Power-law quantum distributions in protoneutron stars

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Abstract. We investigate the bulk properties of protoneutron stars in the framework of a relativistic mean field theory based on nonextensive statistical mechanics, originally proposed by C. Tsallis and characterized by power-law quantum distributions. We study the relevance of nonextensive statistical effects on the β-stable equation of state at fixed entropy per baryon, for nucleonic and hyperonic matter. We concentrate our analysis in the maximum heating and entropy per baryon $s = 2$ stage and $T \approx 40 \div 80$ MeV. This is the phase, at high temperature and high baryon density, in which the presence of nonextensive effects may alter more sensibly the thermodynamical and mechanical properties of the protoneutron star. We show that nonextensive power-law effects could play a crucial role in the structure and in the evolution of the protoneutron stars also for small deviations from the standard Boltzmann-Gibbs statistics.

1. Introduction

A protoneutron star (PNS) is born immediately after the gravitational collapse of a massive star ($M \approx 10 \div 20 \, M_\odot$) and in the first seconds of its evolution it is a very hot, lepton rich and β-stable object, with a temperature of a few tens of MeV and a lepton concentration typical of the pre-supernova matter [1].

It is well known that the knowledge of the nuclear equation of state (EOS) plays a crucial role in the determination of the structure and in the evolution of the PNS [2]. The processes related to strong interaction should in principle be described by quantum chromodynamics. However, in the energy density range reached in the compact stars, strongly non-perturbative effects in the complex theory of QCD are not negligible. In the absence of a converging method to approach QCD at finite density one often turns to effective and phenomenological model investigations.

In the last years there is an increasing evidence that the nonextensive statistical mechanics, originally proposed by C. Tsallis and characterized by power-law quantum equilibrium distributions, can be considered as an appropriate physical and mathematical basis to deal with physical phenomena where strong dynamical correlations, long-range interactions, anomalous diffusion and microscopic memory effects take place [3, 4, 5, 6, 7].

In this framework, several authors have outlined the relevance of nonextensive statistical mechanics effects in high energy physics and astrophysical problems [8, 10, 9, 11, 12, 13, 14, 15, 16, 17]. This feature results to be particular valid in the central core of such compact object, where hadrons may be strongly correlated and long range interaction may take place.

The existence of nonextensive statistical effects, should sensibly affect the finite temperature and density nuclear EOS [18, 19, 20, 21]. In fact, by varying temperature and density, the EOS reflects in terms of the macroscopic thermodynamical variables the microscopic interactions in different nuclear matter regimes.

In this work we limit ourselves to consider only small variation from the standard Boltzmann-Gibbs (BG) statistics (from $q = 0.97$ to $q = 1.03$). In particular, we concentrate our study in the stage immediately after the supernova explosion (called deleptonizazion era), when the PSN assumes the maximum heating and entropy per baryon ($s = 2$) and the presence of nonextensive effects may alter more sensibly the thermodynamical and mechanical proprieties of the PNS.

2. Nonextensive hadronic equation of state

The hadronic EOS is calculated in the framework of the nonextensive statistical mechanics introduced by Tsallis [3, 4, 5]. The nonextensive statistics represents a physical mathematical tool in several physical fields and it is based on the following definition of q-deformed entropy functional

$$S_q[f] = \frac{1}{q-1}\left(1 - \int [f(\mathbf{x})]^q \, d\Omega\right), \left(\int f(\mathbf{x}) \, d\Omega = 1\right), \tag{1}$$

where $f(\mathbf{x})$ stands for a normalized probability distribution, $\mathbf{x}$ and $d\Omega$ denoting, respectively, a generic point and the volume element in the corresponding phase space. The nonextensive statistics is, therefore, a generalization of the common BG statistical mechanics and for $q \to 1$ it reduces to the standard BG entropy. Furthermore, nonextensive statistical effects vanishes approaching to zero temperature.

In this context, the nonextensive statistics entails a sensible difference on the power-law particle distribution shape in the high energy region with respect to the standard statistic. In particular, for a dilute gas of particles and for small deviations from the standard statistical mechanics ($q \approx 1$), it can be written as [22]

$$n_i = \frac{1}{\tilde{e}_q(\beta(E_i - \mu_i)) \pm 1}, \tag{2}$$

where $\beta = 1/T$ and the sign (± 1) is for fermions and bosons respectively. Following Ref. [22], in Eq.(2), for $q > 1$, we have

$$\tilde{e}_q(x) = \begin{cases} [1 + (q-1)x]^{1/(q-1)} & \text{if } x > 0 \text{ ;} \\ [1 + (1-q)x]^{1/(1-q)} & \text{if } x \leq 0 \text{ ,} \end{cases} \tag{3}$$

whereas, for $q < 1$,

$$\tilde{e}_q(x) = \begin{cases} [1 + (q-1)x]^{1/(q-1)} & \text{if } x \leq 0 \text{ ;} \\ [1 + (1-q)x]^{1/(1-q)} & \text{if } x > 0 \text{ .} \end{cases} \tag{4}$$

