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Some results concerning n-σ-centralizing mappings in semiprime rings

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Abstract Let $n \geq 1$ be a fixed integer. Let R be a semiprime ring and S an additive subgroup of R, σ, τ two endomorphisms of R and $F : R \to R$ an additive mapping of R. In the present paper, we prove that

(1) if R is $(n+1)!$-torsion free, S is $(n+1)$-power closed and $[F(x), \sigma(x)^n] \in Z(R)$ for all $x \in S$, then $[F(x), \sigma(x)^n] = 0$ for all $x \in S$;
(2) if R is 3!-torsion free, S is square closed and $[[F(x), \sigma(x)], \sigma(x)] \in Z(R)$ for all $x \in S$, then $[[F(x), \sigma(x)], \sigma(x)] = 0$ for all $x \in S$.

We also consider a number of applications in semiprime rings with derivations, (σ, τ)-derivations and generalized derivations, and extend some known results in the literature.

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الملخص

ليكن $n \geq 1$ عدداً صحيحاً ثابتاً. لتكن R حلقة نصف أولية وَ S زمرة جزئية جمعية من R، وَ σ, τ تشاكلين داخليين لِـ R وَ $F: R \to R$ راسماً جمعياً لِـ R. في هذه الورقة نثبت أنه

(1) إذا كانت R بدون التواء- $(n+1)!$، وَ S مغلقة تحت قوى-$(n+1)$، وَ $[F(x), \sigma(x)^n] \in Z(R)$ لجميع قيم $x \in S$، فإن $[F(x), \sigma(x)^n] = 0$ لجميع قيم $x \in S$.
(2) إذا كانت R بدون التواء- $3!$، وَ S مغلقة تحت التربيع، وَ $[[F(x), \sigma(x)], \sigma(x)] \in Z(R)$ لجميع قيم $x \in S$، فإن $[[F(x), \sigma(x)], \sigma(x)] = 0$ لجميع قيم $x \in S$.

نعتبر أيضاً عدداً من التطبيقات للحلقات نصف الأولية مع اشتقاقات، وَ اشتقاقات-(σ, τ) وَ اشتقاقات مُعَمَّمَة، ونمدد بعض النتائج المعروفة مسبقاً.

1 Introduction

Let R be an associative ring. Let n be a fixed positive integer. A ring R is said to be n-torsion free if, for $x \in R$, $nx = 0$ implies $x = 0$. For $x, y \in R$, the commutator of x, y is denoted by the symbol $[x, y]$ and is defined by $[x, y] = xy - yx$.

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Recall that R is prime if $aRb = 0$ implies either $a = 0$ or $b = 0$, and is semiprime if $aRa = 0$ implies $a = 0$. An additive mapping $D : R \to R$ is called a derivation, if $D(xy) = D(x)y + xD(y)$ holds for all $x, y \in R$. An additive mapping $F : R \to R$ is called a generalized derivation if there exists a derivation $D : R \to R$ such that $F(xy) = F(x)y + xD(y)$ holds for all $x, y \in R$. By this definition, every derivation is a generalized derivation of R, but the converse is not true in general.

Let S be a nonempty subset of R and n a positive integer. S is said to be n-power closed, if $x^n \in S$ for all $x \in S$. A mapping f from R to R is called n-centralizing (resp., n-commuting) on S, if $[f(x), x^n] \in Z(R)$ for all $x \in S$ (resp., $[f(x), x^n] = 0$ for all $x \in S$). Many authors have studied the commuting and centralizing mappings in prime and semiprime rings. This work was initiated by Posner [16] who proved that a prime ring R admitting a nonzero centralizing derivation is commutative. Mayne [15] proved the analogous result for centralizing automorphisms. Since then a number of authors have extended these results of Posner and Mayne in several directions (see [1,2,4–8,11,12,14,18,19]). In these papers, the maps considered are derivations, endomorphisms, generalized derivations or any arbitrary additive maps in prime or semiprime rings. In [6], Brešar gave the complete structure of additive commuting maps on prime rings. More precisely, Brešar proved the following two striking results:

