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A generalized theoretical methodology to forecast flow coefficient, head coefficient and efficiency of Pumps-as-Turbines (PaTs)

Massimiliano Renzi^{a,*}, Mosè Rossi^a

^aFree University of Bozen-Bolzano, Faculty of Science and Technology, Piazza Università 1, Bolzano – 39100, Italy

Abstract

Among the most used hydraulic machines in the small-scale hydropower sector, Pump-as-Turbine (PaT) technology is suitable for both practical and economical aspects. These machines are already profitably applied in remote and rural zones for electricity production and in energy recovery applications in both civil and industrial plants, like Water Distribution Networks (WDNs) and chemical plants. Several studies aimed to provide theoretical formulas able to forecast flow rate and head at the Best Efficiency Point (BEP) in turbine mode obtaining, however, contrasting results and a lack of generalization. In this work, a generalized theoretical methodology for forecasting the flow rate, the head and the efficiency of PaTs at their BEP is studied. Specific correlations between the non-dimensional parameters of PaTs in pump and in turbine mode are presented and discussed. The accuracy of the presented methodology is compared to the ones available in literature showing a good generalization capability and a significant improvement in forecasting the behaviour of the PaT, starting from the available performance characteristics in pump mode.

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1. Introduction

Among renewable sources, water was one of the firsts to be used for electricity production. In developed countries, most of the available geodetic altitudes were almost exploited and, for this reason, the hydropower sector is moving from the large-scale production to the small-scale one [1]. Small-scale hydropower is mainly used for two purposes: the first one is to supply electricity, which is produced by using small water resources [2], to people that live in remote and rural zones, while the second one is to recover energy from different civil plants like Water

* Corresponding author.

E-mail address: Massimiliano.Renzi@unibz.it

Distribution Networks (WDNs) [3], wastewater plants [4] and other civil systems. Most of the used technologies in the small-scale hydropower sector derive from the ones used in the large-scale hydropower, even though they are rearranged taking into account different issues arising in the new installation sites. Among them, Pump-as-Turbine (PaT) technology is suitable for both power generation and energy recovery thanks to its favorable economic aspects [5, 6]. The most challenging part of studying PaTs is the evaluation of their BEP values together with off-design operating conditions when they are operating in turbine mode. Regarding the forecast of BEPs values, different researchers like Alatorre & Frenk [7], Schmiedl [8], and Grover [9] provided analytical formulas able to forecast BEPs values of flow rate, head and efficiency by knowing the ones in pump mode, which are typically easily available from the manufacturers of pumps. These results were derived by empirical correlations that were developed through laboratory tests. Regarding the forecast related to off-design operating conditions, a generalized trend has to be defined in order to obtain an objective and reliable prediction. The same authors of this paper [10] provided an analytical methodology able to forecast PaTs' off-design operating conditions in turbine mode regardless not only the geometry, kinematic and dynamic behaviour of the hydraulic machines (fluid dynamic similarity), but also the typology (normalization process). However, this method can be used only if BEP values (flow rate, head and efficiency) in turbine mode are already known. Another methodology, based on Artificial Neural Networks (ANNs), has been developed to forecast the PaT performance in design and off-design conditions, including the BEP, using, as input, the main operating data in pump mode [11]. To the authors' knowledge, two different theoretical methods were recently developed for forecasting flow rate and head of PaTs operating in turbine mode. Huang et al. [12] derived a methodology of rotor characteristic in turbine mode able to predict BEP values of flow rate and head in both pump and turbine mode: this method was subsequently verified by the means of experimental tests on three different pumps showing more precise results instead of using the most common formulas available in literature. On the other hand, Tan et al. [13] used experimental data of four different pumps, considering a wide range of both specific speed and diameters values, for developing a relationship between specific speed values in pump and in turbine mode, as well as specific diameter ones. The aim of this paper is to use experimental data of 32 PaTs collected by the same authors of this paper in a previous work [10] and other 27 experimental tests collected by Stefanizzi et al. [14] for improving the model proposed by Tan et al. [13]. The novelty of the work regards the development of correlations based on non-dimensional parameters that are able to forecast also the efficiency of PaTs in turbine mode, thus strengthening the forecasting capability of the proposed methodology. Finally, this model was validated with experimental data of a PaT that was not included in the development of the model: results were also compared with three other forecast's methodologies available in literature to assess the accuracy of the proposed model.

