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Solving stochastic assignment to transportation networks with TVs and AVs

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Abstract

This paper focuses on solving stochastic assignment with several types of vehicles, for instance advanced and traditional vehicles, competing for the same arcs and jointly participating to congestion. In urban transportation networks paths likely overlap, thus two path choice models, derived from Random Utility Theory, are analyzed: Probit and Gammit, properly modeling path overlap through covariance between path perceived utilities. Since for these two models no closed form is available for choice probabilities, two specifications of Montecarlo algorithms for assignment to uncongested networks are presented: the efficiency of the commonly used Mersenne Twister Pseudo-Random Number Generator is compared with a PRNG based on Sobol (quasi-random) numbers. Then, several MSA-based algorithms for equilibrium assignment ot congested networks are analyzed: some step size strategies are proposed and compared with existing ones aiming at improving practical rate of convergence. Sufficient convergence conditions are presented for equilibrium assignment with arc cost flow functions with symmetric or asymmetric Jacobian matrix. Results of applications are also discussed to support theoretical results.

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*Keywords:*stochastic assignment; multi-vehicle assignment, equilibrium assinment.

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1. Introduction

Innovative advanced vehicles (AVs), including connected, automated, and autonomous vehicles, are fast developing and several prototypes are currently experimented in real urban areas. But, for several years mixed traffic including traditional vehicles (TVs) is expected, hence enhanced methods to support transportation project assessment and evaluation are needed (see for instance Cantarella and di Febbraro, 2017a).

Methods for travel demand assignment to a transportation network are the basic tool for transportation system analysis, useful to compute the arc flow (and cost) pattern in any design scenario (see Cascetta, 2009 for a general review of these topics).

For congested transportation networks, where arc flows depend on arc costs, equilibrium assignment searches for mutually consistent arc flows and costs. Equilibrium assignment was first introduced, under steady-state conditions, by Wardrop (1952), who named it User Equilibrium (UE), following what we may now call a deterministic utility approach to routing behaviour modelling. The UE assignment was analytically addressed by Beckmann et al. (1956) that formulate an optimization model and demonstrate the existence and the uniqueness of the solution.

A more general kind of equilibrium assignment was introduced by Daganzo and Sheffi (1977) who named it Stochastic User Equilibrium (SUE), following a random utility approach, that is the user path choice behaviour is described through probabilistic choice models derived from Random Utility Theory (RUT), as introduced by Domencich and McFadden (1975). SUE (and UE) assignment was formulated as fixed-point problem in Daganzo (1983) through the inverse of the cost function.

General fixed-point models for equilibrium assignment have been proposed in Cantarella (1997) by combining together the arc flow vector function, describing the assignment to an uncongested network, and the arc cost vector function; these models can be easily extended to deal with several types of assignment. Sufficient conditions for solution existence and uniqueness can easily be stated, requiring mild assumptions, mainly continuity and monotonicity of the involved functions; weaker uniqueness conditions have also been stated, not available for optimization models and/or Wardrop UE.

In a recent paper, Cantarella and Di Febbraro (2017b) showed how existing fixed-point models for stochastic equilibrium assignment can be extended to transportation networks where several types of vehicles compete for the same arcs and jointly participate to congestion.

This paper proposes extensions to multi-vehicle assignment of existing solution algorithms for stochastic assignment uncongested and congested urban networks, where paths likely overlap. Probit and Gammit path choice models, used in this paper, assume that path perceived utilities are distributed according to a MVNormal or MVGamma random variable respectively, they permit to properly model path overlapping through the perceived utility covariance matrix. But, in this case no closed form is available for choice probabilities, thus Montecarlo techniques based on Pseudo-Random Number Generator (PRNG) are commonly used.

This paper is organized as follows: sections 2 and 3 present models and algorithms, respectively, for stochastic assignment, then section 4 reports some results of applications of the proposed approach to the well-known Sioux Falls network; in section 5 main results are commented and some research perspectives are reported.

2. Models for stochastic assignment

This section describes a modelling approach for stochastic assignment with multi vehicle types by extending the one in Cantarella and Di Febbraro (2017b). Stochastic assignment to uncongested networks is discussed below in sub-section 2.1; whilst Stochastic user equilibrium assignment to congested networks is discussed in the next sub-section 2.2.

Users are distinguished with respect to o-d pair they are travelling from/to, user category (users with common socio-economic and behavioural features) and type of used vehicle (traditional, connected, automated, autonomous, …). Demand flows are assumed constant and route choice is the only user choice behaviour affected by network performances, or more properly by congestion. Transportation supply is modelled through a flow network, say a graph with a transportation cost and a flow associated to each arc. A route connecting an Origin Destination pairs is described by a path. [Presented results still hold if more general definitions of routes are used, such as hyperpaths.]