Theorem 1.1 [6, Proposition 3.1] *Let R be a 2-torsion free semiprime ring and U be a Jordan subring of R. If an additive mapping F of R into itself is centralizing on U, then F is commuting on U.*

Theorem 1.2 [6, Theorem 3.2] *Let R be a prime ring. If an additive mapping F of R is commuting on R, then there exists $\lambda \in C$ and an additive mapping $\xi : R \to C$, such that $F(x) = \lambda x + \xi(x)$ for all $x \in R$.*

Moreover, Bell & Martindale [3] and Deng & Bell [7] studied the centralizing and n-centralizing derivations on some subsets of semiprime rings.

In [7], Deng and Bell proved the following theorems:

Theorem 1.3 *Let n be a fixed positive integer, let R be an $n!$-torsion free semiprime ring and I a nonzero left ideal of R. If R admits a nonzero derivation D such that $D(I) \neq 0$ and n-centralizing on I, then R contains a nonzero central ideal.*

Theorem 1.4 *Let R be a 6-torsion free semiprime ring and I a nonzero left ideal of R. If R admits a nonzero derivation D such that $D(I) \neq 0$ and the map $x \mapsto [D(x), x]$ is centralizing on I, then R contains a nonzero central ideal.*

Recently, Dhara and Ali [9] have studied these results replacing derivation D with a generalized derivation F of R.

The notion of n-commuting and n-centralizing maps is extended to n-σ-commuting and n-σ-centralizing maps. Let S be a nonempty subset of R, n a positive integer and σ an endomorphism of R. The mapping $f : R \to R$ is said to be n-σ-commuting (n-σ-centralizing) on S if $[f(x), \sigma(x)^n] = 0$ for all $x \in S$ (resp., $[f(x), \sigma(x)^n] \in Z(R)$ for all $x \in S$).

For convenience, we shall write 1-σ-commuting and 1-σ-centralizing maps as σ-commuting and σ-centralizing maps, respectively.

In [13], Lee studied the σ-commuting maps in semiprime rings and determine the complete structure of σ-commuting maps. In the present article, we study the n-σ-centralizing maps and we show in $(n+1)!$-torsion free semiprime rings R that any n-σ-centralizing additive map is n-σ-commuting in an $(n+1)$-power closed additive subgroup of R. This result can be applied to extend some recent results related to derivations and generalized derivations [1,10,17] to the central case.

2 Main Results

We begin with the following theorem.

Theorem 2.1 *Let $n \geq 1$ be a fixed integer. Let R be an $(n+1)!$-torsion free semiprime ring, σ an endomorphism of R and S an $(n+1)$-power closed additive subgroup of R. If $F : R \to R$ is an additive mapping such that $[F(x), \sigma(x)^n] \in Z(R)$ for all $x \in S$, then $[F(x), \sigma(x)^n] = 0$ for all $x \in S$.*

Proof Let $x \in S$ and $t = [F(x), \sigma(x)^n]$. Then $t \in Z(R)$. Linearizing our hypothesis $[F(x), \sigma(x)^n] \in Z(R)$, we get

$$[F(y), \sigma(x)^n] + [F(x), \sigma(x)^{n-1}\sigma(y) + \sigma(x)^{n-2}\sigma(y)\sigma(x) + \cdots + \sigma(y)\sigma(x)^{n-1}] \in (R)$$

for all $x, y \in S$. Putting $y = x^{n+1}$, the above relation yields

$$[F(x^{n+1}), \sigma(x)^n] + [F(x), \sigma(x)^{2n} + \sigma(x)^{2n} + \cdots + \sigma(x)^{2n}] \in Z(R),$$

