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# Automated Operational Modal Analysis of an arch bridge considering the influence of the parametric methods inputs

Gabriele Marrongelli<sup>a\*</sup>, Filipe Magalhães<sup>b</sup>, Álvaro Cunha<sup>b</sup>

<sup>a</sup> *Department of Architecture, Built environment and Construction Engineering (ABC), Politecnico di Milano, Piazza Leonardo da Vinci 32, 20133 Milan, Italy*

<sup>b</sup> *Construct – ViBest, Faculty of Engineering (FEUP), University of Porto, R. Dr. Roberto Frias, 4200-465 Porto, Portugal*

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## Abstract

This paper presents a strategy to efficiently obtain modal parameters estimates with parametric methods using a set of values for their input parameters. This is illustrated in detail with the application of the SSI-COV method, by the construction of tri-dimensional stabilization diagrams that consider varying modal orders and varying lengths for the correlation matrices, which are then automatically analyzed using clustering techniques. As case study, it is used an arch bridge built in 1940 with an 80 meter long non-reinforced concrete arch. The developed routines are applied to the datasets collected during a quite extensive ambient vibration test. The work also highlights how the natural frequencies and more particularly the modal damping values may depend on the choice of the parametric method input parameters.

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## 1. Introduction

Dynamic testing under operational conditions and automatic identification of modal parameters (i.e. natural frequencies, modal damping ratios and mode shapes) have steadily rising in popularity over recent decades. The increasing diffusion of long term dynamic monitoring systems for structural assessment as well as the success of different damage detection algorithms are driving the strong interest towards automated procedures of output-only

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\* Corresponding author.  
E-mail address: [gabriele.marrongelli@polimi.it](mailto:gabriele.marrongelli@polimi.it)

modal identification. Nowadays, different approaches of automated procedures apt to identify modal parameters in operational conditions have been developed, often based on the Stochastic Subspace Identification (SSI) method [1]. The main objective is to automatically estimate modal parameters using just the structural response measured under ambient excitation. The large attention currently received by SSI-methods probably depends on the fact that these procedures are apt to accurately identify weakly excited and closely space modes and are especially suited to be automated [2]. Presently, SSI procedures can be implemented in two classic forms [3]: covariance driven (SSI-Cov) and data driven (SSI-Data). Various strategies have been implemented for the SSI outputs interpretation, considering that two main parameters affect the results: a) the order of the model,  $n$ ; b) the number of output block rows used to build the Toeplitz block matrix (SSI-Cov),  $i$ , or the size of the block Hankel matrix (SSI-Data), as used in [4]. Furthermore, literature studies demonstrate that among modal parameters estimates the damping ratios are more sensitive to both the variation of  $n$  and of  $i$ . For this reason, when the identification of accurate modal estimates is important, the parametric methods should be applied simultaneously considering alternative values for  $i$  and  $n$ , as demonstrated for instance in [6] and [9].

The present work intends to provide an efficient approach (with few manual operations) based on a parametric method apt to perform the automatic identification of the modal parameters simultaneously considering a range of values for  $n$  and  $i$ . The simultaneous consideration of both variables in the construction of 3D stabilization diagrams is the most innovative aspect of the present work. The adopted procedure, which is herein discussed, is illustrated with the application on the Covariance driven Stochastic Subspace Identification (SSI-Cov) method, but it can be applied together with other parametric methods.

After a brief description of the selected case study, an arch bridge which crosses Tua River in the north of Portugal, the SSI-Cov method is concisely described. Then, its practical application based on the use of stabilization diagrams is detailed. Afterwards, the clustering technique used to group poles with physical meaning is presented. Finally, the results obtained in the selected case study are presented and discussed.

## 2. Brief description of the case study

The adopted case study is an arch bridge constructed in Portugal during the Second World War period. The bridge main structural element is a non-reinforced concrete arch with a span of 80 m and a rise until the crown of 20 m. This supports the deck through columns that are spaced 4.5 m apart (see Fig. 1). The deck is formed by 4 longitudinal beams with variable inertia that support the pavement slab.



Fig. 1– Picture of the bridge taken from upstream, with the indication of the sections instrumented during the AVT (reference section in black).

Due to the construction of a dam in the vicinity of the bridge, this was equipped with a dynamic monitoring system [6]. In order to support the design of the dynamic monitoring system, an ambient vibration test was performed covering 19 sections of the deck (see Fig. 1), with measurements at both edges (upstream and downstream) of the majority of the sections. In each instrumented point, 16 min long acceleration time series were collected using seismographs with tri-axial force-balance sensors. Two seismographs were placed in fixed positions (reference sensors), whereas, three other seismographs were moved from setup to setup to cover all the sections marked in Fig. 1 [6].

