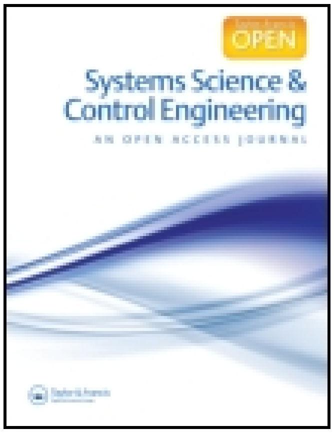

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A feedback model for the evolution of civilizations

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The paper proposes a simple feedback model capable of explaining the evolution of various civilizations as determined by historians and scientists. The forward path of this feedback model consists of a first-order system, accounting for an accumulation process, in series with a pure time delay, and its feedback path consists of a constant possibly preceded by a filter. To account for an eventual decline, a smoothed derivative term can also be added. It is shown how the evolution pattern depends on few model parameters susceptible to interesting interpretations, thus providing a powerful "tool for thought". The relation of the suggested model with the Phillips model of a closed economy is also pointed out.

Keywords: feedback control; time-delay systems; complex systems

1. Introduction

Since the seminal monumental work of A.J. Toynbee "A Study of History", published in 12 volumes between 1934 and 1961, of which an excellent abridged version by D.C. Somervell is available (Toynbee, 1970), the analysis of the evolution of civilizations has attracted a continual interest (Blaha, 2004; Braudel, 1995; Napolitani, 2001; Targowski, 2004). The present authors, too, have been involved in this kind of studies with the intent of using concepts and ideas of dynamic system and control theory to describe the behaviour of social entities (Lepschy & Viaro, 2003), following a path that had been initiated by Lepschy and Milo (1976). The main result of the mentioned papers has been a mathematical description of the considered phenomena in terms of linear ordinary differential equations whose dependent variable is a suitable function of time. A similar approach has been taken by Blaha (2002).

This paper tries to provide some additional insight into the "mechanism" that generates the civilization evolution along the lines of Lepschy and Viaro (2004). Therefore, the accent is on qualitative behaviour, i.e. the evolution patterns, rather than on quantitative data fitting, also because measuring the level of a civilization or determining the value of a suitable comprehensive societal indicator is not an easy task and would require a prior agreement among researches.

Instead, quantitative models have long been employed in the study of the general aspects of a national or regional economy because, in this case, commonly accepted indices have been defined and large time series data of economic variables can easily be found. This fact, among others, justifies the great success of macroeconomic models such as those proposed by A.W. Phillips and his successors (Phillips, 1954, 1957; Turnovsky, 2011). It is apparent that these models refer to phenomena that are related to the level of civilization. This connection suggests the use of similar models to account for the dynamics of civilizations, too. Another reason in favour of this choice is the almost cyclic character of many macroeconomic phenomena, a pattern that has been observed in a number of civilizations as well, even if with very different time scales (Blaha, 2002; Napolitani, 2001). By way of example, Figure 1 shows the evolution of the Nile River civilization according to Blaha (2004).

Toynbee (1970) did not represent graphically this kind of oscillatory behaviour, but spoke of sequences of alternating "routs" and "rallies", the most typical pattern being formed by a first period of rather rapid growth followed by a breakdown (an event marking the end of growth) and then by four routs separated by three rallies (in their own words, "the normal rhythm seems to be rout–rally–rout–rally–rout–rally–rout: three and a half beats").