that is,

$$[F(x^{n+1}), \sigma(x)^n] + 2nt\sigma(x)^n \in Z(R). \tag{1}$$

Let $z = [F(x^{n+1}), \sigma(x)^n] + 2nt\sigma(x)^n$. Since $x^{n+1} \in S$, we replace x with x^{n+1} in our assumption and get $[F(x^{n+1}), \sigma(x)^{(n+1)n}] \in Z(R)$. Now we compute $[F(x^{n+1}), \sigma(x)^{(n+1)n}] = \sum_{i=0}^{n} \sigma(x)^{ni}[F(x^{n+1}), \sigma(x)^n]$ $\sigma(x)^{n(n-i)}$. Since $[F(x^{n+1}), \sigma(x)^n] = z - 2nt\sigma(x)^n$, we have that $[F(x^{n+1}), \sigma(x)^{(n+1)n}] = \sum_{i=0}^{n} \sigma(x)^{ni}(z - 2nt\sigma(x)^n)\sigma(x)^{n(n-i)} = \sum_{i=0}^{n}(z\sigma(x)^{n^2} - 2nt\sigma(x)^{n+n^2}) = (n+1)(z\sigma(x)^{n^2} - 2nt\sigma(x)^{n+n^2}) \in Z(R)$. But R is $(n+1)$-torsion free, so that

$$z\sigma(x)^{n^2} - 2nt\sigma(x)^{n+n^2} \in Z(R). \tag{2}$$

Now commuting $\sigma(x)^{kn}$ with $F(x)$ successively, we get

$$[F(x), \sigma(x)^{kn}] = [F(x), \underbrace{\sigma(x)^n.\sigma(x)^n.\ldots.\sigma(x)^n}_{k \text{ times}}] = kt\sigma(x)^{(k-1)n}$$

and

$$[F(x), [F(x), \sigma(x)^{kn}]] = kt[F(x), \sigma(x)^{(k-1)n}] = k(k-1)t^2\sigma(x)^{(k-2)n}$$
$$= \frac{k!}{(k-2)!}t^2\sigma(x)^{(k-2)n}.$$

Thus commuting $\sigma(x)^{kn}$ with $F(x)$ successively m-times yields

$$[F(x), \ldots, [F(x), \sigma(x)^{kn}]] = \frac{k!}{(k-m)!}t^m\sigma(x)^{(k-m)n}.$$

Using this fact, we can write, successively commuting both sides of (2) n-times with $F(x)$, that

$$(n!)zt^n - 2n(n!)t.t^n\sigma(x)^n = 0.$$

Again, commuting with $F(x)$, we have

$$-2n(n)!t^{n+2} = 0.$$

As the R is $(n+1)!$-torsion free, $t^{n+2} = 0$. Since center of semiprime ring contains no nonzero nilpotent elements, we have $t = 0$, as desired. □

Theorem 2.2 *Let R be a* 3!*-torsion free semiprime ring, σ an endomorphism of R, S an additive subgroup of R such that $u^2 \in S$ for all $u \in S$ and $F : R \to R$ an additive mapping. If the map $x \mapsto [F(x), \sigma(x)]$ is σ-centralizing on S, then this map is σ-commuting on S.*

Proof Let $x \in S$ and $t = [[F(x), \sigma(x)], \sigma(x)]$. Then $t \in Z(R)$. Since R is 2-torsion free, linearizing our hypothesis we obtain

$$[[F(y), \sigma(x)], \sigma(x)] + [[F(x), \sigma(y)], \sigma(x)] + [[F(x), \sigma(x)], \sigma(y)] \in Z(R)$$

for all $x, y \in S$. Replacing x^2 with y in the above relation, we get

$$[[F(x^2), \sigma(x)], \sigma(x)] + [[F(x), \sigma(x)^2], \sigma(x)] + [[F(x), \sigma(x)], \sigma(x)^2] \in Z(R).$$