In this work, we study the proprieties of hot and dense nuclear medium, using the nonlinear Walecka model (GM3 parameter set), based on a relativistic mean-field model (RMF) approach, where the nuclear force is mediated by the exchange of virtual isoscalar-scalar (σ), isoscalar-vector (ω) and isovector-vector (ρ) mesons fields [23, 24, 25, 26, 27].

The vector (ρ_B) and scalar (ρ_S) baryon density are given respectively by [28]

$$\rho_B = 2 \sum_{i=B} \int \frac{d^3k}{(2\pi)^3} [n_i(k) - \overline{n}_i(k)], \tag{5}$$

$$\rho_S = 2 \sum_{i=B} \int \frac{d^3k}{(2\pi)^3} \frac{M_i^*}{E_i^*} [n_i^q(k) + \overline{n}_i^q(k)], \tag{6}$$

where $n_i(k)$ and $\overline{n}_i(k)$ are the q-deformed fermion particle and antiparticle distributions function given in Eq.(2); For $q > 1$ and $\beta(E_i^* - |\mu_i^*|) > 0$, we have, for example,

$$n_i(k) = \frac{1}{[1 + (q-1)\,\beta(E_i^*(k) - \mu_i^*)]^{1/(q-1)} + 1}\,. \tag{7}$$

The nucleon effective energy is defined, as usual, by $E_i^*(k) = \sqrt{k^2 + M_i^{*2}}$, where $M_i^* = M_i - g_{\sigma B}\sigma$ and the effective chemical potentials is: $\mu_i^* = \mu_i - g_{\omega B}\omega - \tau_{3iB}g_{\rho B}\rho$.

The β-stability condition and the charge neutrality are given by

$$\mu_\Lambda = \mu_{\Sigma^0} = \mu_{\Xi^0} = \mu_n\,, \tag{8}$$

$$\mu_{\Sigma^-} = \mu_{\Xi^-} = \mu_n + \mu_e\,, \tag{9}$$

$$\mu_p = \mu_{\Sigma^+} = \mu_n - \mu_e\,, \tag{10}$$

$$\rho_p + \rho_{\Sigma^+} - \rho_{\Sigma^-} - \rho_{\Xi^-} - \rho_e = 0\,. \tag{11}$$

The total entropy per baryon is calculated as

$$s = \frac{1}{T\rho_B}(S_B + S_l)\,, \tag{12}$$

with

$$S_B = P_B + \epsilon_B - \sum_{i=B}\mu_i\rho_i\,, \tag{13}$$

$$S_l = P_l + \epsilon_l - \sum_{i=l}\mu_i\rho_i\,, \tag{14}$$

where the sums runs over all the baryons and leptons species. In the considered stage ($s = 2, Y_\nu = 0$), neutrinos have already escaped from the PNS and the lepton fraction ($Y_L = Y_e + Y_{\nu_e} = (\rho_e + \rho_{\nu_e})/\rho_B$) reduces to $Y_L = Y_e$, where ρ_e, ρ_{ν_e} and ρ_B are the electron, neutrino and baryon number densities, respectively.

Finally, the thermodynamical quantities can be obtained from the thermodynamic potential in the standard way. More explicitly, the baryon pressure P_B and the energy density ϵ_B can be written as

$$P_B = \frac{2}{3}\sum_i \int \frac{\mathrm{d}^3k}{(2\pi)^3}\frac{k^2}{E_i^*(k)}[n_i^q(k) + \overline{n}_i^q(k)] - \frac{1}{2}m_\sigma^2\sigma^2 - U(\sigma) + \frac{1}{2}m_\omega^2\omega^2 + \frac{1}{2}m_\rho^2\rho^2\,, \tag{15}$$

$$\epsilon_B = 2\sum_i \int \frac{\mathrm{d}^3k}{(2\pi)^3}E_i^*(k)[n_i^q(k) + \overline{n}_i^q(k)] + \frac{1}{2}m_\sigma^2\sigma^2 + U(\sigma) + \frac{1}{2}m_\omega^2\omega^2 + \frac{1}{2}m_\rho^2\rho^2\,. \tag{16}$$

The implementation of the hyperon degrees of freedom comes from determination of the corresponding meson-hyperon coupling constants that have been fitted to hypernuclear properties and their specific values are taken from Ref. [26].