2. Research and methods

2.1. Data collection and elaboration

Data of PaTs' BEP values in both pump and turbine modes (32+27 PaTs) were collected from literature and elaborated, by applying the non-dimensional analysis, for evaluating the main non-dimensional parameters; in addition, also hydraulic efficiency, specific speed and specific diameter are considered. The ranges of the different non-dimensional parameters used in this study are quite wide in order to get generalized results: the flow coefficient (ϕ) ranges between 0.001 and 0.094, the head coefficient (ψ) ranges between 0.05 and 0.48, the efficiency (η) ranges between 0.35 and 0.89, the power coefficient (Λ) ranges between 0.0004 and 0.0086, the specific speed (N_s) ranges between 0.1 and 2.39 and, finally, the specific diameter (D_s) ranges between 1.68 and 16.53. A non-dimensional analysis is commonly used for extending the obtained results to other hydraulic machines that operate in the same fluid dynamic similarity conditions.

2.2. Formulas' rearrangement and theoretical approach

After all the aforementioned parameters were evaluated, specific speed values of PaTs in pump (p) and in turbine (t) mode, as well as the specific diameter ones, were correlated: a clear linear trend was observed all over the considered range for these two quantities. Therefore, knowing the data of N_{Sp} and D_{Sp} , both N_{St} and D_{St} can be easily forecasted

using two different linear equations. Both N_s and D_s are evaluated, using the flow coefficient (ϕ) and the head coefficient (ψ), by the means of equations (1) and (2), respectively.

$$N_s = \sqrt{\Phi} / \sqrt[4]{\Psi^3} \quad (1)$$

$$D_s = \sqrt[4]{\Psi} / \sqrt{\Phi} \quad (2)$$

Rearranging both equation (1) and equation (2), the head coefficient (ψ) in turbine mode (t), knowing both N_{St} and D_{St} , is evaluated as equation (3) shows:

$$\Psi_t = 1 / (N_{St} \cdot D_{St})^2 \quad (3)$$

Finally, the flow coefficient (ϕ) is evaluated by coupling equation (3) with equation (1) or equation (2), independently. After performing this method and transforming the non-dimensional parameters to the respective dimensional magnitudes, both flow rate and head at BEP in turbine mode are predicted. These two values are not enough to define the overall BEP of a PaT; indeed, the efficiency (η) forecast is still missing. Until now, most of the researchers that provided the theoretical formulas for forecasting the flow rate and the head at BEP in turbine mode used the same hydraulic efficiency (η) at BEP in pump mode also in in turbine mode, assuming that both of them are equal, independently of the operation mode. However, this assumption might be a limitation and lead to inaccurate results. To the authors' knowledge, only three authors supplied a method for its evaluation in turbine mode. By the means of the non-dimensional analysis, different attempts, using correlations related to two non-dimensional parameters, were performed for finding an equation with two variables able to forecast the efficiency (η) of the PaT. Among them, the best result, in terms of Mean Squared Error (MSE) minimization, was obtained by using the specific speed (N_s) and the efficiency (η) in pump mode as independent variables, as will be better described in the following section.

3. Results and comments

3.1. Correlations used for forecasting the PaT's performance

The main correlations used to forecast both N_{St} and D_{St} of PaTs operating in turbine mode are depicted in this paragraph. Data related to 32+27 PaTs were used for developing relations like $N_{St} = f(N_{Sp})$ and $D_{St} = f(D_{Sp})$. Using these formulas, both N_{St} and D_{St} of a PaT are predicted maintaining an acceptable accordance with the real values. Fig.1 shows the trend of N_{St} as a function of N_{Sp} ; the obtained R^2 -value is equal to 0.9488 and equation (4) describes the obtained trend.

$$N_{St} = 0.9051 \cdot N_{Sp} \quad (4)$$

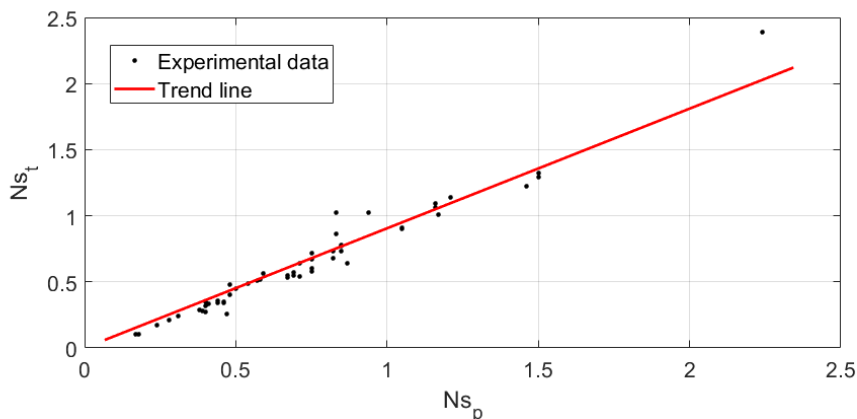


Fig. 1. Correlation of the specific speed of PaTs in pump (N_{Sp}) and in turbine (N_{St}) mode