But $[[F(x), \sigma(x)^2], \sigma(x)] = [[F(x), \sigma(x)], \sigma(x)^2] = 2t\sigma(x)$, so that the last relation reduces to

$$[[F(x^2), \sigma(x)], \sigma(x)] + 4t\sigma(x) \in Z(R).$$

Set $z = [[F(x^2), \sigma(x)], \sigma(x)] + 4t\sigma(x) \in Z(R)$. By our hypothesis, we can write $[[F(x^2), \sigma(x)^2], \sigma(x)^2] \in Z(R)$ for all $x \in S$. This yields

$$[[F(x^2), \sigma(x)^2], \sigma(x)^2] = [[F(x^2), \sigma(x)], \sigma(x)]\sigma(x)^2 + 2\sigma(x)[[F(x^2), \sigma(x)], \sigma(x)]\sigma(x) + \sigma(x)^2[[F(x^2), \sigma(x)], \sigma(x)].$$

Since $[[F(x^2), \sigma(x)], \sigma(x)] = z - 4t\sigma(x)$, we have from above that $[[F(x^2), \sigma(x)^2], \sigma(x)^2] = (z - 4t\sigma(x))\sigma(x)^2 + 2\sigma(x)(z - 4t\sigma(x))\sigma(x) + \sigma(x)^2(z - 4t\sigma(x)) = -16t\sigma(x)^3 + 4z\sigma(x)^2 \in Z(R)$. This implies $[[F(x), \sigma(x)], -16t\sigma(x)^3 + 4z\sigma(x)^2] = 0$. Now using the fact that $[[F(x), \sigma(x)], \sigma(x)^k] = kt\sigma(x)^{k-1}$, where $k \geq 1$ any integer, we get $-48t^2\sigma(x)^2 + 8zt\sigma(x) = 0$. Again this implies $[[F(x), \sigma(x)], -48t^2\sigma(x)^2 + 8zt\sigma(x)] = 0$. This gives $-96t^3\sigma(x) + 8zt^2 = 0$. Thus we have $0 = [[F(x), \sigma(x)], -96t^3\sigma(x) + 8zt^2] = -96t^3[[F(x), \sigma(x)], \sigma(x)] = -96t^4$. Since R is 3!-torsion free, we have $t^4 = 0$. As the center of semiprime ring contains no nonzero nilpotent elements, we conclude $t = 0$. This completes the proof. □

3 Application to generalized (σ, τ)-derivation

Let σ and τ be two endomorphisms of R. By a (σ, τ)-derivation D, we mean an additive mapping $D : R \to R$ satisfying the condition $D(xy) = D(x)\sigma(y) + \tau(x)D(y)$ for all $x, y \in R$. An additive mapping $G : R \to R$ is said to be a generalized (σ, τ)-derivation if there exists a (σ, τ)-derivation D such that $G(xy) = G(x)\sigma(y) + \tau(x)D(y)$ holds for all $x, y \in R$.

Recently in [1], Ali and Chaudhry proved that if R is a semiprime ring and G a generalized (σ, τ)-derivation of R associated with the (σ, τ)-derivation D of R, such that $[G(x), \sigma(x)] = 0$ for all $x \in R$, then $D(R)[R, R] = 0$ and $D(R) \subseteq Z(R)$, where σ and τ are two automorphisms of R.

Using Theorem 2.1, the above result is extended to central case. Moreover, the situation studied is when σ and τ are two epimorphisms of R.

Theorem 3.1 *Let R be a 2-torsion free semiprime ring, σ and τ be two epimorphisms of R. Suppose that G is a generalized (σ, τ)-derivation of R associated with the (σ, τ)-derivation D of R. If $[G(x), \sigma(x)] \in Z(R)$ for all $x \in R$, then $D(R)$ is contained in a central ideal of R.*