It is thus reasonable to model the above-mentioned behaviour by means of a damped harmonic oscillator (Blaha, 2002, 2004) or, more generally, by a second-order system, perhaps in cascade with a smoothed derivative action (Lepschy & Viaro, 2003) to account also for overdamped responses with terminal decay, which is the case in the abortive civilizations (Toynbee, 1970). This

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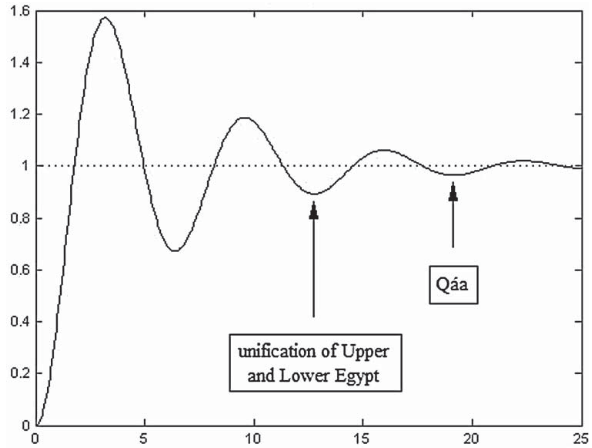

Figure 1. Evolution of the Nile River civilization according to Blaha (2004). The ordinate represents a suitably defined "societal level"; the abscissa spans 1000 years starting from the beginning of the fourth millennium B.C. (one time unit corresponds to 40 years). For example, the second minimum precedes the unification of upper and lower Egypt, and the third minimum precedes King Qáa's upheaval.

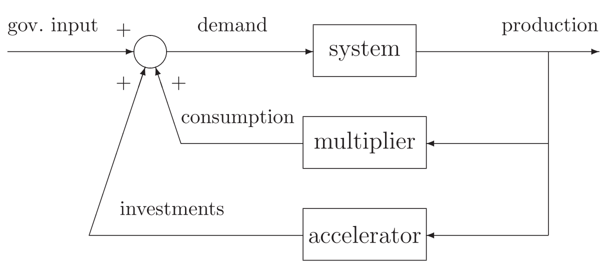

Figure 2. Phillips model of a closed economy (no import and export).

purely descriptive approach, however, does not convey insight into the way the processes operate.

The system dynamics approach to the modelling and simulation of complex aggregates (Forrester, 1968; Meadows, 2008) has revealed the crucial role of accumulation and delay phenomena inside feedback loops (often *intrinsic*), a feature that is typical of compartmental systems, too (Cobelli, Lepschy, Romanin Jacur, & Viaro, 1986). Also the celebrated macroeconomic Phillips model (see Figure 2), which draws on the fundamental works by Samuelson (1939) and Hicks (1950), has such a structure, with an additional extrinsic feedback to control the system output (production). Observe that the feedback connection can generate a variety of overall system behaviours, both oscillatory and aperiodic, from even the simplest component parts, such as gains, integrators, pure time delays or first-order lags. To obtain the desired response, it is enough to change one or two model parameters like the marginal propensity to consume in Samuelson's multiplier–accelerator model or the proportional gain of a standard controller in a classic control system. Precisely for this reason, the behaviour of social systems has been qualified as "counterintuitive" by Forrester (1971).

This paper suggests the use of a feedback model to explain the behaviour of various civilizations. In its simplest version, the forward path contains a first-order system (e.g. an integrator) followed by a pure time delay, and the feedback path consists of a constant. As is known, this configuration is typically adopted to approximate the behaviour of complex distributed systems (Lepschy, Mian, & Viaro, 1987; Taiwo, 1997; Taiwo, Effanga, & Odusanya, 1999) or to design robust controllers (Krajewski, Lepschy, Miani, & Viaro, 2005). Section 2 presents the feedback model in detail and depicts its possible responses. Section 3 shows that the model can simulate both oscillatory behaviours, such as the one in Figure 1, and non-oscillatory behaviours, such as those characterizing the fossilized, arrested and abortive civilizations, to use the words of Toynbee. Some concluding remarks are drawn in Section 4.

2. Feedback model

As already said, the purpose of the suggested models is to account in a simple way for the mechanism that generates a given response. Therefore, no exact fit of observed or estimated data is expected from them, since this would require a much greater model complexity at the expense of insight and understanding. On the other hand, it would not make much sense to fit exactly the degree of a civilization whose definition is still debated (a notable attempt in this direction, which entails unwieldy historic and anthropologic analyses, has been made by Blaha (2002) who proposes a measure of "societal level" based on several physical, social, cultural and psychological factors).