2.1. Stochastic Assignment to Uncongested Networks

The arc costs are generally different with respect to the vehicle type *m* (but not with respect to o-d pair *i*) to reflect different performances, and we assume that the arc cost per vehicle type are given by an affine transformation of the arc generic costs, thus:

$$\boldsymbol{c}_m = \chi_m \, \boldsymbol{c} + \boldsymbol{c}_{o,m} \qquad \forall m \qquad (1)$$

where (from right to left):

$\boldsymbol{c}_{o,m} \geq 0$ is the arc specific cost vector for vehicle type *m*, such as monetary costs, ...; it is equal to infinity (that is a large value) for an arc not available for vehicle type *m*;

$\boldsymbol{c} \geq 0$ is the arc generic cost vector, common to all vehicle types;

$\chi_m > 0$ is a (dimensionless) vehicle type specific coefficient;

$\boldsymbol{c}_m \geq 0$ the arc total cost vector for vehicle type *m*.

Under steady-state conditions the route costs for each combination of o-d pair *i*, user category *j* and vehicle type *m* can be obtained from the corresponding arc total costs through an affine transformation from the arc space to the route space defined by the transpose of arc-route incidence matrix:

$$\boldsymbol{w}_{ijm} = \boldsymbol{B}_{ijm}{}^{T} \cdot \boldsymbol{c}_m + \boldsymbol{w}_{o,ijm} \qquad \forall i, j, m \qquad (2)$$

where (from right to left):

$\boldsymbol{w}_{o,ijm} \geq 0$ is the vector of route specific or non-additive costs for o-d pair *i*, user category *j*, vehicle type *m*, such as fees, tolls, ... ;

$\boldsymbol{B}_{ijm}$ is the arc-route incidence matrix for o-d pair *i*, user category *j*, vehicle type *m*, with entries $b_{ak} = 1$ if arc a belongs to route *k*, $b_{ak} = 0$ otherwise;

$\boldsymbol{w}_{ijm} \geq 0$ is the vector of route total costs for o-d pair *i*, user category *j*, vehicle type *m*.

All costs are assumed measured by a common unit, usually travel time or money, through duly homogenization of different attributes, if the case.

The utility function for o-d pair *i*, user category *j* and vehicle type *m* is almost always specified through an affine transformation of costs both in research analysis and in practical applications:

$$\boldsymbol{v}_{ijm} = -\beta_{jm} \, \boldsymbol{w}_{ijm} + \boldsymbol{v}_{o,ijm} \qquad \forall i, j, m \qquad (3)$$

where (from right to left):

$\mathbf{v}_{o,ijm}$ is the vector of route systematic utilities for o-d pair *i*, user category *j*, vehicle type *m*, independent of route costs;

$\beta_{im} > 0$ is the utility scale parameter such that the term $\beta_{im} \cdot \boldsymbol{w}_{ijm}$ is dimensionless to be consistent with utility;

$\mathbf{v}_{ijm}$ is the vector of route total systematic utilities for o-d pair *i*, user category *j*, vehicle type *m*.

Route choice behaviour for of o-d pair *i*, user category *j* and vehicle type *m* can be modelled by applying any discrete choice modelling theory. Let

$\mathbf{p}_{ijm} \geq 0$ be the vector of route choice proportions for o-d pair i, user category *j*, vehicle type *m*, with $\boldsymbol{1}^T \boldsymbol{p}_{ijm} = 1$;

Route choice proportions depend on route utility through the choice function, which models user routing behaviour:

$$\boldsymbol{p}_{ijm} = \boldsymbol{p}_{ijm}(\boldsymbol{v}_{ijm};\ \theta_{jm},\ \eta_{jm}) \qquad \forall i, j, m \qquad (4)$$

where $\theta_{jm} \geq 0$ is the utility dispersion parameter and η_{jm} indicates any other parameter, to be calibrated against real data; these parameters are usually not distinguished per o-d pair.

Most often equation (4) is an application of Random Utility Theory (Domencich and McFadden, 1974), assuming that: (i) each user travelling between o-d pair i, belonging to user category j and using vehicle type m associates to each available route a perceived utility, (ii) chooses the maximum perceived utility route, (iii) the perceived utility is modelled as a continuous r.v. variable, with mean being the systematic utility, due to some sources of uncertainty.

Thus the choice proportion of an alternative is given by the probability that its perceived utility is equal to maximum among all alternatives. When the perceived utility co-variance matrix is non singular, probabilistic route choice functions are obtained.

