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The volume-based Strain Energy Density approach applied to static and fatigue strength assessments of notched and welded structures

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Abstract

A large bulk of experimental data from static tests of sharp and blunt V-notches and from fatigue tests of welded joints are presented in an unified way by using the mean value of the Strain Energy Density (SED) over a given finite-size volume surrounding the highly stressed regions. When the notch is blunt, the control area assumes a crescent shape and R_0 is its width as measured along the notch bisector line. In plane problems, when cracks or pointed V-notches are considered, the volume becomes a circle or a circular sector, respectively, with R_0 being their radius. R_0 depends on material fracture toughness, ultimate tensile strength and Poisson's ratio in the case of static loads; it depends on the fatigue strength $\Delta\sigma_A$ of the butt ground welded joints and the Notch Stress Intensity Factor (NSIF) range ΔK_1 in the case of welded joints under high cycle fatigue loading (with $\Delta\sigma_A$ and ΔK_1 valid for $5x10^6$ cycles). Dealing with static tests, about nine hundred experimental data as taken from the recent literature are involved in the synthesis. The strong variability of the non-dimensional radius R/R_0, ranging from about zero to about 1000, makes the check of the approach based on the mean value of the SED severe. In parallel, dealing with welded joints, nine hundred experimental data are here summarised in terms of the local SED.

"Keywords: Strain Energy Density; control radius; finite size volume; notch; welded joints; brittle fracture"

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1. Introduction

For many years the Strain Energy Density (SED) has been used to formulate failure criteria for materials exhibiting both ductile and brittle behavior. Since Beltrami (1885) to nowadays the SED has been found being a powerful tool to assess the static and fatigue behavior of notched and unnotched components in structural engineering. Different SED-based approaches were formulated by many researchers. First, Gillemot and collaborators[1, 2] experimentally determined the critical Absorbed Specific Fracture Energy for various engineering materials by using smooth and notched components, with the brittle fracture of welded materials under static loads being also considered. The point-wise criterion formalised by Sih gave a sound theoretical basis to Gillemot's experimental findings[3, 4]. Sih proposed the strain energy density parameter S, which is the product of the strain energy density and a small

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distance from the point of singularity. Failure was thought of as controlled by a critical value of S, whereas the direction of crack propagation was determined by imposing a minimum condition on S. Sih's criterion was used in different fields and strongly supported by a number of researchers, among the others by Gdoutos[5]. Recently, the volume energy function has been scaled from macro to micro to take into account the micro-cracks with a stronger stress singularity[6, 7]. Dealing with Sih's energy criterion, a forthcoming contribution of the present authors will apply Neuber's fictitious rounding approach to V-sharp notches under in-plane shear loading; the direction of provisional crack growth will be obtained by imposing the minimum condition on S . The concept of strain energy density has been reported in the literature in order to predict the fatigue behavior of notches both under uniaxial and multi-axial stresses[8, 9]. In particular Ellyin proposed a *fatigue master life curve* based on the use of the plastic strain energy per cycle as evaluated from the cyclic *hysteresis loop* and the positive part of the elastic strain energy density[9]. The two views, Ellyin's cyclic hysteresis loop concept evaluating the plastic energy for tensile specimens and Sih's criterion evaluating the local accumulated SED near the crack tip, although formally different, are strictly connected and both tied to Gillemot's concept of Absorbed Specific Fracture Energy. Neuber's concept of elementary structural volume[10] was used by Lazzarin and collaborators to formalize a SED approach applied to finite size volumes[11-13]. The control radius R_0 of the volume, over which the energy has to be averaged, depends on the ultimate tensile strength, the fracture toughness and Poisson's ratio in the case of static loads, whereas it depends on the unnotched specimen's fatigue limit, the threshold stress intensity factor range and the Poisson's ratio under high cycle fatigue loads[11, 12]. The approach was successfully used under both static and fatigue loading conditions to assess the strength of notched and welded structures subjected to predominant mode I and also to mixed mode loading[11-15]. The main aim of the present paper is to summarize the analytical frame of the SED method combined with the material control volume and to present a synthesis of more than 1800 experimental data (about 900 from static tests, 900 from fatigue tests) considering very different materials with a control radius, R_0, ranging from 0. 4 to 500 μm.