Based on the considerations of Section 1, the models are structured as closed-loop systems with an intrinsic feedback, possibly followed by a smoothed differentiator (see Figure 3). By regarding $A(s)\,e^{-t_d s}$ as the transfer function of a plant to be controlled, $B(s)$ as that of a sensor, k and h as design parameters, and $F(s)$ as a suitable filter, this is also the reference structure for the design of a two-degrees-of-freedom control system (where, however, $F(s)$ is placed before the feedback loop and is therefore called "prefilter").

Often, in this case, it is assumed that the controlled process in the forward path is approximated by a first-order-lag-plus-time-delay (or, in short, FOLTD) model. This choice is motivated by the fact that such a model

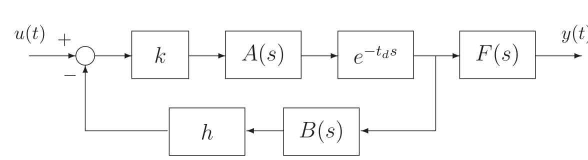

Figure 3. General feedback model of a civilization: $A(s)$ accounts for an accumulation process and $e^{-t_d s}$ for a time delay of duration t_d; $B(s)$ represents a filter and $F(s)$ a differentiator; k and h are constant gains.

accounts well for the dynamics of high-order processes with many real left half-plane poles and a large pole-zero excess, and allows the designer to evaluate easily the control system robustness strictly related to the stability margins (Krajewski et al., 2005).

It is reasonable to adopt a similar approximation in the study of civilizations which are the result of many interconnected processes. To this end, the transfer function of the (unit-gain) undelayed part of the FOLTD model (the block denoted by $A(s)$ preceded by a static gain k in Figure 3) can be taken as either

$$A(s) = \frac{1}{1 + T_a s}, \tag{1}$$

where the time constant T_a is obviously much longer than that arising in control system design, or just as

$$A(s) = \frac{1}{s}, \tag{2}$$

which is the transfer function of a pure integrator.

The adopted system structure lends itself to a natural interpretation. In fact, the level of a civilization clearly depends on the accumulated knowledge and experience (output of the block $A(s)$). But this accumulated stock gives rise only with a certain delay t_d (corresponding to the block denoted by the transfer function $e^{-t_d s}$ in Figure 3) to the exploitation of resources capable of increasing the quality of life.

In turn, the augmented consumption deprives the system of resources and, thus, reduces the original thrust injected to the system (represented by input $u(t)$). Such a depriving action accounts for the *negative* feedback channel along which a block of constant positive gain h (suitably small to guarantee sustainability) is placed.

If the subtraction of resources is not immediately effective, the feedback path may also contain an additional filter whose transfer function $B(s)$ could be

$$B(s) = \frac{1}{1 + T_b s}. \tag{3}$$

For simplicity, in the following, $B(s) = 1$, which, in technological systems, corresponds to neglecting the sensor dynamics, usually much faster than the process dynamics ($T_b \ll T_a$). In the present context, this means that all of the loop delays are incorporated into the forward path.

The transfer function of the overall system, when the transfer function $F(s)$ of the additional block in cascade with the closed loop is set equal to 1, turns out to be

$$W(s) = \frac{k\, e^{-t_d s}}{1 + T_a s + kh\, e^{-t_d s}} \tag{4}$$

for $A(s)$ given by Equation (3) (and $B(s) = 1$), and

$$W(s) = \frac{k\, e^{-t_d s}}{s + kh\, e^{-t_d s}} \tag{5}$$

for $A(s)$ given by Equation (2) (and again $B(s) = 1$).