In Fig. 1 and in Fig. 2, we report some of the most important effects in the structure and in the thermodynamical properties of the PNS in the framework of nonextensive statistical mechanics, during the maximum heating phase ($s = 2$ and $Y_\nu = 0$). In the left panel of Fig. 1, we show the variation of the maximum baryonic mass as a function of the central baryon density ρ_c, for different values of q, both for nucleonic (np) and hyperonic (npH) stars. We observe a remarkable increase (reduction) of the maximum baryonic mass when $q > 1$ ($q < 1$). The corresponding mass-radius relation (right panel) is also sensibly modified in presence of a nonextensive statistics.
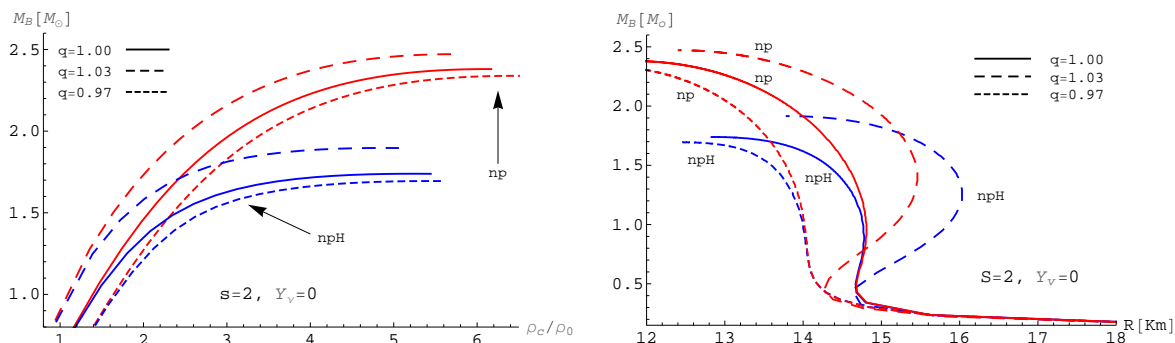
Figure 1. Maximum baryonic mass M_B in units of solar mass $M_\odot$ (left panel) and the corresponding stellar radius profile (right panel), as a function of the central baryon density ρ_c (in units of the nuclear saturation density ρ_0) for different values of q and for nucleons and hyperons stars.

The above behaviors are substantially due to the softening of the EOS in presence sub-extensive statistics ($q < 1$), together with a remarkable reduction of the maximum temperature both for nucleonic and hyperonic PNS, as reported in Fig. 2. Contrariwise, when $q > 1$, we observe an increase of the maximum temperature. These remarkable differences in the stellar temperature have important consequences in the PNS evolution and, consequently, in the cooling of the PNS, making it longer when $q > 1$, and shorter when $q < 1$, with important astrophysical implications. All of these effects are also present when neutrinos are trapped in the PNS ($s = 1$ and $Y_L = 0.4$), however, due to the lower temperature achieved in this phase, they are less pronounced.
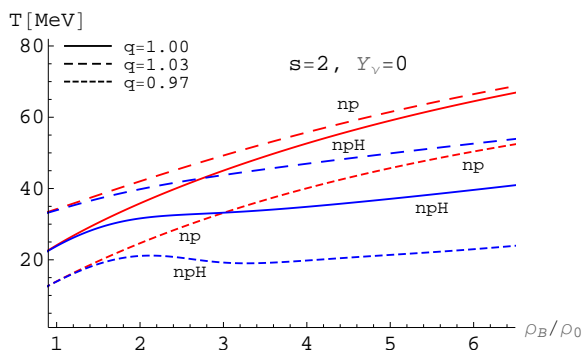
Figure 2. Temperature as a function of the baryon density (in units of the nuclear saturation density ρ_0) for nucleonic and hyperonic PNS, for different values of q.

In conclusion, we have shown that the nonextensive statistical mechanics, characterized by power-law quantum distributions, can play a crucial role on the physical properties of the PNS and can be considered as an effective mathematical basis to investigate the complex structure and evolution of the PNS. From a phenomenological point of view, we have considered the nonextensive index q as a free parameter, even if, in principle, it should depend on the physical conditions inside the PNS, on the fluctuation of the temperature and be related to microscopic quantities (such as, for example, the mean interparticle interaction length). The variation of the maximum temperature, together with a significant variation of the maximum mass and stellar radius, could be considered as a relevant phenomenological behavior in order to observe some astrophysical evidences of nonextensive statistical effects in PNS.

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