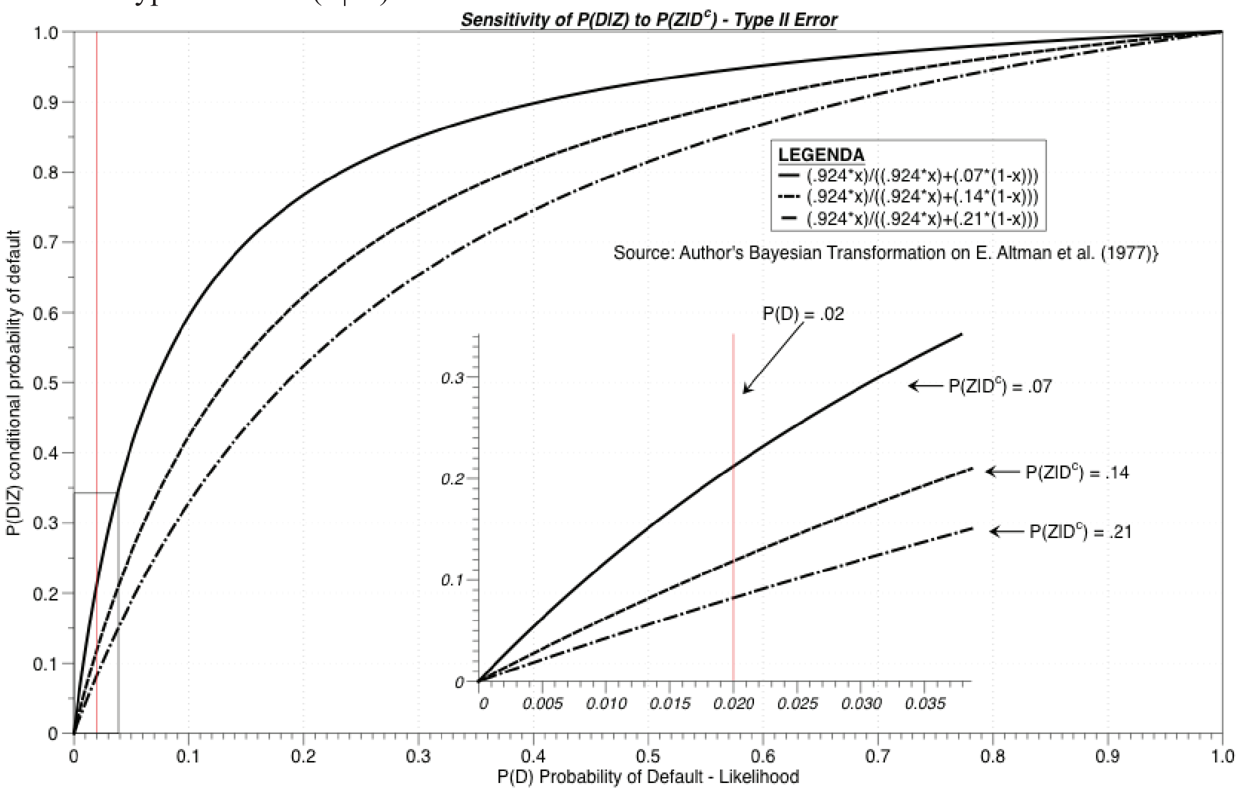
In this sense it appears to be also a very effective managerial (clinical) tool to diagnose the conditional *health* of a corporation, with a $P(Dc|Zc) = 99.8\%$ (almost 10 out of 10).

Bayesian model averaging (BMA) in forecasting has been analyzed and described in detail by Stock and Watson (2004). When idiosyncratic credit default prediction models are combined, the forecast is a weighted average of the individual model outcomes, where the weights depend on the accuracy of the individual forecasts. In BMA the weights are computed as *posterior* probabilities that the model is correct. In addition, the individual forecasts in BMA are model-based and the *posterior* means of the variable are forecast, conditioned on the selected model. Thus BMA extends forecast combining to a fully Bayesian setting, where the forecasts themselves are Bayes forecasts, given the model (and some parametric *priors*) (Stock and Watson, 2004).