Demand conservation relation for o-d pair i, user category j, vehicle type m assures that flows of all connecting routes sum up to demand flow, say $\mathbf{1}^T \mathbf{h}_{ijm} = d_{ijm}$, it can be expressed as:

$$\mathbf{h}_{ijm} = d_{ijm}\, \mathbf{p}_{ijm} \qquad \forall i, j, m \tag{5}$$

where (from right to left):

$d_{ijm} \geq 0$ is the demand flow for o-d pair i, user category j, vehicle type m, measured in users per time unit;

$\mathbf{h}_{ijm} \geq 0$ is the route flow vector for o-d pair i, user category j, vehicle type m, measured in users per time unit.

Under steady-state conditions the arc flows due to each combination of o-d pair i, user category j and vehicle type m can be obtained from the route flows through a linear transformation from the route space to the arc space defined by the arc-route incidence matrix:

$$\mathbf{f}_{ijm} = (\nu_m / \mu_m)\, \mathbf{B}_{ijm} \cdot \mathbf{h}_{ijm} \qquad \forall i, j, m \tag{6.a}$$

where (from right to left):

$\mu_m > 0$ is the occupancy factor for vehicle type m, measured in users per vehicle, so that arc flows are measured in vehicles per time unit;

$\nu_m \geqslant 0$ is the equivalence coefficient measured in vehicles of type m per TV, so that all arc flows of any vehicle type are measured in TVs per time unit;

$\mathbf{f}^{ijm} \geq \mathbf{0}$ is the arc total flow vector for o-d pair i, user category j, vehicle type m, measured in TVs per time unit.

Having assumed that all arc flows are measured in TVs per time unit, the arc total flows are given by the sum over all o-d pairs, user categories, vehicle types of arc flows plus the arc base flows:

$$\mathbf{f} = \Sigma_{ij}\ \Sigma_m \mathbf{f}_{ijm} + \mathbf{f}_b \tag{6.b}$$

where (from right to left):

$\mathbf{f}_b \geq \mathbf{0}$ is the arc base flow vector, arc flows not resulting by user choice behaviour;

$\mathbf{f} \geq \mathbf{0}$ is the arc total flow vector.

The arc-route flow consistency relation is given by combining equations (6.a) and (6.b):

$$\mathbf{f} = \Sigma_{ij}\, \Sigma_m\, (\nu_m / \mu_m)\, \mathbf{B}_{ijm} \cdot \mathbf{h}_{ijm} + \mathbf{f}_b \tag{6.c}$$

A relation between arc flows and (common) arc costs can be obtained by combining together equations 1 - 6 leading to the arc flow function:

$$\mathbf{f}(\mathbf{c};\, \mathbf{d}) = \Sigma_{ij}\ \Sigma_m\, (\nu_m / \mu_m)\, d_{ijm}\, \mathbf{B}_{ijm} \cdot \mathbf{p}_{ijm}(-\beta_{jm}\, (\mathbf{B}_{ijm}{}^T \cdot (\chi_m\, \mathbf{c} + \mathbf{c}_{o,m}) + \mathbf{w}_{o,ijm}) + \mathbf{v}_{o,ijm}) + \mathbf{f}_b$$

where $\mathbf{d} \geq \mathbf{0}$ is the vector of demand flows, with entries d_{ijm}.

Without any loss of generality

- the vector $\boldsymbol{w}_{o,ijm}$ can be multiplied by parameter $-\beta_{jm}$ and included in the vector $\boldsymbol{v}_{o,ijm}$,
- the ratio ν_m / μ_m can be considered as a single parameter ν_m.

This way the so-called standard arc flow function is obtained:

$$\boldsymbol{f}(\boldsymbol{c};\, \boldsymbol{d}) = \textstyle\sum_{ij} \sum_m \nu_m\, d_{ijm}\, \boldsymbol{B}_{ijm} \cdot \boldsymbol{p}_{ijm}(-\beta_{jm}\, \boldsymbol{B}_{ijm}^T \cdot (\chi_m\, \boldsymbol{c} + \boldsymbol{c}_{o,m}) + \boldsymbol{v}_{o,ijm}) + \boldsymbol{f}_b \tag{7}$$

Main input data of the arc flow function are arc costs, $\boldsymbol{c}$ and $\boldsymbol{c}_{o,m}$, and demand flows, $\boldsymbol{d}$, parameters ν_m, β_{jm}, χ_m, for each vehicle type *m* (to be calibrated against real data), and parameters of the choice function (4) - omitted for simplicity's sake - together with the vectors of systematic utilities independent from arc costs, $\boldsymbol{v}_{o,ijm}$, and of the arc base flows, $\boldsymbol{f}_b$.

Let *n* be the number of arcs, the arc flow function gets values in the feasible arc flow set: $S_f \subseteq \mathbb{R}^n_{;+}$, which is non-empty (if the network is connected), compact (since closed and bounded, if only elementary paths are considered), convex. The arc flow function is continuous and continuous differentiable with respect to (common) arc costs if all the choice functions are continuous and continuously differentiable; moreover, it is monotone non decreasing with respect to (common) arc costs with symmetric (semi-definite negative) Jacobian if all the choice functions are monotone increasing with respect to systematic utility with symmetric (semi-definite positive) Jacobian.