### 3. Background on covariance driven stochastic subspace identification (SSI-Cov)

The SSI procedures are particularly suitable for modal parameters estimation of bridges, footbridges and historical structures, as well as in the present case study: a non-reinforced arch bridge. This is based on the fitting of a state-space model to the experimental data. Assuming the excitation as a white noise, the stochastic state-space model, in its discrete form, is represented by the following equations:

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{A} \cdot \mathbf{x}_k + \mathbf{w}_k \\ \mathbf{y}_k &= \mathbf{C} \cdot \mathbf{x}_k + \mathbf{v}_k \end{aligned}$$

Where  $\mathbf{x}_k$  and  $\mathbf{y}_k$  are respectively the state vector and the vector containing the output measurements, at the time instant  $k$ .  $\mathbf{w}_k$  and  $\mathbf{v}_k$  are vectors that represent the noise due to modelling inaccuracies and the noise content of the measurements. Matrix  $\mathbf{A}$  is the system matrix that describes all the dynamic information of the system and  $\mathbf{C}$  is the corresponding output matrix. The goal of the SSI-Cov method is to obtain modal estimates starting from the covariance matrices of the measured structural response time series, which are organized in a Toeplitz matrix. The identification of the modal parameters through the matrices  $\mathbf{A}$  and  $\mathbf{C}$  is performed with a singular value decomposition (SVD) of the Toeplitz matrix and the resolution of a least-square equation (using the Moore-Penrose pseudo-inverse). Once the identification of the state space model is performed, the modal parameters are easily extracted from the matrices  $\mathbf{A}$  and  $\mathbf{C}$ .

The identification of the modal parameters using a SSI procedure requires the definition of the model order that better characterizes the dynamic behavior of the structure. For simple or academic experimental structures it is relatively easy to find it, but for real structures it is impossible to predict a priori the order of the model that better fits the experimental data. The best way to overcome this problem is to estimate the modal parameters using several models with different orders within a previously defined interval. However, the use of models with a high orders leads to the appearance of spurious modes associated to the noise content of measurements, which are not representative of the structural behavior. The separation of the physical modes from the spurious modes plays a crucial role in the dynamic identification. The most popular approach used to recognize and remove such modes is the creation of a *stabilization diagram* [3] based on the identified poles (each pole represents a modal estimation). The construction of the stabilization diagrams is based on the fact that poles associated with physical modes present consistent modal properties for several model orders, these are classified as stable poles. Their identification relies on the definition of limits for the variation of the modal parameters.

### 4. Automatic identification: proposed methodology

The stabilization diagram by itself does not solve the problem of identification of modal parameters because it is just a graphical tool that helps the user in the selection of the best model that represents all structural modes. For this reason, in the context of continuous dynamic monitoring or just for a more efficient and objective identification of the modal parameters without user interaction, after the elimination of the majority of spurious mode estimates, with a stabilization diagram, a further procedure able to group all the estimates related to the same physical mode needs to be applied. This can be accomplished in different ways [7], as for instance with hierarchical clustering tools, such as developed in [3], or with the construction of histograms, such as in [8].

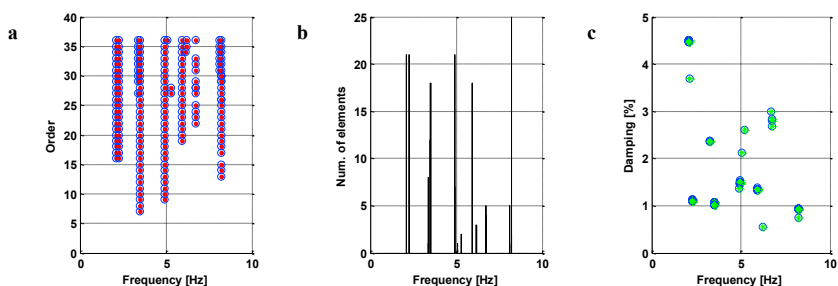


Fig. 2 – (a) stabilization diagram; (b) clustering results; (c) frequency VS damping – (Setup 1).

The methodology used in this work is based on the last approach, which produces a double set of histograms from the vertical alignments of stable poles on stabilization diagrams.

