JOURNAL OF TRAFFIC AND TRANSPORTATION ENGINEERING (ENGLISH EDITION) 2016; 3 (5): 390–397

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Original Research Paper

Numerical visco-elastoplastic constitutive modelization of creep recovery tests on hot mix asphalt

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ARTICLE INFO

Article history:
Available online 3 October 2016

Keywords:
Hot mix asphalt
Visco-elastoplastic model
Hardening law
Creep-recovery test

ABSTRACT

This paper discusses a visco-elastoplastic constitutive model to analyze the creep deformability of asphalt concretes at high service temperatures, finalized to improve the interpretation of permanent deformation phenomenon and performance design of road pavements. A three dimensional constitutive visco-elastoplastic model is introduced, in tensor as well as in numerical form. The associated uniaxial model is used to arrange a plastic element in series with the viscoelastic component. The latter is defined by an elastic spring placed in parallel with three Maxwell elements. Three different hardening laws, namely isotropic, kinematic and mixed hardening, are included in the constitutive model to compare the creep deformability. The proposed constitutive model has been calibrated and validated on the basis of uniaxial creep-recovery test results at 40 °C. This is performed with a high performance hot mix asphalt concrete (HP-HMA) at different stresses and loading times. Depending on the hardening law considered, permanent deformation data predicted by the proposed model results are reasonably consistent with the experimental creep-recovery data. A rational constitutive model that is physically congruent with the creep phenomenon of asphalt concretes was developed and calibrated to achieve a deeper understanding of the stress-strain response of such materials. The fundamental relevance of an appropriate plastic response modeling, in the study of the creep behavior of asphalt concretes for highway and road pavements.

1. Introduction

The accumulation of permanent deformations at high service temperatures, which can be observed in the form of longitudinal hollows with raised edges (conventionally called ruts), is a severe and relevant failure mode of the road flexible pavements. Rational study of such degradation requires the use of an adequate constitutive model to properly analyze the complex stress-strain response under the traffic

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Peer review under responsibility of Periodical Offices of Chang'an University.
http://dx.doi.org/10.1016/j.jtte.2016.09.009

loading of asphalt concretes used in road pavement surfacing.

The creep response of asphalt concretes under a constant or cyclic compressive load is categorized as the visco-elasto-plastic type and experimentally verified in several studies (Collop et al., 2003; Costanzi and Cebon, 2014; Erkens et al., 2002; Giunta and Pisano, 2006; Olard and Di Benedetto, 2003). It can be studied using rheological models (defined by elastic, viscous and plastic elements) arranged in more or less complex configurations according to the required accuracy for analysis and the possibility to solve the analytical complexity related to the number of basic elements considered (Collop et al., 2003; Costanzi and Cebon, 2014; Giunta and Pisano, 2006; Olard and Di Benedetto, 2003). Concepts typical of continuum thermomechanics, such as the Helmholtz free energy and the dissipative inequality of Clausius–Duhem, represent a significant evolution in the modeling of the mechanical behavior of bituminous materials (Drescher et al., 2010; Erkens et al., 2002; Krishnan and Rajagopal, 2005; Krishnan et al., 2006; Pasetto and Baldo, 2007, 2015a, 2015b). Other theories have been also investigated over the years, including continuum damage mechanics (Lee and Kim, 1998; Masad et al., 2005), fractional models (Celauro et al., 2012; Oeser et al., 2007, 2008; Pellinen et al., 2007), and the distinct element method (Abbas et al., 2007; Collop et al., 2004; Dai and You, 2007; Dondi et al., 2012).

To complete the mathematical formulation of an overall constitutive model in this study, a plastic flow law based on inviscid plasticity and characterized by three different hardening laws is presented after a brief summary of a viscoelastic model that represents the fundamental framework of considered constitutive equations. The main goal is to provide significant improvements to the mathematical formulations proposed in previous studies (Pasetto and Baldo, 2007, 2015b), to obtain reliable numerical predictions of the creep response of asphalt concretes. This is a fundamental requirement in the rational design of road pavements.

The constitutive modeling framework is integrated using an original numerical procedure for the calibration and validation of the proposed model. Creep recovery tests are performed on a high performance hot mix asphalt concrete (HP-HMA) for road flexible pavements to collect the necessary experimental data for the calibration and validation of the proposed visco-elastoplastic model.