All usually adopted probabilistic choice functions give strictly positive probabilities, and are continuous and continuously differentiable with respect to systematic utility; moreover, if the parameters of the perceived utility pdf do not depend on systematic utility values, the resulting choice function, called *invariant*, is monotone increasing with respect to systematic utility with symmetric (semi-definite positive) Jacobian (Cantarella, 1997), and choice probabilities depend on differences between systematic utility values only.

The arc flow function (7) is a general model of stochastic assignment to an uncongested network with multi-vehicle types, or SUNv for short (after Cascetta, 2009, v denoting the extension to multi-vehicle types).

2.2. Stochastic Equilibrium Assignment to Congested Networks

For congested transportation networks, where arc flows depend on arc costs, equilibrium assignment searches for mutually consistent arc flows and costs. Arc generic costs depend on the arc total flows through the arc cost function, which models user driving behaviour:

$$\boldsymbol{c} = c(\boldsymbol{f};\, \boldsymbol{\kappa},\, \boldsymbol{\mu}) \qquad \forall \boldsymbol{f} \tag{8}$$

where $\boldsymbol{\kappa} > \boldsymbol{0}$ is the vector of arc capacities and $\boldsymbol{\mu} > \boldsymbol{0}$ indicates the vector of any other parameter; they are omitted in the following for simplicity's sake.

Equilibrium assignment can effectively be expressed by fixed-point models given by the arc flow function (7) and the arc cost function (8):

$$\boldsymbol{f}^* = \boldsymbol{f}(\boldsymbol{c}^*;\, \boldsymbol{d}) \in S_f \subseteq \mathbb{R}^n \tag{9}$$

$$\boldsymbol{c}^* = \boldsymbol{c}(\boldsymbol{f}^*) \in \boldsymbol{c}(S_f) \subseteq \mathbb{R}^n \tag{10}$$

Other equivalent models can be formulated with respect to route variables. An equivalent formulation with respect to flows (or costs) only is often used in literature (Cantarella, 1997), which can be obtained by explicitly including equation (10) into equation (9):

$$\boldsymbol{f}^* = \boldsymbol{f}(\boldsymbol{c}(\boldsymbol{f}^*);\, \boldsymbol{d}) \in S_f \subseteq \mathbb{R}^n \tag{11}$$

or vice versa equation (9) into (10):

$$\boldsymbol{c}^* = \boldsymbol{c}(\boldsymbol{f}(\boldsymbol{c}^*;\ \boldsymbol{d})) \in \boldsymbol{c}(S_f) \subseteq \mathbb{R}^n \tag{12}$$

Existence is guaranteed if both the arc flow function and the arc cost function are continuous (and the network is connected), applying Brouwer theorem to model (11). With reference to model (9, 10) for a monotone decreasing arc flow function, if the arc cost function is monotone strictly increasing uniqueness is guaranteed. Uniqueness conditions can be weakened for strictly positive invariant probabilistic route choice functions only requiring that arc cost function is monotone increasing (but not necessarily strictly monotone). Anyhow uniqueness of arc flows also guarantees uniqueness of arc costs as well as route flows and costs, and of flows and cost per o-d pair, user category, vehicle type. Weaker (sufficient) conditions for uniqueness have been recently derived; a full discussion of this topic is out the scope of this paper, it suffices mentioning that monotonicity of the arc cost function is not needed to assure uniqueness.

3. Solution algorithms for stochastic assignment

This section describes a solution approach for stochastic assignment to transportation networks with multi vehicle types. Stochastic assignment to uncongested networks is discussed in sub-section 3.1; Stochastic equilibrium assignment to congested networks is discussed in the sub-section 3.2.

3.1. Stochastic Assignment to Uncongested Networks

The solution of the stochastic assignment with multi-vehicle types to uncongested networks based on Probit or Gammit choice models, say the computation of the arc flow function (7), $\boldsymbol{f}(\boldsymbol{c};\ \boldsymbol{d})$, requires an enhancement of the well-known Montecarlo algorithm (after Sheffi, 1985). Some preliminary considerations need to be discussed before the main steps of the algorithm can be described.