As Fig.1 shows, the trend line, which is described by a linear equation, fits accurately the collected experimental data. Its R²-value is very high, so an increase of the equation’s order determines minimal improvements of the R²-value (+1.6%); therefore, it was decided to use a linear equation. Fig.2 shows the trend line of D_{St} as a function of D_{Sp} that is described by equation (5), whose R²-value is 0.986.

$$D_{St} = 0.9436 \cdot D_{Sp} \tag{5}$$

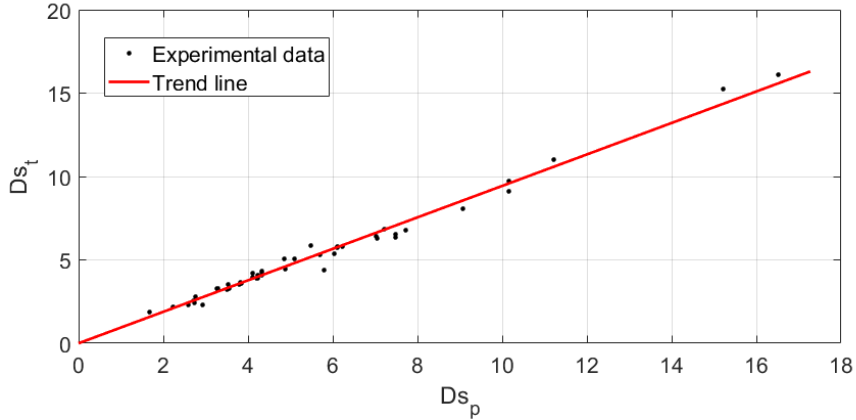


Fig. 2. Correlation of the specific diameter of PaTs in pump (D_{Sp}) and in turbine (D_{St}) mode

The red line in Fig.2 has the same trend recorded in Fig.1. Like in the previous case, the trend line is described by a linear equation. An increase of the equation’s order does not lead to a significant rise of the R²-value (+0.2%); thus, the linear trend is considered sufficiently reliable. It is worth to notice that, until now, the theoretical models available in literature forecast only the flow rate and the head of PaTs at BEP in turbine mode. However, these two physical magnitudes are not enough to characterize globally the BEP because the prediction of the PaTs’ efficiency (η) in turbine mode is still missing. As reported in Section 2, several combinations of non-dimensional parameters were evaluated to define a law able to forecast the efficiency (η) of PaTs in turbine mode. Among them, the best results were obtained by expressing η_t as a function of N_{Sp} and η_p that were used as independent variables. Fig.3 shows the trend of η_t as a function of N_{Sp} and η_p by means of a second order equation using a 3D plot. The R²-value corresponds to 0.7857 and equation (6) draws the obtained trend. Only the range of N_{Sp} is displayed in Fig.3 because, in this zone, there is the highest percentage of the experimental points available form literature.

$$\eta_t = 0.7933 \cdot N_{Sp} + 0.605 \cdot \eta_p - 0.09246 \cdot N_{Sp}^2 - 0.8254 \cdot (N_{Sp} \cdot \eta_p) + 0.3936 \cdot \eta_p^2 \tag{6}$$

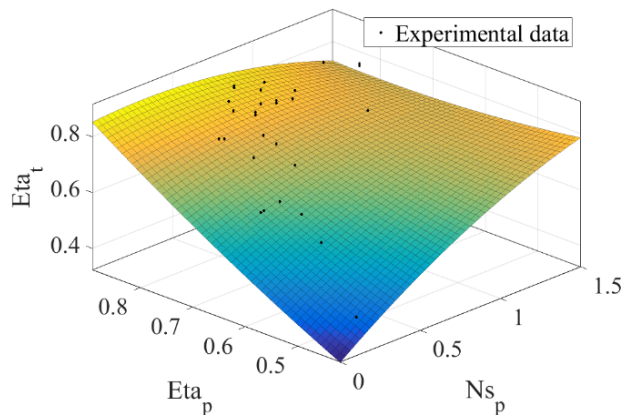


Fig. 3. 3D plot of η_t as a function of N_{Sp} and η_p

Analysing the results obtained by the correlation expressed in equation (6), the obtained R^2 -value is considerably lower than the other two recorded by the means of equation (4) and equation (5), leading to a lower reliability of the forecast. However, as the hydraulic efficiency is linked not only to the operating principles of the machine but also to its specific design and typology, the result is considered satisfactory. Certainly, an increase of the reliability of the overall model can be performed by increasing the number of experimental data considered in the model.