2. Materials and methods

2.1. *Aggregates and bitumen*

The high performance hot mix asphalt concrete considered in this investigation is produced using both conventional aggregates, namely crushed limestone and filler, and Electric Arc Furnace steel slags provided from different steel plants located in Northeastern Italy. The limestone, as well as the artificial aggregates (EAF slags), were supplied in three grading fractions: 0/5, 5/10 and 10/15 mm. A bituminous binder modified with Styrene-Butadiene-Styrene (SBS) polymer is used in the laboratory investigation. A penetration of 44 mm/10 (EN 1426), softening point of 77 °C (EN 1427) and Fraass breaking point of −12 °C (EN 12593), were recorded in the laboratory characterization of the SBS modified bitumen. Its manufacturer has certified an elastic recovery value at 25 °C (EN 13398) higher than 50%.

2.2. *Asphalt concrete*

The design of the HP-HMA grading curve was developed according to Specifications of the Italian Association of Pavements Technologists (SITEB) (2000), with a total amount of steel slag fixed at 25% and nominal maximum aggregate size of 12 mm. Fig. 1 shows the design grading curve of the asphalt concrete. Table 1 reports the optimum bitumen content and the most relevant engineering characteristics of the mix, as well as its indirect tensile strength value (EN 12697-23).

A complete discussion of the chemical and physical-geotechnical characteristics of the aggregates, as well as the mix design procedure and performances of the mix, can be found in two previous papers by Pasetto and Baldo (2006, 2008).

2.3. *Methods*

To investigate the creep response of HP-HMA mix, tests of creep recovery (i.e., constant load uniaxial tests) were performed on Marshall cylindrical samples according to the British Standard 598, Part 111. However, different values were selected for the loading period, stress and testing temperature to properly support the constitutive model's calibration and validation.

Loading and unloading times were chosen, with the total testing time equal to 120 s. The shorter loading time was fixed at 10 s, the second one at 20 s, and the longest one at 30 s. Therefore, the corresponding unloading times were set at 110, 100 and 90 s, respectively.

For each testing time, in addition to the standard stress level of 100 kPa, two other stress values (i.e. 300 and 500 kPa) were used to cover wider and more severe creep conditions. All creep tests were conducted at 40 °C.

The creep recovery test was chosen for its calibration and validation of the constitutive model, as it is relatively easier for mathematical modelization. It was also chosen due to its wide use in the road labs.

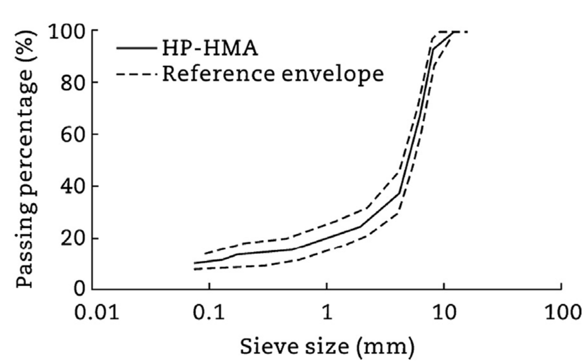

Fig. 1 – HP-HMA grading curve and SITEB reference envelope.

Table 1 – Physical and mechanical properties of HP-HMA concrete.

Properties	Value
Bulk density (kg/m^3)	2532
Voids content after compaction (%)	3.20
ITS @ 25 °C (MPa)	1.33
Optimum bitumen content (%)	5.50

3. Theory and calculation

3.1. *Summary of viscoelastic constitutive model*

Different studies have demonstrated that the mechanical response of a viscoelastic "material point" can be expressed in terms of the Cauchy stress tensor (Pasetto and Baldo, 2015b; Simo and Hughes, 1998).