Specification of the Probit or Gammit choice models. Following the most common approach, for each combination of o-d pair i, user category j and vehicle type m the route perceived utility is described by a Multi-Variate Normal or Gamma random variable specified through an affine transformation of arc perceived disutility independently distributed as Normal or Gamma random variables, with mean given by the known arc costs. Let

- c_a be the generic cost for arc a, an entry of vector $\boldsymbol{c}$;
- x_a be the zero flow cost for arc a;
- τ be the dispersion parameter;
- Y_a be the generic cost for arc a considered as a Normal or Gamma random variable with mean c_a, and variance τx_a;
- $\boldsymbol{Y}$ be the arc generic cost random vector with entries Y_a;
- $\boldsymbol{Z}_m = \chi_m \boldsymbol{Y} + \boldsymbol{c}_{o,m}$ be the arc cost random vector for vehicle type m;
- $\boldsymbol{U}_{ijm}$ be the perceived utility random vector for o-d pair i, user category j, vehicle type m, with mean given by the systematic utility v_{ijm} and covariance matrix Σ_U.

If random variables Y_a are assumed pair wise independent Normal or Gamma random variables with mean c_a, and variance τx_a, $\boldsymbol{Z}_m$ turns out a Multi-Variate Normal or Gamma random variable with mean vector $\boldsymbol{c}_m = (\chi_m \boldsymbol{c} + \boldsymbol{c}_{o,m})$ and diagonal co-variance matrix Σ_Z with entries $\tau\,(\chi_m x_a + c_{o,ma})$.

Moreover, assuming vectors $\boldsymbol{w}_{o,ijm}$ and $\boldsymbol{v}_{o,ijm}$ null for simplicity's sake, combining together equations (1), (2) and (3) yields:

$$\boldsymbol{v}_{ijm} = -\beta_{jm} \boldsymbol{B}_{ijm}{}^T \cdot (\chi_m \boldsymbol{c} + \boldsymbol{c}_{o,m})$$

Therefore

$$\boldsymbol{U}_{ijm} = -\beta_{jm}\, \boldsymbol{B}_{ijm}{}^{T} \cdot \boldsymbol{Z}_{m}$$

is a Multi-Variate Normal or Gamma random variable with mean vector $\boldsymbol{v}_{ijm}$ and co-variance matrix $\Sigma_U = \beta_{jm}\, \boldsymbol{B}_{ijm}{}^{T} \cdot \Sigma_Z \cdot \boldsymbol{B}_{ijm}$; according to this result the variance of a route perceived utility is proportional to route (zero flow) cost and the covariance between two routes proportional to common (zero flow) cost.

Specification of the enhanced Montecarlo algorithm. At each iteration k of the Montecarlo algorithm:

1. an arc generic cost vector $\boldsymbol{y}^k$ is generated as a pseudo-realization of random vector $\boldsymbol{Y}$;
2. for each vehicle type m arc: costs $\boldsymbol{z}_m{}^k = \chi_m\, \boldsymbol{y}^k + \boldsymbol{c}_{o,m}$ are computed;
3. for each for o-d pair i, user category j: demand flows are assigned to shortest paths using arc costs $\boldsymbol{z}_m{}^k$ leading to arc flows $\boldsymbol{f}_{ijm}{}^k$;
4. total arc flows for vehicle type m are computed: $\boldsymbol{f}_m{}^k = \Sigma_m\, \boldsymbol{f}_{ijm}$;
5. total arc flows are computed: $\boldsymbol{f}^k = \Sigma_m\, \boldsymbol{f}_m{}^k$;
6. total arc flows $\boldsymbol{f}^k$ are averaged with those of the previous iterations.

The algorithm is stopped when a pre-fixed maximum number of iterations is reached. it provides an unbiased estimation of the arc flow pattern consistent with the Probit / Gammit route choice behaviour model as specified above.

3.2. Stochastic Equilibrium Assignment to Congested Networks

Algorithms based on the Method of Successive Averages (MSA) (Sheffi and Powell, 1982; Daganzo, 1983) are the most used ones to solve fixed-point models for SUE assignment, since they can accommodate any choice model from RUT, and are suitable for large scale applications. Their basic iteration requires the computation of the cost function (8) to get arc costs from arc flows and the computation of the arc flow function (7) to get arc flows from arc costs, as described in the above sub-section for Probit or Gammit choice models (algorithms are also available for other choice models).

Generally MSA-based algorithms do not provide the equilibrium arc flows in a finite number of iterations, but only a succession of arc flow patterns; sufficient conditions for theoretical convergence of such a succession may be stated through Blum theorem.

Applying the Method of Successive Averages (MSA) to model (11) the MSA-FA solution algorithm is obtained based on the recursive equation:

$$\boldsymbol{f}^{k} = \boldsymbol{f}^{k-1} + (1/k)\, [\boldsymbol{f}(\boldsymbol{c}(\boldsymbol{f}^{k-1})) - \boldsymbol{f}^{k-1}] \tag{13}$$

it may be proved converging if the Jacobian of the arc cost function is symmetric Cantarella (1997).