2. Expressions for the Strain Energy Density in a control volume

The SED approach is based on the idea that under tensile stresses failure occurs when $\overline{W} = W_c$, where the critical value W_c varies from material to material. If the material behaviour is ideally brittle, then W_c can be evaluated by simply using the conventional ultimate tensile strength σ_t, so that $W_c = \sigma_t^2 / 2E$, where E is Young's modulus. Under plane strain conditions, R_0 can be expressed in terms of the fracture toughness, K_{IC}, the ultimate strength, σ_t, and Poisson's ratio[16], ν:

$$R_0 = \frac{(1+\nu)(5-8\nu)}{4\pi}\left(\frac{K_{IC}}{\sigma_t}\right)^2 \tag{1}$$

In the presence of pointed V-notch under mode 1 loading it is possible to determine the total strain energy over the sector of radius R_0 and, then, the mean value of the elastic SED. The final relationship is

$$\overline{W_1} = \frac{I_1}{4\,E\,\lambda_1\,(\pi-\alpha)}\left(\frac{K_1}{R_0^{1-\lambda_1}}\right)^2 \quad , \tag{2}$$

where λ_1 is Williams' eigenvalue and K_1 the mode I notch stress intensity factor. The parameter I_1 is different under plane stress and plane strain conditions and depends on Poisson's ratio. Eq. (2) has been extended to pointed V-notches in mixed mode[11, 15]. Dealing with welded joints under plane strain conditions, the elastic strain energy density range, ΔW, can be evaluated by updating Eq. (2) with the stress intensity factor range ΔK_1 . R_0 was carefully identified with reference to conventional arc welding processes. R_0 for welded joints made of structural steels and aluminium alloys was found to be 0.28 mm and 0.12 mm, respectively[12]. Different values of R_0 might characterise welded joints obtained from high-power processes, in particular from automated laser beam welding.

Dealing with blunt notches, it is possible to determine the total strain energy over the crescent shape volume and then the mean value of the SED, by using the elastic maximum notch stress. The mean value of SED can be expressed as a function of the stress at the notch tip, σ_{tip}. More precisely:

$$\overline{W}_1 = F(2\alpha)\times H\left(2\alpha,\frac{R_0}{R}\right)\times\frac{\sigma_{tip}^2}{E} \qquad (3)$$

where $F(2\alpha)$ depends on the notch opening angle and H is a function of R_0/R ratio, the notch opening angle, 2α, and Poisson's ratio[11-13]. Under mixed mode loading the problem becomes more complex than under mode I loading, mainly because the maximum elastic stress is outside the notch bisector line and its position varies as a function of the mode I to mode II stress ratio. Dealing with U notches and taking advantage of equivalent local mode I, Eq.(3) has been updated by substituting σ_{tip} with the maximum value of the principal stress along the notch edge being the function H very close to that obtained under mode I loading[14]. When the notch opening angle is different from zero the most convenient choice is to directly evaluate the SED from FE models.