$$\boldsymbol{\sigma} = \frac{\partial W^0}{\partial \boldsymbol{\varepsilon}} - \sum_{i=1}^{n} \boldsymbol{q}_i = \boldsymbol{\sigma}^0 - \sum_{i=1}^{n} \boldsymbol{q}_i \tag{1}$$

where W^0 (ε) = ½ (ε:D_0:ε) is the deformation energy representative of the instantaneous elastic response, assuming the hypothesis of linear elastic behavior, D_0 is the tensor of the instantaneous elastic modulus, ε represents the strain tensor, $\boldsymbol{q}_i$ is a set of internal variables, with each one assigned to a viscous process, $\boldsymbol{\sigma}^0$ is the instantaneous elastic stress. Introducing a relative stiffness of γ_i and relaxation time of τ_i for each viscous process, as well as a stiffness of γ_∞ to the equilibrium such that $\gamma_\infty + \sum_{i=1}^{n}\gamma_i = 1$, the internal variables can be expressed as follows (Pasetto and Baldo, 2015b; Simo and Hughes, 1998)

$$\boldsymbol{q}_i(t) = \frac{\gamma_i}{\tau_i} \int_{-\infty}^{t} \exp\left(-\frac{t-s}{\tau_i}\right) \boldsymbol{\sigma}^0(s)\mathrm{d}s \tag{2}$$

where t is the time, s is a variable of the convolution integral.

3.2. *Formulation of the visco-elastoplastic model*

The outlined constitutive equations represent the general formulation of a visco-elastic model (Simo and Hughes, 1998). However, it has to be properly integrated by the theory of plasticity, in order to describe the development of permanent strain (Pasetto and Baldo, 2007, 2015b).

The introduction of a permanent plastic strain ε^p, developed by a bituminous mixture as a consequence of a load application, requires to rewrite the elastic energy as W^0 (ε^{ve}) = ½ ($\varepsilon - \varepsilon^p$):$D_0$:($\varepsilon - \varepsilon^p$), where $\varepsilon^{ve} = \varepsilon - \varepsilon^p$ is the recoverable viscoelastic strain component. Therefore the Cauchy stress tensor can be expressed as

$$\begin{cases} \boldsymbol{\sigma}(\boldsymbol{\epsilon}, \boldsymbol{\epsilon}^p, \boldsymbol{q}_i; t) = \mathbf{D}_0[\boldsymbol{\epsilon}(t) - \boldsymbol{\epsilon}^p(t)] - \sum_{i=1}^{n} \frac{\gamma_i}{\tau_i} \int_{-\infty}^{t} \exp\left(-\frac{t-s}{\tau_i}\right) \boldsymbol{\sigma}^0(s)\mathrm{d}s \\ \boldsymbol{\sigma}^0(t) = \mathbf{D}_0[\boldsymbol{\epsilon}(t) - \boldsymbol{\epsilon}^p(t)] \end{cases} \tag{3}$$

In order to properly formulate a visco-elastoplastic model, a yield criterion and evolution law are required for the plastic strains. The mechanical behavior of bituminous mixtures under compressive or tensile stresses is different (Erkens et al., 2002). Moreover, the deformability of the asphalt concretes is characterized by volumetric as well as deviatoric strain. However, the creep response under monotonic compressive loading conditions of an asphaltic material can be analyzed by the Von Mises yield criterion coupled with an appropriated hardening law. A previous study of the isotropic hardening by Pasetto and Baldo (2015b) has been successfully implemented. In addition to the isotropic hardening, two new hardening laws (i.e., kinematic and mixed type) are considered in this study. Hence, subsequent yield conditions are assumed.

$$\phi(\boldsymbol{\sigma}, \kappa) = F(\boldsymbol{\sigma}) - \sigma_y(\kappa) = 0 \tag{4}$$

$$\phi(\boldsymbol{\sigma}, \boldsymbol{\alpha}) = F(\boldsymbol{\sigma} - \boldsymbol{\alpha}) - \sigma_y = 0 \tag{5}$$

$$\phi(\boldsymbol{\sigma}, \boldsymbol{\alpha}, \kappa) = F(\boldsymbol{\sigma} - \boldsymbol{\alpha}) - \sigma_y(\kappa) = 0 \tag{6}$$

where $\varphi(\boldsymbol{\sigma},\kappa)$ describes a plastic potential, which is a function expressing the criticality of the stress state in relation to the plastic flow processes, σ_y represents a function that expresses the maximum value that $F(\boldsymbol{\sigma})$ can assume without any ulterior plastic creep developing, κ is a hardening parameter (basically a scalar quantity), α is the back stress that represents the center of the elastic domain (it is a tensorial parameter, associated to the kinematic hardening).