On the other hand applying the MSA to model (12) the MSA-CA solution algorithm is obtained based on the recursive equation:

$$\boldsymbol{c}^{k} = \boldsymbol{c}^{k-1} + (1/k)\, [\boldsymbol{c}(\boldsymbol{f}(\boldsymbol{c}^{k-1})) - \boldsymbol{c}^{k-1}] \tag{14}$$

it may be proved converging if the Jacobian of the arc flow function is symmetric as shown in Cantarella (1997).

A convergence index often used to stop MSA-FA is the average absolute difference over flows:

$$(\Sigma_a\, |\, f(\boldsymbol{c}(\boldsymbol{f}^{k-1}))_a{}^{k} - f_a{}^{k-1} |\, / f_a{}^{k-1}) \,/\, n$$

where n is the number of arcs, to be compared with a given error threshold ε. A similar index, based on arc costs, may be defined for MSA-CA. Others indices may be defined based on the maximum difference, possibly excluding arcs with very low flows.

4. Numerical applications

In this section some results of several experiments of solution of the SUE assignment problem are discussed considering the well known Sioux Falls test network, used in several works to test models and algorithms (source: http://www.bgu.ac.il/~bargera/tntp/, where input data, such as zero flow costs and demand flows are also reported). It consists of 24 nodes (assumed all as origin/destination nodes), 76 arcs and 552 origin/destination pairs, as shown in figure 1 below.

In the applications discussed below one user class and two vehicles, 1 = TVs and 2 = AVs, are considered. Total demand flows are split between the two vehicle type according different proportions: 100%-0%, 90%-10%, 70%-30%, 50%-50%, 30%-70%, 10%-90%, 0%-100%. Both Probit and Gammit choice models are considered. Other parameters are given in the following table 1 assuming AVs more effective regarding costs and effect on congestion, and with less choice dispersion. For simplicity's sake: $\beta_{jm} = 1$, $\mathbf{v}_{o,ijm} = 0$, and $\mathbf{f}_b = 0$.

Table 1. Parameters per vehicle type

m	ν_m	χ_m	τ_m
1	1.0	1.0	50
2	0.8	0.9	0.5

The cost-flow function for each arc is specified by the separable polynomial BPR (1964) travel time function, with both parameters equal to 2.

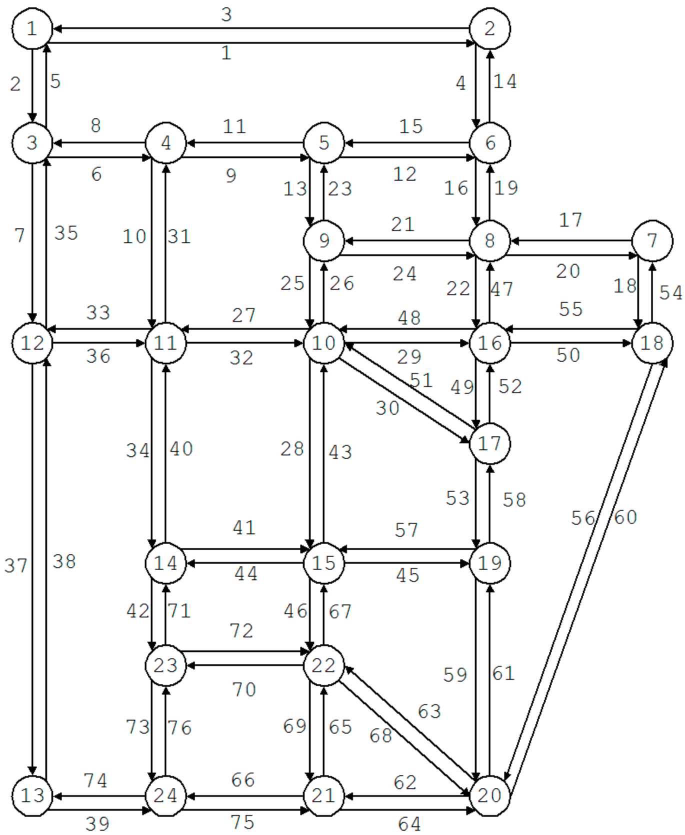

Fig. 1. Sioux Falls test network (Source: http://www.bgu.ac.il/~bargera/tntp/).

All results have been obtained assuming the number of iterations of Montecarlo algorithm equal to 60, and a threshold error = 0.01 for MSA-FA (with maximum number of iterations equal to 500).

Two main scenarios are considered:

A) all arcs are open to both vehicle types, no arc specific cost is considered;
B) arcs (10,16), (16,10), (10,17), (17,10), (16,17) and (17,16) are closed to TVs, thus corresponding arc specific costs are equal to infinity.