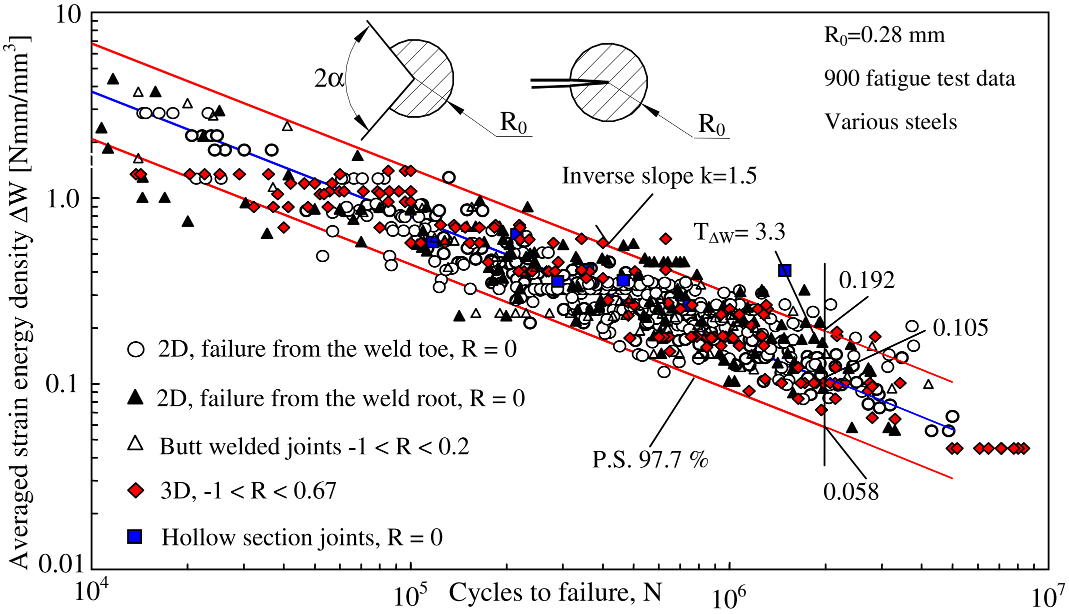

Fig. 1. Fatigue strength of welded joints as a function of the averaged local strain energy density; *R* is the nominal load ratio

3. Synthesis based on SED in a control volume

The mean value of the strain energy density (SED) in a circular sector of radius R_0 located at the fatigue crack initiation sites has been used to summarise fatigue strength data from steel welded joints of complex geometry (Fig. 1). About 900 experimental data reconverted in terms of the SED have been compared with a theoretical scatter band already reported in the literature[11, 12] to summarise fatigue strength properties of fillet and butt welded joints. A good agreement has been found. Dealing with static loading, the local SED values are normalised to the critical SED values (as determined from unnotched specimens) and plotted as a function of the R/R_0 ratio. A scatterband is obtained whose mean value does not depend on R/R_0, whereas the ratio between the upper and the lower limits were found to be about equal to 1.3/0.8=1.6 (Fig. 2). The strong variability of the non-dimensional radius R/R_0 (notch root radius to control volume radius ratio, ranging here from about zero to about 1000) makes the check of the approach based on the mean value of the local SED on a material-dependent control volume stringent.