In other words, if the commercial bank utilizes several scoring and rating systems and is e.g. P(Dj) = 60% confident that the borrower will default, the ZETA Analysis will condition the a *priori* probabilities of default P(Dj|•) of several other tests on Z so that:

$$P\left(D_j \mid Z\right) = \frac{P\left(Z \mid D_j\right)P\left(D_j\right)}{\sum_{i=1}^{n} P\left(Z \mid D_i\right)P\left(D_i\right)} \cong 95\%$$

Exhibit B: Relationship between decisor's conditional P(D|Z) and ZETA Analysis P(Z|D) at P(D) = 0.02 and Type II Error $P(Z|D^c) = 0.07$



Robustness

E. Altman's ZETA (1977) bankruptcy classification model is based on the sound and empirically tested scientific principles of discriminant analysis which comply with the ROBUST (Reporting Of Bayes Used in clinical STudies) criteria of conditional Bayesian analysis (Sung 2005; Spiegelhalter et al., 1999a; Spiegelhalter et al., 1999b; Spiegelhalter et al., 2000).

In particular:

- the prior distribution is specified – normal distribution;
- the prior distribution is justified – historical bankruptcy data;
- Type I and II errors are specified;
- the statistical model is specified – discriminant analysis;
- the statistical technique is specified;
- the central tendency tests have been performed;

- the standard deviation and confidence intervals have been specified.

Conclusions

The authors of this paper conclude that a standard Bayesian transformation of the ZETA bankruptcy prediction methodology introduced by Altman in 1968-1977 allows for a continuous *a posteriori* update of conditional Type I and II errors due to variation in the systemic likelihood of default. The Bayesian transformation can be used *both* to condition the loan manager's *prior* decision (generally based on Basel II-compliant Internal Rating Based system or Credit Agency's Rating) and to update such a decision on the basis of any *posterior* hypothesis (based on actuarial frequentist assumptions of conditional hazard rates) regarding the creditworthiness and the probability of default of an underlying pool of securities.

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Discussion and further survey of economic literature on Bayesian modeling

In standard Bayesian analysis, the parameters of a given model are treated as random variables, distributed according to a *prior* distribution. In Bayesian Model Averaging (BMA), the binary variables indicating whether a given model is true are also treated as random variables and distributed according to some *prior* distribution (Stock and Watson, 2004).

If the utilization of Bayesian average modeling is undisputed in epidemiology (Peackock, 2011; Ashby, 2006; Bland, 1998) with a wider use advocated (Lilford, 1996), the applications of BMA to economic forecasting have been quite recent (Stock and Watson, 2004) and have been mostly utilized to predict stock market growth and fluctuations where a continuous update of prior predictors is necessary.

Min and Zellner (1990) have used Bayesian and non-Bayesian methods for combining models and forecasts with applications to forecasting international growth; Avramov (2002) has used BMA to analyze stock return predictability and model uncertainty; Cremers (2002) – to analyze stock return predictability and Wright (2004; 2008) – to forecast inflation by Bayesian model averaging and exchange rate forecasts.

Macroeconomic aggregates have been analyzed by Koop and Potter (2003) who focused on forecasting GDP and the change of inflation.

In general, Bayesian statistics, as opposed to frequentist statistics, have shown some limits in advanced applications, in particular in the substitution of credible intervals (posterior interval) with confidence intervals (CIs) (Gelman et al., 2004; Spiegelhalter et al., 2004), and medical statistics require in general that both techniques be utilized (Deeks at al., 2004; Jones et al., 2007; Kovacs et al., 2001).

Further integration with the wide experiences collected by epidemiologists in analyzing extremely varying disease pools and the associated diagnostics can be useful in developing useful testing for the prediction of bankruptcy (strep throats!).

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