4.1. Scenario A) analysis

Table 2 shows convergence errors and MSA-FA iterations against TVs%-AVs% for the two choice models, Probit and Gammit. Convergence with an error of 0.010 or less is always reached, but one case where after 500 iterations the MSA-FA reached an error 0.016.

Figure 2 shows the percentage gain of total costs for TVs and AVs with respect to total cost with all TVs (1366 or 1312 Msec for Probit and Gammit, respectively) against AVs%, with quite similar results for both choice models. Percentage gain values are computed scaling by the percentage of vehicle types and by parameter χ_m for AVs to avoid any bias effect. As it may be expected introducing AVs benefits both vehicle types up to more than 25%, presumably due to reduction of congestion as modeled by parameter ν_m.

Graphs in Figure 2 (and in Figure 3 below) may be used to define the Minimum Penetration Rate (MPR) of AVs to guarantee a given percentage gain for TVs and AVs. For instance a 10% gain for both vehicle types is obtained with MPR = 30%, in this scenario.

4.2. Scenario B) analysis

Table 3 shows convergence errors and MSA iterations vs. TVs%-AVs% for the two choice models, Probit and Gammit. Convergence with an error of 0.010 or less is always reached, but three cases where after 500 iterations the MSA-Fa reached a maximum error equal to 0.016.

Figure 3 shows the percentage gain of total costs for TVs and AVs with respect to total cost with all TVs (2231 or 2189 Msec for Probit and Gammit, respectively) against AVs%, with very similar results for both choice models. As it may be expected, reserving some arcs to AVs only leads to greater benefit for them, still some benefits are observed for TVs too.

5. Conclusions

In this paper an approach to stochastic assignment with multi-vehicle types has been proposed, presenting models and algorithms for uncongested and congested networks. Vehicle types may be distinguished with respect to several parameters, to be calibrated against observed data, as well choice functions. Moreover, The proposed method allows us a consistent definition of AVs Minimum Penetration Rate for a given percentage gain.

Numerical applications to a reference network show that the proposed approach leads to a method effective for practical applications. Application refers to two vehicle types only, TVs and AVs, but the proposed method can be applied for any set of vehicle types. Probit- or Gammit- based stochastic assignment show similar results, thus Gammit should be preferred in this case since no user perceives a positive utility for any paths.

Several topics seem worth of further research effort: the effects of the number of Montecarlo iterations over convergence, the use of Sobol quasi-random numbers, the application of advanced MSA-based algorithms, as well as the discussion of real scale examples.

Table 2. scenario A: convergence vs. TVs % and AVs %

TVs%	AVs%	choice models	SUE iterations	SUN iterations	convergence error
100%	0%	PROBIT	155	9300	0,010
		GAMMIT	500	30000	0,016
90%	10%	PROBIT	126	7560	0,010
		GAMMIT	177	10620	0,010
70%	30%	PROBIT	210	12600	0,009
		GAMMIT	296	17760	0,010
50%	50%	PROBIT	107	6420	0,010
		GAMMIT	131	7860	0,008
30%	70%	PROBIT	121	7260	0,009
		GAMMIT	68	4080	0,009
10%	90%	PROBIT	167	10020	0,007
		GAMMIT	90	5400	0,009
0%	100%	PROBIT	221	13260	0,010
		GAMMIT	128	7680	0,008