While the isotropic hardening in Eq. (4) represents the conventional approach for a monotonic load condition, the kinematic hardening in Eq. (5) and the mixed type hardening of Eq. (6) are more appropriate for cyclic load conditions. Although only monotonic load conditions were considered in the experimental creep recovery tests and therefore the isotropic hardening law could have been sufficient, it has been investigated the different response of the model for each of the hardening laws implemented, in order to appreciate the differences in the prediction accuracy.

To express the dependence of yield conditions on the rate of deformation and plastic flows due to creep, the function $F(\boldsymbol{\sigma})$ depends on the instantaneous elastic stress $\boldsymbol{\sigma}^0(t) = D_0[\varepsilon(t) - \varepsilon^p(t)]$, rather than on the effective stress $\boldsymbol{\sigma}$. In addition to yield conditions, a plastic strain flow rule must be introduced to properly define the inelastic behavior of a visco-elastoplastic material, such as asphalt concrete. This plastic flow law can be obtained by applying the fundamental assumption in the mathematical theory of plasticity (Simo and Hughes, 1998), which is described the following equation.

$$\dot{\varepsilon}^p = \dot{\lambda}\boldsymbol{m} \tag{7}$$

where $\dot{\lambda}$ expresses the plastic multiplier, $\boldsymbol{m}$ represents the direction of the plastic flow. Within the framework of associated plasticity, the introduction of the Kuhn-Tucker loading-unloading conditions, along with the consistency condition (Pasetto and Baldo, 2015b; Simo and Hughes, 1998), allows to obtain, after the appropriate mathematical elaborations, the subsequent fundamental expressions for the isotropic, the kinematic and the mixed type hardening, respectively.

$$\dot{\lambda} = \frac{\boldsymbol{m}:\mathrm{D}_0:\dot{\varepsilon}}{\boldsymbol{m}:\mathrm{D}_0:\boldsymbol{m}+K} \tag{8}$$

$$\dot{\lambda} = \frac{\boldsymbol{m}:\mathrm{D}_0:\dot{\varepsilon}}{\boldsymbol{m}:\mathrm{D}_0:\boldsymbol{m}+c\boldsymbol{m}:\boldsymbol{m}} \tag{9}$$

$$\dot{\lambda} = \frac{\boldsymbol{m}:\mathrm{D}_0:\dot{\varepsilon}}{\boldsymbol{m}:\mathrm{D}_0:\boldsymbol{m}+c\boldsymbol{m}:\boldsymbol{m}+K} \tag{10}$$

where $K = -\frac{1}{\dot{\lambda}}\partial\phi/\partial\kappa\dot{\kappa}$ is the hardening modulus, c is a parameter related to the hardening modulus ($c = 2/3K$) that describes the evolution of the back stress according to $\dot{\boldsymbol{\alpha}} = c\dot{\varepsilon}^{\mathrm{p}}$. Considering the derivative of Eq. (3), namely $\dot{\boldsymbol{\sigma}} = \mathrm{D}_0(\dot{\varepsilon}-\dot{\varepsilon}^{\mathrm{p}}) - \sum\dot{\boldsymbol{q}}_{\mathrm{i}}$, it can be written that $\dot{\boldsymbol{\sigma}} = \mathrm{D}^{\mathrm{ep}}\dot{\varepsilon} - \sum\dot{\boldsymbol{q}}_{\mathrm{i}}$, in which $\boldsymbol{D}^{\mathrm{ep}}$ is the constitutive elasto-plastic tensor, that assumes the respective expressions for isotropic, kinematic and mixed type hardening.