Proof By Theorem 2.1, we have that G is σ-commuting on R, that is,

$$[G(x), \sigma(x)] = 0 \tag{3}$$

for all $x \in R$. By linearizing, above relation gives

$$[G(y), \sigma(x)] + [G(x), \sigma(y)] = 0 \tag{4}$$

for all $x, y \in R$. Replacing yx for y in (4), we get

$$[G(y)\sigma(x) + \tau(y)D(x), \sigma(x)] + [G(x), \sigma(y)\sigma(x)] = 0 \tag{5}$$

for all $x, y \in R$ which implies

$$[G(y), \sigma(x)]\sigma(x) + [\tau(y)D(x), \sigma(x)] + [G(x), \sigma(y)]\sigma(x) + \sigma(y)[G(x), \sigma(x)] = 0 \tag{6}$$

for all $x, y \in R$. Using (3) and (4), from above we get

$$[\tau(y)D(x), \sigma(x)] = 0$$

for all $x, y \in R$. Since τ is an epimorphisms of R, we have $[RD(x), \sigma(x)] = 0$ for all $x \in R$. This implies $0 = [R^2D(x), \sigma(x)] = R[RD(x), \sigma(x)] + [R, \sigma(x)]RD(x) = [R, \sigma(x)]RD(x)$, again implying $[R, \sigma(x)]R[D(x), \sigma(x)] = 0$ for all $x \in R$. In particular $[D(x), \sigma(x)]R[D(x), \sigma(x)] = 0$ for all $x \in R$. Since R is semiprime ring, $[D(x), \sigma(x)] = 0$ for all $x \in R$. Then by [13, Corollary 2], we conclude that $D(R)$ is contained in a central ideal of R. □

Corollary 3.2 *Let R be a 2-torsion free prime ring, σ and τ be two epimorphisms of R. Suppose that G is a generalized (σ, τ)-derivation of R associated with the nonzero (σ, τ)-derivation D of R. If $[G(x), \sigma(x)] \in Z(R)$ for all $x \in R$, then R is commutative.*

4 Application to pair of derivations

In a recent paper [10], Fosner and Vukman proved the following: If R is a 2-torsion free semiprime ring and $f : R \to R$ an additive mapping satisfying $[f(x), x^2] = 0$ for all $x \in R$, then $[f(x), x] = 0$ for all $x \in R$. As an application of this result, they proved that if $[D^2(x) + G(x), x^2] = 0$ for all $x \in R$, where $D, G : R \to R$ are two derivations, then D and G both maps R into its center.

Now we apply Theorem 2.1 to extend these results of [10] to central case.

Theorem 4.1 *If R is a 3!-torsion free semiprime ring and $f : R \to R$ an additive mapping satisfying $[f(x), x^2] \in Z(R)$ for all $x \in R$, then $[f(x), x] = 0$ for all $x \in R$.*

Proof By Theorem 2.1, since R is a 3!-torsion free semiprime ring, $[f(x), x^2] \in Z(R)$ for all $x \in R$ implies $[f(x), x^2] = 0$ for all $x \in R$. Then by [10, Theorem 2], $[f(x), x] = 0$ for all $x \in R$. □

We generalize the second results of derivations as follows:

Theorem 4.2 *Let R be an $n!$-torsion free semiprime ring, I an ideal of R and $D, G : R \to R$ two derivations such that $D(I) \neq 0$ and $G(I) \neq 0$. If $[D^2(x)+G(x), x^n] = 0$ for all $x \in I$, then $D(I)$ and $G(I)$ are contained in nonzero central ideals of R.*

Proof Linearizing the given identity, we get

$$[D^2(y) + G(y), x^n] + [D^2(x) + G(x), yx^{n-1} + \cdots + x^{n-1}y] = 0 \tag{7}$$

for all $x, y \in I$. Replacing y with yx, we get

$$[(D^2(y) + G(y))x + 2D(y)D(x) + yD^2(x) + yG(x), x^n] \\ + [D^2(x) + G(x), (yx^{n-1} + \cdots + x^{n-1}y)x] = 0,$$

that is,

$$[D^2(y) + G(y), x^n]x + 2[D(y)D(x), x^n] + [y(D^2(x) + G(x)), x^n] \\ + [D^2(x) + G(x), (yx^{n-1} + \cdots + x^{n-1}y)]x + (yx^{n-1} + \cdots + x^{n-1}y)[D^2(x) + G(x), x] = 0.$$