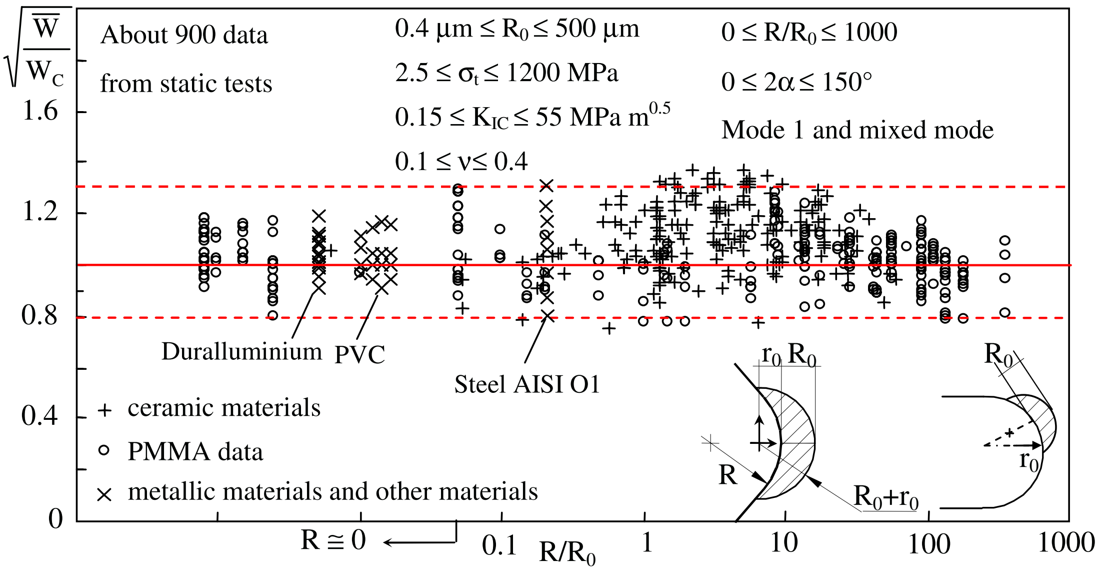

Fig. 2. Synthesis of data taken from the literature[17,18]. Different materials are summarised, among the others AISI O1 and duralluminium

Conclusions

The mean value of the strain energy density (SED) in a circular sector of radius R_0 located at the crack initiation sites has succesfully been used to summarise about 1800 data from static and fatigue failure related to materials characterized by very different critical radius ranging from micro to macro size (from 0. 4 μm to 500 μm).

References

1. Gillemot LF. Brittle fracture of welded materials. In: Commonwealth Welding Conference; 1965, C.7. 353-358.
2. Gillemot LF, Czoboly E, Havas I. Fracture mechanics applications of absorbed specific fracture energy: notch and unnotched specimens Theor Appl Fract Mech 1985; **4**: 39-45.
3. Sih GC. Strain-energy-density factor applied to mixed mode crack problems. Int J Fract 1974; **10**:305-32.
4. Sih GC. *Mechanics of Fracture Initiation and Propagation*, Dordrecht: Kluwer Academic Publisher; 1991.
5. Gdoutos EE. *Fracture Mechanics Criteria and Applications*, Dodrecht: Kluwer Academic Publishers; 1990.
6. Sih GC, Tang XS. Scaling of volume energy density function reflecting damage by singularities at macro-, meso- and microscopic level. Theor Appl Fract Mech 2005; **43**: 211–231.
7. Sih GC. *Multiscaling in molecular and continuum mechanics: interaction of time and size from macro to nano*. Dordrecht: Springer; 2007.
8. Glinka G. Energy density approach to calculation of inelastic strain-stress near notches and cracks Engng Fract Mech 1985; **22**: 485-508.
9. Ellyin F. *Fatigue Damage, Crack Growth and Life Prediction*, London: Chapman & Hall; 1997.
10. Neuber H. *Theory of notch stresses*. Berlin: Springer-Verlag; 1958.
11. Lazzarin P, Zambardi R. A finite-volume-energy based approach to predict the static and fatigue behaviour of components with sharp V-shaped notches. Int J Fract 2001; **112**: 275-298.
12. Livieri P, Lazzarin P. Fatigue strength of steel and aluminium welded joints based on generalised stress intensity factors and local strain energy values. Int J Fract 2005; **133**: 247-276.
13. Lazzarin P, Berto F. Some expressions for the strain energy in a finite volume surrounding the root of blunt V-notches. Int J Fract 2005; **135**:161-185.
14. Berto F, Lazzarin P, Gómez FJ, Elices M. Fracture assessment of U-notches under mixed mode loading: two procedures based on the 'Equivalent local mode I' concept. Int J Fract 2007; **148**: 415-433.
15. Lazzarin P, Livieri P, Berto F, Zappalorto M. Local strain energy density and fatigue strength of welded joints under uniaxial and multiaxial loading, Engng Fract Mech 2008, 75, 1875-1889.
16. Yosibash Z, Bussiba Ar, Gilad I. Failure criteria for brittle elastic materials. Int J Fract 2004; **125**:307-333.
17. Gómez FJ, Guinea GV, Elices M. Failure criteria for linear elastic materials with U-notches. Int J Fract 2006; **141**: 99-113.
18. Gomez F.J., Elices M. Fracture loads for ceramic samples with rounded notches, Engng Fract Mech 2006; **73** :880-894.