$$\mathrm{D}^{\mathrm{ep}} = \left[1-\frac{\boldsymbol{m}:\mathrm{D}_0:\boldsymbol{m}}{\boldsymbol{m}:\mathrm{D}_0:\boldsymbol{m}+K}\right]\boldsymbol{D}_0 \tag{11}$$

$$\mathrm{D}^{\mathrm{ep}} = \left[1-\frac{\boldsymbol{m}:\mathrm{D}_0:\boldsymbol{m}}{\boldsymbol{m}:\mathrm{D}_0:\boldsymbol{m}+c\boldsymbol{m}:\boldsymbol{m}}\right]\mathrm{D}_0 \tag{12}$$

$$\mathrm{D}^{\mathrm{ep}} = \left[1-\frac{\boldsymbol{m}:\mathrm{D}_0:\boldsymbol{m}}{\boldsymbol{m}:\mathrm{D}_0:\boldsymbol{m}+c\boldsymbol{m}:\boldsymbol{m}+K}\right]\mathrm{D}_0 \tag{13}$$

Irrespective of the plastic multiplier formulation and elasto-plastic tensor expression, the following equations for the visco-elastoplastic (VEP) constitutive model are determined.

$$\boldsymbol{\sigma}(t) = \int_{-\infty}^{t}\mathrm{D}^{\mathrm{ep}}\dot{\varepsilon}\mathrm{d}s - \sum_{i=1}^{n}\boldsymbol{q}_{\mathrm{i}}(t) \tag{14}$$

$$\boldsymbol{q}_{\mathrm{i}}(t) = \frac{\gamma_{\mathrm{i}}}{\tau_{\mathrm{i}}}\int_{-\infty}^{t}\exp\left(-\frac{t-s}{\tau_{\mathrm{i}}}\right)\boldsymbol{\sigma}^0(s)\mathrm{d}s \tag{15}$$

The next section discusses the calibration and validation procedures based on numerical equations that govern the one-dimensional model obtained by the VEP constitutive framework. The analysis focuses only along the load direction of the experimental uniaxial creep recovery tests.

3.3. *Calibration and validation of the constitutive model*

The calibration and validation procedures are based on the one-dimensional formulation of the VEP constitutive model, with the hypothesis of isotropic and homogeneous material.

The one-dimensional constitutive model can be represented in terms of micromechanical elements by a plastic slider in series with a viscoelastic component. The latter is given by an elastic spring in parallel with three Maxwell elements (Fig. 2). After introducing a parameter for the elastic spring of the viscoelastic component (elastic modulus E_∞) and a pair of parameters for each Maxwell element to be used (elastic modulus E_i and viscosity η_i), it is possible to

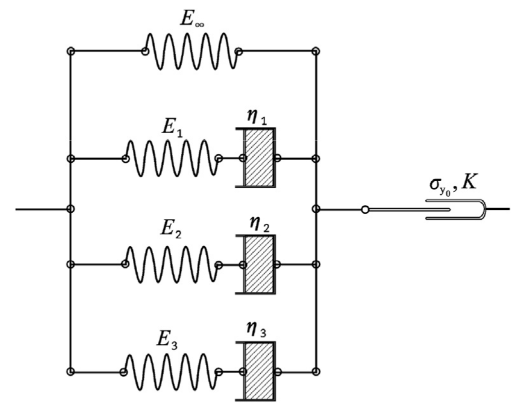

Fig. 2 – One-dimensional schematization of the constitutive model.

define $E_0 = E_\infty + \sum_{i=1}^{n}E_i$, as well as $\gamma_i = E_i/E_0$ and $\gamma_\infty = E_\infty/E_0$. Moreover, the relaxation time associated with each viscous process is defined as $\tau_i = \eta_i/E_i$.

Preliminary calibrations were performed to identify the best-suited number of Maxwell elements that properly reproduces the different stages of viscous relaxation. These analyses prove that three Maxwell elements are required to achieve satisfactory results. The model is thus characterized by seven parameters for the viscoelastic component and two parameters for the plastic slider (yield stress σ_{y_0} and hardening modulus K). Hence, nine total parameters are involved in the one-dimensional constitutive model.