Fig. 2b represents the clusters obtained grouping all stable poles of the stabilization diagram shown in Fig. 2a. Each cluster is described by a vertical line that intercepts the x-axis at the mean natural frequency of all the mode estimates belonging to the cluster, and its height is equal to the number of elements inside the cluster. The clusters that stand out because of their larger size represent the poles which should have a physical meaning. The SSI method is recursively applied for increasing model order, the poles that do not respect the limits of the accepted variations previously defined are removed from the stabilization diagram. In fact, Fig. 2a shows just stable poles. Consequently, the poles which have similar natural frequencies are grouped. It may happen that poles representing the same mode are split in close consecutive clusters. But, the algorithm is capable to recognize this situation and to group poles belonging to consecutive clusters corresponding to the same mode (in the present application the imposed condition was a  $MAC > 0.95$ ). The algorithm includes the adoption of increasing values of  $i$ , producing three dimensional stabilization diagrams, as the one depicted in Fig. 3a, which are post-processed in the same way (Fig. 3b). Once the clustering procedure is executed for each dataset, the average results obtained by each analysis are stored separately. They are re-grouped together in order to get the average values in terms of natural frequencies, damping ratios, and reconstructing modal shapes only in the end, when all collected dataset have been processed.

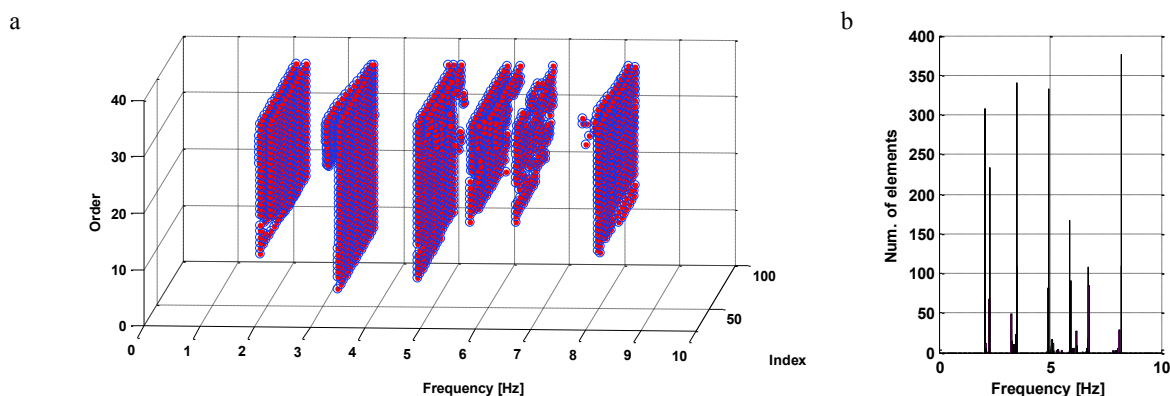


Fig. 3 – (a) 3D-stabilization diagram using different values of index “ $i$ ” (from 25 to 100); (b) clustering results.

## 5. Application of the proposed methodology

The data collected during all the setups performed during the ambient vibration test described in section 2 was processed with SSI-Cov algorithm. The methodology described in section 4 is applied in order to highlight the dependence between the modal estimates and the input parameters of the parametric method, in this case, the parameter  $i$ , used to define the number of block rows of the Toeplitz matrix. Normally, this parameter is tuned in order to obtain a good quality of the estimates, but the prediction of an accurate value of this parameter is still a challenging task. For each set of data associated with each experimental setup, one three-dimensional stabilization diagram was constructed and automatically interpreted. Fig. 3a shows the 3D-stabilization diagram associated with *Setup 3*. As clearly visible, the stable poles obtained are located in vertical “planes”, in which every pole respects the allowed variations. In the present application the following values were imposed: natural frequency variation  $< 1\%$ ; damping ratio variation  $< 5\%$  and MAC coefficient between two consecutive mode shapes  $> 0.98$ .

As shown in Fig. 3b, the post-processing of these 3D stabilization diagrams produces clusters with a high number of elements in correspondence to these “planes”, which represent the structural modes. Furthermore, several clusters containing very few elements, associated with spurious modes, are removed from the analyses. Once each setup has been analyzed, all poles belonging to those clusters in which mean natural frequency estimates do not differ more than 4% and MAC ratios are higher than 0.9, have been grouped into a final cluster that is representative of the same structural mode. In the adopted frequency range (0-10 Hz), four vertical bending modes and three lateral bending modes have been clearly detected, as depicted in Fig. 4 and Fig. 5, respectively.