As $[D^2(x) + G(x), x^n] = 0$ for all $x \in I$ from (7), we get that

$$2[D(y)D(x), x^n] + [y, x^n](D^2(x) + G(x)) + (yx^{n-1} + \cdots + x^{n-1}y)[D^2(x) + G(x), x] = 0. \tag{8}$$

Now, putting $y = xy$ in (8), we have

$$2[(D(x)y + xD(y))D(x), x^n] + x[y, x^n](D^2(x) + G(x)) \\ +x(yx^{n-1} + \cdots + x^{n-1}y)[D^2(x) + G(x), x] = 0. \tag{9}$$

Left multiplying (8) by x and subtracting from (9), we obtain that

$$2[D(x)yD(x), x^n] = 0$$

for all $x, y \in I$. Since R is 2-torsion free, from above relation we have

$$D(x)yD(x)x^n - x^n D(x)yD(x) = 0 \tag{10}$$

for all $x \in I$. Replacing y with $yD(x)z$, we get

$$D(x)yD(x)zD(x)x^n - x^n D(x)yD(x)zD(x) = 0. \tag{11}$$

By (10), this can be written as

$$D(x)yx^n D(x)zD(x) - D(x)yD(x)x^n zD(x) = 0,$$

which is

$$D(x)y[D(x), x^n]zD(x) = 0$$

for all $x \in I$. This implies

$$[D(x), x^n]y[D(x), x^n]z[D(x), x^n] = 0.$$

That is $([D(x), x^n]I)^3 = 0$. Since R is semiprime ring, $[D(x), x^n]I = 0$. Thus $[D(x), x^n] \subseteq I \cap ann(I) = 0$. Then by [11], $D(I)$ is contained in a nonzero central ideal of R. Thus $D(I) \subseteq Z(R)$ and so $D^2(I) \subseteq Z(R)$. Therefore, our hypothesis gives

$$[G(x), x^n] = 0$$

for all $x \in I$. By same argument, $G(I)$ is contained in a nonzero central ideal of R. □

Now, applying Theorem 2.1, we have the following:

Theorem 4.3 *Let R be an $(n+1)!$-torsion free semiprime ring, I an ideal of R and $D, G : R \to R$ two derivations such that $D(I) \neq 0$ and $G(I) \neq 0$. If $[D^2(x) + G(x), x^n] \in Z(R)$ for all $x \in I$, then D and G both are contained in nonzero central ideals of R.*

Corollary 4.4 *Let R be an $(n+1)!$-torsion free prime ring, I an ideal of R and $D, G : R \to R$ two derivations. If $[D^2(x) + G(x), x^n] \in Z(R)$ for all $x \in I$, then either $D = G = 0$ or R is commutative.*

5 Application to pair of generalized derivations

In a recent paper [17], Rehman and De Filippis proved the following:

Theorem 5.1 *Let n be a fixed positive integer, and let R be a semiprime $n!$-torsion free ring. If R admits generalized derivations F and G associated with nonzero derivations f and g, respectively, such that $[F^2(x)+G(x), x^n] = 0$ for all $x \in R$, then one of the following holds:*

(1) *R contains a nonzero central ideal;*
(2) *$f = 0$, $g(R) \subseteq Z(R)$, and there exist $a, b \in U$ such that $F(x) = ax$, $G(x) = bx + g(x)$ for all $x \in R$, and $a^2 + b \in C$, where C is the extended centroid of R.*

Now, applying Theorem 2.1, we can state the result for the central case as follows:

Theorem 5.2 *Let n be a fixed positive integer, and let R be a semiprime and $(n+1)!$-torsion free ring. If R admits generalized derivations F and G associated with nonzero derivations f and g respectively, such that $[F^2(x) + G(x), x^n] \in Z(R)$ for all $x \in R$, then one of the following holds:*

(1) *R contains a nonzero central ideal;*
(2) *$f = 0$, $g(R) \subseteq Z(R)$, and there exist $a, b \in U$ such that $F(x) = ax$, $G(x) = bx + g(x)$ for all $x \in R$, and $a^2 + b \in C$, where C is the extended centroid of R.*

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