The numerical formulation of the model's viscoelastic portion already has been developed in a previous study by Pasetto and Baldo (2015b). Since the applied load is given in a creep recovery test, an equation of the strain as a function of time is provided. With respect to that formulation, the corresponding one-dimensional expression is determined by adopting the quantity E_0 for $\boldsymbol{D}_0$ and introducing the uniaxial stress and strain in substitution of the stress and strain tensors. Therefore, the following equations are defined as below, in which Δt_p represents the time increment, computed as difference between the current time step t_{p+1} and the previous time step t_p.

$$q_i(t_{p+1}) = \exp\left(-\frac{\Delta t_p}{\tau_i}\right)q_i(t_p) + \frac{1}{2}\Delta t_p\frac{\gamma_i}{\tau_i}\left[\sigma^0(t_{p+1}) + \exp\left(-\frac{\Delta t_p}{\tau_i}\right)\sigma^0(t_p)\right] \tag{16}$$

$$\varepsilon^{\mathrm{ve}}(t_{p+1}) = \left[1-\frac{1}{2}\Delta t_p\sum_{i=1}^{n}\frac{\gamma_i}{\tau_i}\right]^{-1}\cdot\left\{E_0^{-1}\left[\sigma(t_{p+1}) + \sum_{i=1}^{n}\exp\left(-\frac{\Delta t_p}{\tau_i}\right)q_i(t_p)\right] + \frac{1}{2}\Delta t_p\left[\sum_{i=1}^{n}\frac{\gamma_i}{\tau_i}\exp\left(-\frac{\Delta t_p}{\tau_i}\right)\right]\varepsilon^{\mathrm{ve}}(t_p)\right\} \tag{17}$$

Within the framework of the VEP model, it is worth defining the limit condition for each hardening law (i.e., isotropic, kinematic and mixed type) to verify the yielding condition in the material

$$\begin{cases} F(t_{p+1}) = E_0\epsilon^{ve}(t_{p+1}) \\ \sigma_y(t_{p+1}) = \sigma_{y_0} + K\lambda(t_{p+1}) \end{cases} \tag{18}$$

$$\begin{cases} F(t_{p+1}) = \frac{E_0}{3}\epsilon^{ve}(t_{p+1}) + \frac{2}{3}\sigma_{y_0} \\ \sigma_y(t_{p+1}) = \sigma_{y_0} \end{cases} \tag{19}$$

$$\begin{cases} F(t_{p+1}) = \frac{E_0}{3}\epsilon^{ve}(t_{p+1}) + \frac{2}{3}\sigma_{y_0} \\ \sigma_y(t_{p+1}) = \sigma_{y_0} + K\lambda(t_{p+1}) \end{cases} \tag{20}$$

where σ_{y_0} is the initial yield stress.

If $F(t_{p+1}) < \sigma_y(t_p)$, the yielding condition has not been achieved at time step t_{p+1} and thus

$$\varepsilon^p(t_{p+1}) = \varepsilon(t_p) \tag{21}$$

$$\lambda(t_{p+1}) = \lambda(t_p) \tag{22}$$

Otherwise, the permanent deformation has to be updated to add the increment of plastic deformation, as expressed below.

$$\lambda(t_{p+1}) = \lambda(t_p) + \left(1 - \frac{E_0}{E_0 + K}\right)^{-1} \frac{E_0\left[\varepsilon^{ve}(t_{p+1}) - \varepsilon^{ve}(t_p)\right]}{E_0 + K} \tag{23}$$

$$\lambda(t_{p+1}) = \lambda(t_p) + \left(1 - \frac{E_0}{E_0 + c}\right)^{-1} \frac{3E_0\left[\varepsilon^{ve}(t_{p+1}) - \varepsilon^{ve}(t_p)\right]}{E_0 + c} \tag{24}$$

$$\lambda(t_{p+1}) = \lambda(t_p) + \left(1 - \frac{E_0}{E_0 + c + 9K}\right)^{-1} \frac{3E_0\left[\varepsilon^{ve}(t_{p+1}) - \varepsilon^{ve}(t_p)\right]}{E_0 + c + 9K} \tag{25}$$

Ultimately, the subsequent equations can be written

$$\varepsilon^p(t_{p+1}) = \varepsilon^p(t_p) + \left[\lambda(t_{p+1}) - \lambda(t_p)\right] \tag{26}$$

$$\varepsilon(t_{p+1}) = \varepsilon^{ve}(t_{p+1}) + \varepsilon^p(t_{p+1}) \tag{27}$$

where $\varepsilon^{ve}(t_{p+1})$ is computed by using means of Eq. (17).