With reference to Equation (4), according to the Nyquist criterion stability is ensured if, and only if, the polar plot of the loop function $L(\jmath\omega) = kh\, e^{-\jmath t_d \omega}/(1 + T_a \jmath\omega)$, whose magnitude decreases monotonically with ω, does not encircle the critical point $-1 + \jmath 0$. Clearly, encirclements may occur only if the loop gain $kh > 1$, so that the system is necessarily stable when $kh < 1$. Since the form of the Nyquist diagram depends only on the ratio t_d/T_a, it is not difficult to determine the values of kh that ensure stability for each value of t_d/T_a. This has been done, e.g. in Lepschy et al. (1987) where the loop gains corresponding to oscillatory and non-oscillatory responses have also been determined.

The steady-state or asymptotic value w_{ss} in the response $y(t)$ to a unit step input $u(t)$ is different from zero for both Equations (4) and (5). Precisely, it is equal to

$$w_{ss} = \frac{1}{1 + kh} \tag{6}$$

for Equation (4), and

$$w_{ss} = \frac{1}{h} \tag{7}$$

for Equation (5). Figures 4 and 5 show a number of step responses obtainable from the transfer functions (4) and (5), respectively.

Oscillations are present only when hk exceeds a certain value. For instance, in the case of system (4) and Figure 4 in which $t_d/T_a = 0.4$, oscillations occur for $hk > 0.7$ (and the system is unstable for $kh > 4.59$), and, in the case of system (5) and Figure 5 in which $t_d = 0.4$, oscillations occur for $hk > 0.98$ (and the system is unstable for $kh > 3.93$).

The overshoot is a function of both kh and t_d/T_a or t_d, respectively. All (stable) transfer functions (4) characterized by the same values of loop gain kh and ratio t_d/T_a exhibit the same overshoot. Instead, given the value of the loop gain, the rise and settling times become longer as

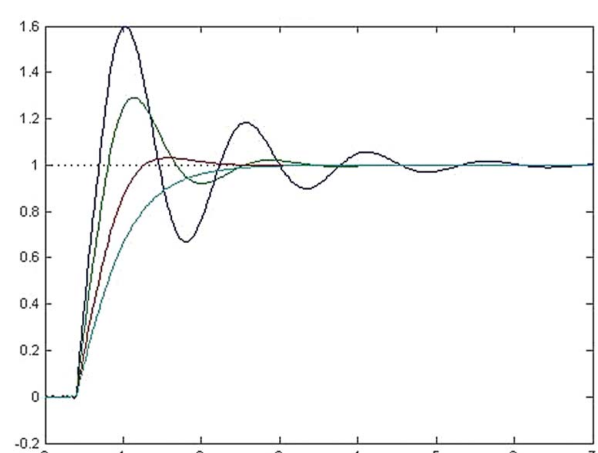

Figure 4. Step responses of the system described by the transfer function (4) for $k = 1$, $T_a = 1$, $t_d = 0.4$ and $h = 0.5, 1, 2, 3$ (normalized to steady state). The overshoot increases with kh.

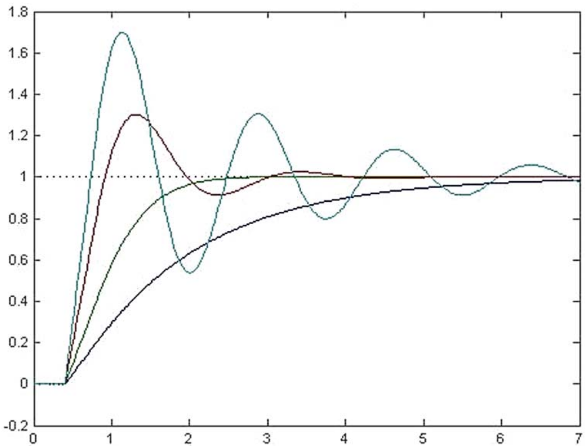

Figure 5. Step responses of the system described by the transfer function (5) for $k = 1$, $t_d = 0.4$ and $h = 0.5, 1, 2, 3$ (normalized to steady state). The overshoot increases with kh.