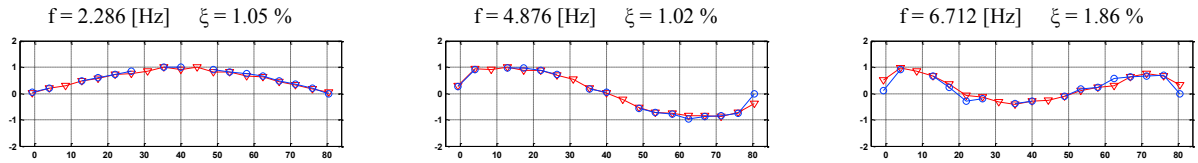


Fig. 4 – First three lateral bending modes (red: downstream, blue: upstream).

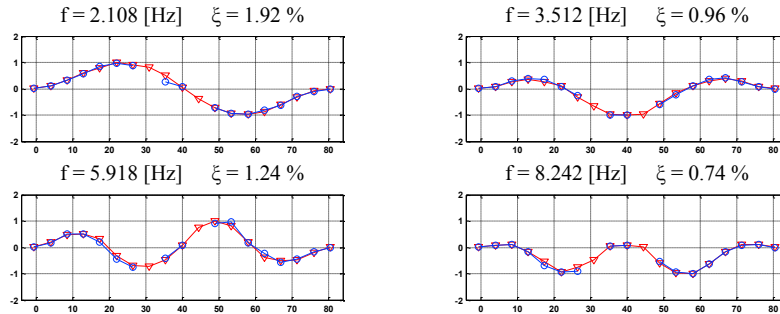


Fig. 5 – First four vertical bending modes (red: downstream, blue: upstream).

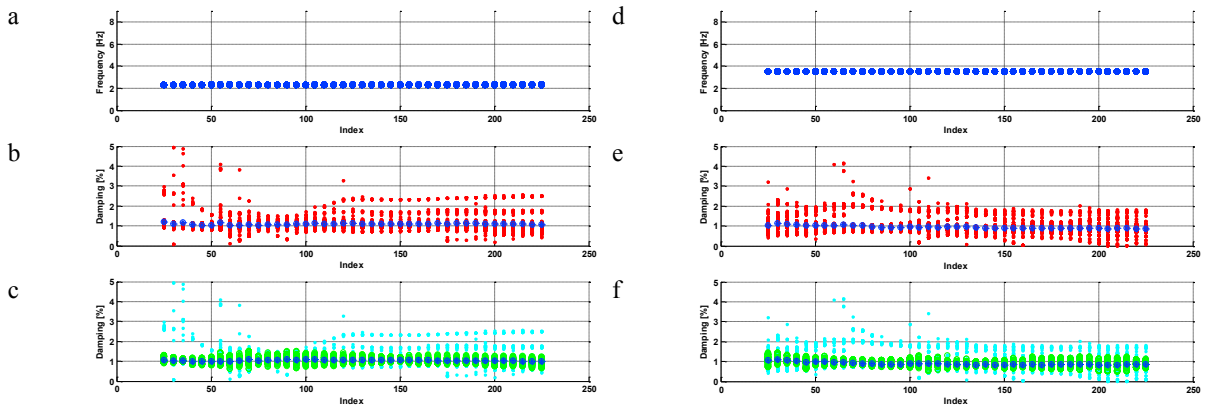


Fig. 6 – a) First lateral bending mode (LB1) b) Second vertical bending mode (VB2), Dependence on parameter  $i$  of natural frequency and modal damping ratio before and after application of the box-plot tool

Fig. 6 shows the most relevant results of the application of the proposed routines under another point of view. The graphics depict the modal parameters evolution with respect to the parameter  $i$  which varies from 25 to 225. As expected the variation of the natural frequency is negligible (Fig. 6a and d), on the other hand, the high scatter of the modal damping ratio (Fig. 6b and d) affects the mean values. In order to reduce its high variability, a control outlier procedure was applied adopting a simple statistical scheme for outlier detection based on the use of the box-plot rule. As a matter of fact, box-plot graphically depicts the results obtained using three quantities: lower quartile (Q1), median (Q2), upper quartile (Q3). In the procedure used in this “outliers check”, the lower and upper quartiles are estimate, and the interquartile range (IQR), which is defined by  $IQR = Q3 - Q1$ . Then, all poles that fall outside the two limits  $Q1 - (1.5 \times IQR)$  and  $Q3 + (1.5 \times IQR)$  are identified as outliers and removed, as shown in Fig. 6c-f. Indeed, the extreme values are removed in order to improve the mean modal damping values, thus obtaining more stable and accurate predictions of this parameter.