The calibration method is based on the iterative use of two sub-procedures. The one-dimensional expression of the constitutive model represents the first sub-procedure (Eq. (27)). It uses the first attempt values of the model's parameters as well as the effective stresses, as input data to compute the total model strains. The model strains are subsequently compared to the total experimental total strains to update the model's parameters. The second sub-procedure relies on an optimization process of the iterative type, which minimizes the objective function $f(x) = \sum_{i=1}^{n}(\varepsilon_i^{EXP} - \varepsilon_i^{MOD})$. In such an expression, ε_i^{EXP} and ε_i^{MOD} are the experimental and model's total strains, respectively. Considered at the time step i, x represents the vector of the model's parameters, n expresses the total number of experimental data. To minimize the objective function, the simplex algorithm is adopted (Nocedal and Wright, 2006). Upon achieving an appropriate tolerance, the optimization process unveils the final model's parameters values. This minimization is developed using a constrained optimization approach. Acceptable ranges are defined for each parameter of the model to avoid the identification of physically meaningless values.

The experimental data used for the model's calibration are provided by the creep test, which was conducted at 100 kPa and 20 s. The other eight experimental creep curves are used for the validation of the model.

4. Results and discussion

The model's parameters values, estimated using the calibration procedure, for each of the hardening laws considered are reported in Table 2. Considering the isotropic hardening as the reference term, the percentage differences between the parameters values determined with the isotropic rule and those obtained using the other two hardening laws are also indicated. The calibration procedure has identified widely different values for each model parameter according to the different type of hardening laws, even if with reference to the same asphalt concrete, with differences up to over 300%.

Among the three viscous processes introduced in the model, the lowest elastic modulus and highest viscosity were obtained for the first one (E_1, η_1), independently from the hardening law considered. The highest viscosity for all the viscous processes was determined with the kinematic hardening.

The values of elastic modulus E_∞ are relatively similar among the different hardening laws. However, the kinematic and mixed type hardenings led to an increment of over 20% with respect to the isotropic hardening rule. Comparing E_∞

Table 2 – Constitutive parameter values.

Parameter	Model – isotropic hardening	Model – kinematic hardening	Isotropic vs kinematic (%)	Model – mixed hardening	Isotropic vs hardening (%)
E_1 (MPa)	19.083	11.849	−61	24.156	+27
η_1 (MPa·s)	229.808	366.157	+59	290.000	+26
E_2 (MPa)	71.231	133.412	+87	70.565	−1
η_2 (MPa·s)	53.507	79.519	+49	50.000	−7
E_3 (MPa)	93.373	20.009	−367	67.552	−38
η_3 (MPa·s)	187.339	274.811	+47	51.000	−267
E_∞ (MPa)	87.313	105.730	+21	108.727	+25
σ_{y_0} (MPa)	0.065	0.100	+54	0.106	+63
K (MPa)	339.730	1285.075	+276	81.099	−319

with E_1, E_2 and E_3, the elastic modulus of the isolated spring resulted higher than that of each single viscous process, but only for the mixed hardening.

For the initial yield stress, an increasing trend can be observed from the isotropic hardening to the mixed type hardening, with a maximum increment of 63%. However, for such a parameter, the value determined with the kinematic law and that obtained by the mixed type hardening resulted very similar.

The hardening modulus is the parameter for which the largest differences can be simultaneously in observed the kinematic and mixed type hardening, with respect to the isotropic hardening.

Lastly, for the five model parameters (E_1, E_2, η_2, η_3, K), the kinematic law and mixed hardening provide opposing trends (i.e., increasing rather than decreasing). It is difficult to establish a general trend for all the parameters at the changing of the hardening law based on any empirical rule. The specific numerical identification of such parameters is required.

Fig. 3 shows the experimental data of total strains compared to those calculated by means of the model, adopting the three sets of optimized parameters, for the calibration curve (100 kPa and 20 s). There is a close overlapping between the three numerical curves, associated to the sets of parameters optimized for the different hardening laws, and the experimental curve. From a qualitative point of view, the curvature of the numerical creep perfectly follows that of the experimental curve for both loading and unloading conditions. For peak strains and permanent strains, the percentage differences between the experimental data and model data, are always lower than 5%. Therefore, the constitutive model, with regards to the specific hot mix asphalt studied, has been properly calibrated for each of the hardening laws. The mixed hardening law led to the lowest differences between the experimental and model data for the permanent strains (0.68%), as well as for the peak strains (1.99%).