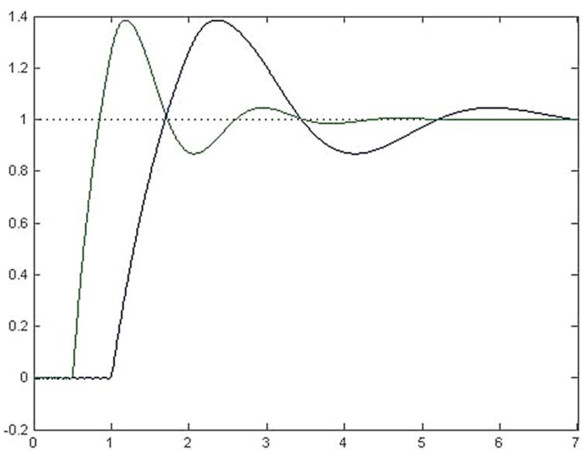

Figure 6. Step responses of the system described by the transfer function (4) for $h = k = 1$ and $t_d = T_a = 0.5$ and for $h = k = 1$ and $t_d = T_a = 1$. The rise and settling times are longer for $T_a = t_d = 1$.

t_d and T_a increase. For instance, the system with $kh = 1$ and $t_d = T_a = 1$ is slower than the system with $kh = 1$ and $t_d = T_a = 0.5$, even if the two systems are characterized by the same overshoot, as shown in Figure 6.

To simulate situations in which the response tends asymptotically to zero because of a progressive depletion or deterioration of resources (Toynbee, 1970, speaks of "disintegration") caused, e.g. by epidemics or war, the block in cascade with the closed-loop system can be assigned the transfer function

$$F(s) = \frac{k_f s}{1 + T_f s}, \tag{8}$$

which exerts a (smoothed) derivative action on the output of the preceding closed loop. In this way, the system response eventually decreases at a rate dependent on the positive time constant T_f, which is usually much longer than the other time constants. Evolutions of this kind are considered in the next section.

In this paper, the evolution is assumed to be "caused" by an input $u(t)$ accounting for a particular event, e.g. an important discovery or invention, occurring at a particular moment. This evolution is, therefore, thought of as a *forced* response in system theory terms. Choosing a step function for the input, as in Figures 4 and 5, seems reasonable, provided the original stimulus does not regress. Different choices may be more appropriate in some cases. A valid candidate is the impulse signal, a sort of *big bang* whose task would be to take instantaneously the system to a suitable initial state, from which the system then *evolves freely* (no input applied for $t > 0$), according to intrinsic laws (by the way, this seems to be indeed Toynbee's idea of the phenomenon). In this case, the output tends to zero if the system is asymptotically stable or to a finite non-zero value if it is stable but not asymptotically. Anyway, independently of which exogenous input is chosen, the transient component of the forced response (Dorato, Lepschy, & Viaro, 1994) and, thus, the *evolution pattern*, is characterized by the same *system modes* as any free evolution.

A final remark concerns the initial response latency equal to t_d before the abrupt output increase. This clearly depends on the fact that the entire time delay in the loop has been attributed to the forward path of the feedback model. If, for instance, the delay element were removed from the forward path and assigned entirely to the feedback path, the response would start increasing from $t = 0$ but would still retain exactly the same shape (shifted to the left by an amount t_d) because the loop function would not change. In the following, the entire delay is assigned to the forward path.

3. Non-normal civilizations

The ability to reproduce the typical damped oscillatory behaviour of a civilization like the one represented in Figure 1, which is characterized by three rather marked decreasing peaks after the first overall maximum (corresponding to the so-called "normal" behaviour according to Toynbee), is apparent from Figures 4 and 5. Therefore, attention is focused first on three different types of non-normal civilizations, namely:

- a civilization that, after a pair of decreasing peaks, decays slowly,
- an arrested civilization characterized by a monotonically increasing time course, and
- an abortive civilization, whose evolution exhibits a bell-shaped form.