Table 1 synthesizes the most relevant results obtained by the application of the implemented routines in the analysis of data collected during the ambient vibration test in which seven modes were identified. Columns 2-3 present the mean values and the standard deviation of the natural frequencies. The low values of the standard deviation demonstrate the good accuracy of the estimates and very small influence of the increasing value of the

parameter  $i$  on this modal parameter. Columns 4-7 describe the mean value and relative standard deviation of the modal damping ratio before ( $\xi$ ) and after ( $\xi^*$ ) the application of outlier check, respectively. As evidenced in the fourth column, the high values of the standard deviation show the strong scatter of this modal parameter estimates, also very influenced by the parameter  $i$ . The results shown in the last column, demonstrate, as expected, that the dispersion obtained after removing the outliers is considerably reduced.

Table 1– Main results in terms of natural frequencies and modal damping ratios

Modes	$f_{\text{mean}}$ (Hz)	$f_{\text{std}}$ (Hz)	$\xi_{\text{mean}}$ (%)	$\xi_{\text{std}}$ (%)	$\xi_{\text{mean}}^*$ (%)	$\xi_{\text{std}}^*$ (%)
1 -- VB1	<b>2.108</b>	0.0429	<b>2.46</b>	0.8757	<b>1.92</b>	0.7554
2 -- LB1	<b>2.286</b>	0.0376	<b>1.15</b>	0.1221	<b>1.05</b>	0.0502
3-- VB2	<b>3.512</b>	0.0194	<b>0.98</b>	0.0305	<b>0.96</b>	0.0147
4 -- LB2	<b>4.876</b>	0.0374	<b>1.14</b>	0.0945	<b>1.02</b>	0.0864
5 -- VB3	<b>5.918</b>	0.0281	<b>1.29</b>	0.1829	<b>1.24</b>	0.0927
6 -- LB3	<b>6.712</b>	0.1849	<b>2.44</b>	0.6375	<b>1.86</b>	0.5206
7 -- VB4	<b>8.242</b>	0.0365	<b>0.81</b>	0.1153	<b>0.74</b>	0.0483

## Conclusions

This paper presents an application of an automated identification algorithm recently implemented to obtain modal estimations from acceleration time series collected during a multi-setup ambient vibration test. The proposed methodology uses the SSI-Cov method and a cluster procedure developed for the automatic analysis of three dimensional stabilization diagram that take into account variations in two input parameters of the identification method. This tool is based on the construction of a double set of histograms capable to group poles with similar characteristics in terms of natural frequencies and modes shapes. The obtained results demonstrate the good performance of the implemented processing routines and highlight the significant dependence of the modal damping ratio estimates on the input parameters. The proposed methodology can be applied with other parametric methods and also in the context of dynamic monitoring projects for the automatic processing of the continuously collected data.

## References

- [1] P. Van Overschee, B. De Moor, Subspace Identification for Linear System: Theory - Implementation - Applications, Conf. Proc. Int. Conf. IEEE Eng. Med. Biol. Soc., vol. 2008, 1996, pp. 4427–30.
- [2] F. Magalhães, Á. Cunha, E. Caetano, Online automatic identification of the modal parameters of a long span arch bridge, Mech. Syst. Signal Process., 23: 2 (2009) 316–329.
- [3] B. Peeters, System identification and damage detection in civil engineering. 2000.
- [4] F. Ubertini, C. Gentile, A. L. Materazzi, Automated modal identification in operational conditions and its application to bridges, Eng. Struct., 46 (2013) 264–278.
- [5] B. A. Pridham J. C. Wilson, A reassessment of dynamic characteristics of the Quincy Bayview Bridge using output-only identification techniques, Earthq. Eng. Struct. Dyn, 34:7 (2005) 787–805.
- [6] F. Magalhães, Á. Cunha, Dynamic testing and continuous monitoring of an arch bridge built in 1940, in 5th International Operational Modal Analysis Conference, IOMAC 2013, 2013.
- [7] A. Cabboi, F. Magalhães, C. Gentile, Á. Cunha, Automated modal identification and tracking: Application to an iron arch bridge, Structural Control and Health Monitoring, 2016.
- [8] M. Scionti, J. P. Lanslots, Stabilisation diagrams: Pole identification using fuzzy clustering techniques, Adv. Eng. Softw., 36:11 2005 768–779.
- [9] V. Zabel, F. Magalhães, C. Bucher, The influence of parameter choice in Operational Modal Analysis: A case study, IMAC in Mains M. (Eds) Topics in Modal Analysis & Testing, Vol. 10, Conference Proceedings of the Society for Experimental Mechanics Series. Springer, Cham, 2016.