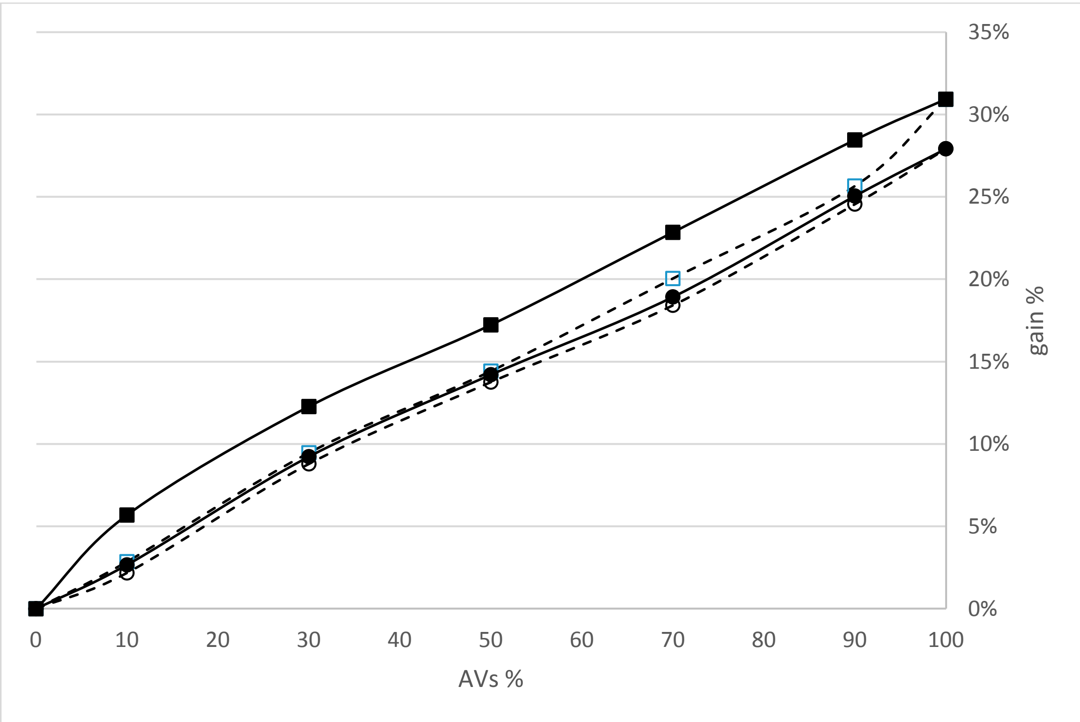

Figure 2. scenario A: total cost gain vs. Avs %-
Squares: Probit, Circles: Gammit; Empty with dashed line: TVs, Full with continuous line: AVs.

Table 3. scenario B: convergence vs. TVs % and AVs %

TVs%	AVs%	choice models	SUE iterations	SUN iterations	convergence error
100%	0%	PROBIT	375	22500	0,010
		GAMMIT	500	30000	0,016
90%	10%	PROBIT	500	30000	0,016
		GAMMIT	500	30000	0,014
70%	30%	PROBIT	116	6960	0,010
		GAMMIT	466	27960	0,009
50%	50%	PROBIT	94	5640	0,009
		GAMMIT	170	10200	0,009
30%	70%	PROBIT	144	8640	0,010
		GAMMIT	90	5400	0,010
10%	90%	PROBIT	154	9240	0,009
		GAMMIT	98	5880	0,009
0%	100%	PROBIT	197	11820	0,010
		GAMMIT	142	8520	0,009

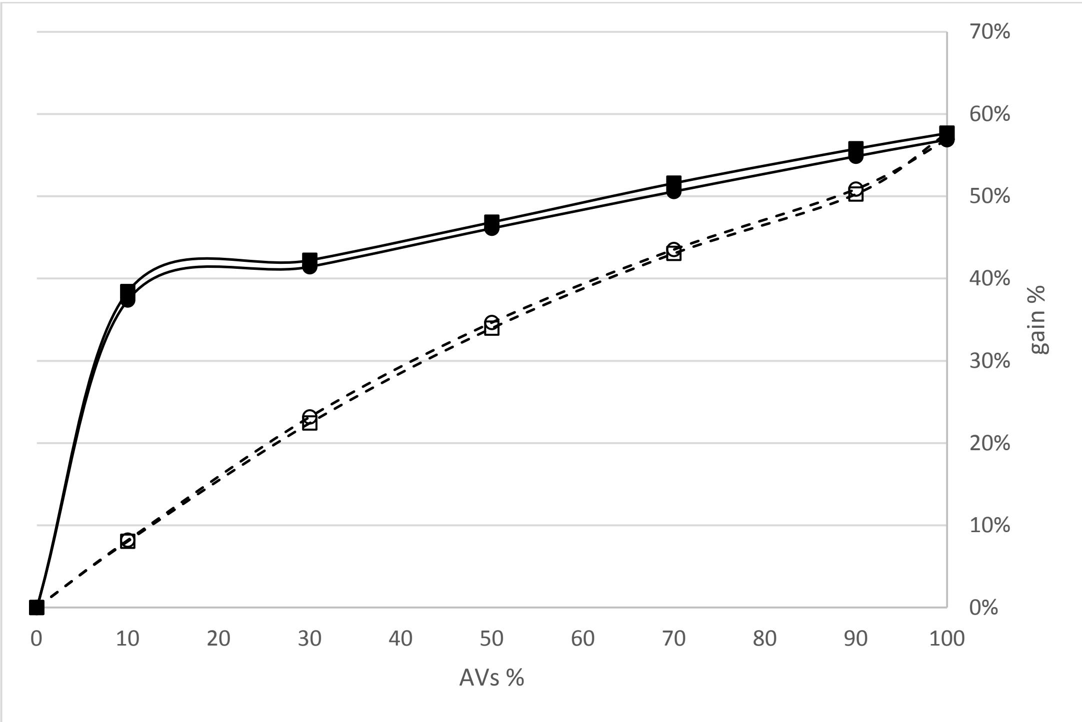

Figure 3. scenario B: total cost gain vs. Avs %-
Squares: Probit, Circles: Gammit; Empty with dashed line: TVs, Full with continuous line: AVs.

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