A careful study of the ancient Greek civilization, with particular regard to mathematics (Napolitani, 2001), shows that, starting from the middle of the fifth century B.C. (immediately before Hippocrates of Chios), the Greek

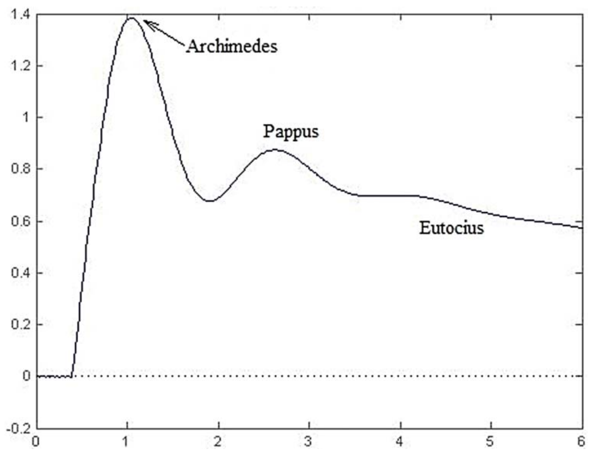

Figure 7. Simulation of the historic development of the Greek scientific civilization from Hippocrates of Chios to Eutocius of Ascalon according to Napolitani (2001).

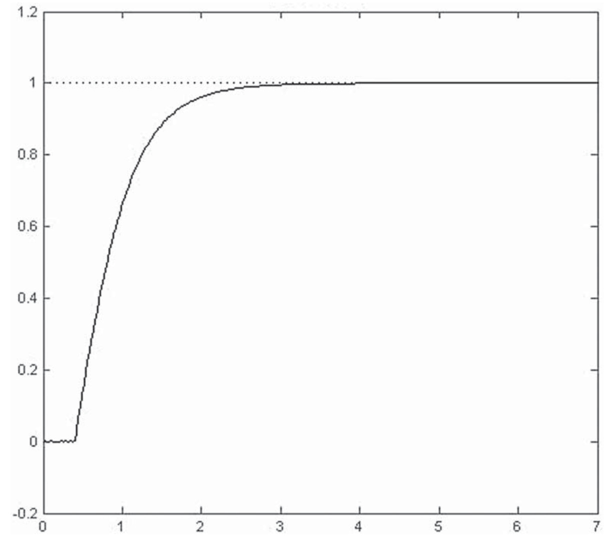

Figure 8. Simulation of an arrested civilization.

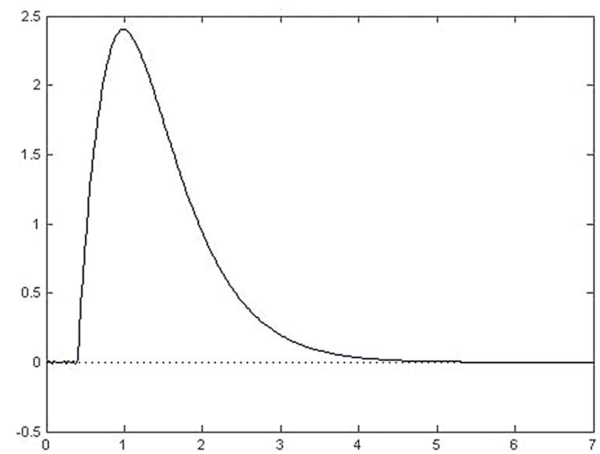

Figure 9. Simulation of an abortive civilization.

scientific culture grew rapidly, reaching a first very pronounced peak at the middle of the third century B.C. (Archimedes of Syracuse, 287–211 B.C.), and then slowly declined until the end of the fifth century A.D. (Eutocius of Ascalon), exhibiting, however, a second flat peak around the end of the third century B.C. (Pappus of Alexandria). Such a behaviour can be obtained, e.g. by placing an additional derivative term $F(s)$, with $k_f = 10$ and $T_f = 10$, in cascade with a function of type (4), with $t_d = 0.4$, $T_a = 1$, $k = 1$, $h = 2.5$. Figure 7 depicts the step response of this model. It reproduces well the curve derived by Napolitani (2001).