3.2. Validation of the model

The experimental data of a PaT, which were not used in the development of the model and were collected by the same authors of this work [3], are used for validating the proposed model. Both real and forecasted BEP values related to the tested hydraulic machine are compared with other methodologies developed by other researchers. The obtained results are listed in Table 1. Table 2 shows the comparison of the experimental values with the forecasted ones obtained by using three other methodologies that are present in literature. It is worth to notice that these three selected methodologies provide also a correlation to forecast the PaT's hydraulic efficiency in turbine mode, allowing to check the reliability of the new proposed formula and to better assess the performance of the machine in turbine mode.

Table 1. Comparison between real and forecasted BEP values considering a tested PaT [3]

NON-DIMENSIONAL COEFFICIENT \ PAT	TESTED PAT IN TURBINE MODE [3]		
	REAL VALUE	FORCEASTED VALUE	ERRORS (%)
Flow coefficient (ϕ)	0.0197	0.0167	-15.23
Head coefficient (ψ)	0.1713	0.1565	-8.64
Efficiency (η)	0.76	0.75	-1.32
Power coefficient (Λ)	0.0026	0.0020	-23.08
Specific speed (N_s)	0.53	0.52	-1.90
Specific diameter (D_s)	4.58	4.87	+6.33

Table 2. Comparison of the accuracy of the models for the PaT's BEP prediction

MODEL	ERRORS (%) AT BEP					
	ϕ	ψ	η	Λ	N_s	D_s
Alatorre & Frenk [8]	-43.78	+11.01	-3.95	-37.59	-30.67	+36.90
Schmiedl [9]	+10.53	+25.96	+3.65	+39.22	-11.58	+0.77
Grover [10]	+57.69	+78.74	-13.12	+181.85	-18.77	-7.92

Results show that the best forecast was obtained on the evaluation of the efficiency (η), while the worst one was obtained on the power coefficient (Λ). The power coefficient (Λ) shows the worst prediction because it is calculated by using equation (7); so, the error propagation contributes to increase the difference between real and predicted values.

$$\Lambda = \eta \cdot \Phi \cdot \Psi \quad (7)$$

However, both specific speed (N_s) and specific diameter (D_s) are forecasted correctly with errors lower than 10%. Compared to the other three methodologies listed in Table 2, the proposed methodology showed better results in terms of ψ , η , Λ and N_s evaluation, while the Schmiedl model presented better results in terms of both Φ and D_s . However, the proposed model showed a better efficiency's forecast, meaning that it is more reliable than the others regarding the forecast of this parameter. Certainly, an increase of PaTs' experimental data would lead to an increase of the reliability of the model itself and, subsequently, to forecasted values closer to the real ones. In addition, an additional validation of the model with other experimental data should be carried out to further confirm these encouraging results.

4. Conclusions

In this paper, a theoretical model used for forecasting BEP values of PaTs was developed by using 32+27 BEP experimental tests available in literature. Non-dimensional analysis was performed for carrying on a general and an objective study of these hydraulic machines. The forecast of both flow coefficient (ϕ) and head coefficient (ψ) was executed by linking PaTs' performance in pump mode with the ones in turbine mode for what concerns the specific speed (N_s) and the specific diameter (D_s), obtaining R^2 -values equal to 0.9488 and 0.986, respectively. Moreover, a novel approach was proposed for the prediction of the hydraulic efficiency (η) at the BEP in turbine mode to supply a complete definition of the PaT's performance: this parameter is typically considered equal to the one in pump mode, while, to the authors' knowledge, only three methodologies supply also an equation for its evaluation in turbine mode. Using both non-dimensional coefficients and magnitudes, an equation for describing η_t as a function of both N_{sp} and η_p was obtained. Finally, a comparison between real and forecasted BEP values, taking into account a tested PaT, was carried out showing that the best forecast was obtained for the evaluation of the hydraulic efficiency (η) (-1.32%), while the worst was obtained for the evaluation of the power coefficient (Λ) (-23.08%). The proposed model was compared with three methodologies that allow to evaluate also the hydraulic efficiency of the PaT: results showed that the proposed methodology led to better results in terms of ψ , η , Λ and N_s evaluation, while slightly worse results are obtained in terms of both Φ and D_s evaluation if compared to Schmiedl's model. However, it showed a better forecast of the hydraulic efficiency evaluation, meaning that using the combination of two independent variables like N_{sp} and η_p lead to a good forecast of the turbine operation mode. The encouraging results obtained by using this model suggested the authors to improve the forecast capability by increasing the number of PaTs' experimental data operating in turbine mode. Moreover, an extension of these data related to the specific speed (N_s) range is also considered for obtaining a more generalized method that takes into account different PaTs' typologies.

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