Arrested civilizations, such as the Polynesian, Eskimo, Nomadic, Ottoman and Spartan ones according to Toynbee (1970), exhibit an initial steep growth that terminates rather abruptly, and then maintain a practically constant value. Such a behaviour can be obtained from Equation (4), e.g. by setting $h = 0.5$, $T_a = 1$ and $t_d = 0.4$. The response of this system, normalized to its steady-state value, is shown in Figure 8.

Abortive civilizations, such as the Far Western Christian, Far Eastern Christian, Scandinavian and Syriac ones according to Toynbee (1970), start like an arrested one but are not capable of maintaining the level reached after the initial growth. They eventually tend to a level similar to the initial one. Such a behaviour, shown in Figure 9, can be obtained by placing the transfer function $F(s) = s/(1 + 0.5s)$ in cascade with Equation (5) for $h = 0.5$ and $t_d = 0.4$.

Ancient civilizations are the elective field of application of the adopted model since historic events reveal their importance only after a rather long time from their occurrence. Moreover, the reliability of the simulations depends crucially on the accuracy with which the model parameters have been determined, and this, in turn, requires the availability of homogeneous data over a long time span. The form and intensity of the input also plays a fundamental role in the presence of globalization phenomena which imply increased material and information exchanges among populations. For these reasons, in the analysis of more recent civilizations, attention should concentrate only on the *evolution patterns*, strictly related to the system modes.

With due awareness of these strong limitations, an attempt can be made to characterize by means of the feedback model with transfer function (4) the evolution of the western civilization from the twelfth century till the end of the twentieth century. Taagepera and Colby (1979) have analysed the course, during this period of time, of a creativity index (Gray, 1966, Figure 2; Taagepera & Colby, 1979, Figure 1) obtained by grading philosophers, painters, sculptors, poets, dramatists, and other writers on an evaluational scale derived from the commentaries of historians and classical scholars. In the mentioned paper by Taagepera and Colby (1979), a combination of two exponentials has been fitted to this collection of data that exhibits approximately a bell-shaped form truncated at the end of the last century with a marked peak before the year 1900. Such an *aperiodic* evolution could be generated by the feedback model with a transfer function (4) for rather small loop gains (so as to avoid oscillatory modes). On the contrary, the more wavy evolution of the Japanese history from the second half of the eighteenth century, as may be inferred from the essay of Akita (1982) seems to require the adoption of larger gains accounting for a more reactive response.

4. Conclusions

The feedback model of Figure 3 can explain the behaviour of all the civilizations studied by Toynbee and his successors. Its component parts are very simple, that is, a retarded first-order system, accounting for an accumulation process, in the forward path, and a constant (possibly multiplied by a first-order filter) in the negative feedback path accounting for subtracted resources. A further derivative block can be placed in cascade with this closed-loop system to account for the possible deterioration of the available resources or a progressive "disintegration" of the population, caused, e.g. by wars or migrations.

Of course, the same results could be obtained by approximating the transcendental transfer function of the delay element, or, better, the product $A(s)\,\mathrm{e}^{-t_d s}$, by means of a (high-order) rational function but, in these authors' opinion, this would make the model less compact without appreciably facilitating the analysis.

The output shape of either Equation (4) or Equation (5) depends on a very small number of parameters, i.e. the gains k and h, the time constant T_a and the time delay t_d. Investigating their relation with sociocultural, technological and psychological factors is a hard task for anthropologists and sociologists and is outside the scope of the present contribution. However, it can safely be said that the capacity to store and transmit information plays a fundamental role in the evolution pattern and duration. It is hoped that, even if the considered feedback model can hardly be used for quantitative analyses and predictions, it may serve as a useful "tool for thought" (Rheingold, 2000).

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