With regards to the validation phase, the largest deviations between the experimental and model data, in terms of both peak and permanent strains, are computed for the stress of 500 kPa and loading time of 30 s. Fig. 4 presents the relative creep curves, while Table 3 reports the numerical data. The quantities $\Delta\varepsilon^{max}_{MOD-EXP}$ and $\Delta\varepsilon^{p}_{MOD-EXP}$ represent the difference between the model strain and the experimental strain, divided by the experimental strain, expressed as percentage, for both the peak and the permanent strain.

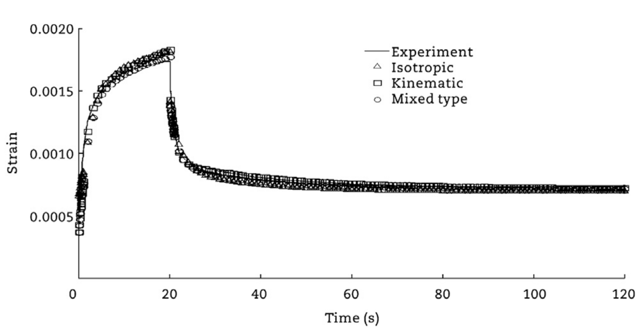

Fig. 3 – Calibration phase – experimental and model curves at 100 kPa and 20 s.

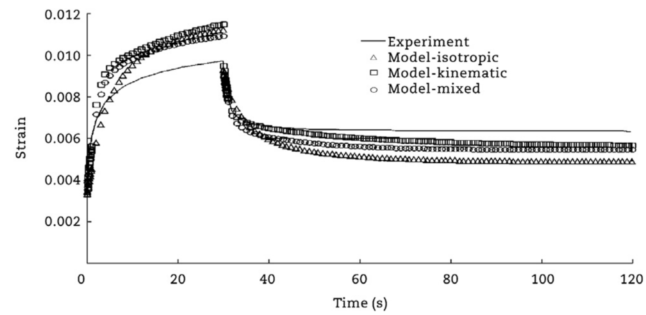

Fig. 4 – Validation phase – experimental and model curves at 500 kPa and 30 s.

Table 3 – Model validation results (MOD from model, EXP from experiment).

Data origin	ε^{max} (%)	ε^{p} (%)	$\Delta\varepsilon^{max}_{MOD-EXP}$(%)	$\Delta\varepsilon^{p}_{MOD-EXP}$(%)
Experimental	0.009737	0.006347	–	–
Model-isotropic	0.011233	0.004886	+15.4	−23.0
Model-kinematic	0.011519	0.005670	+18.3	−10.6
Model-mixed	0.010937	0.005447	+12.3	−14.2

The numerical creep curves maintain, qualitatively, the same trend of the experimental curves for all hardening laws. However, the numerical curves shifted towards higher strain values during loading, whereas the model estimated lower values for the permanent strain. However, focusing the analysis on the peak strain during loading and on the permanent strain during unloading, the greatest percentage difference between the numerical and experimental data is about 18% in terms of maximum strain. The difference never exceeds 23% for the unrecoverable strain, depending on the hardening law considered. Hence, the divergence between the numerical model and experimental results, even if it is not negligible, remains reasonably limited for all the curves analyzed in the validation. The numerical model has been positively validated.

With respect to the isotropic law, the mixed type hardening presented lower differences for both peak and permanent strains. Focusing on the permanent strains, which are linked to the rutting phenomena of flexible pavements, the set of parameters linked to the isotropic hardening were less preferable since they led a higher underestimation of the permanent strains (– 23%).

Even though it is the repeated load condition that should allow appreciating in a more remarkable manner the different response between the hardening models considered, significant deviations can be observed also with the static load condition used in this study.

5. Conclusions

The research study described in this paper identifies the parameters of a visco-elastoplastic constitutive model developed for hot mix asphalt that is typically used for road pavements. The model formulation is characterized by three viscous processes and inviscid plasticity, of the associated type. Three different types of hardening laws (i.e., isotropic, kinematic and mixed) were implemented in the constitutive equations, characterized by nine material parameters, related to viscoelasticity and plasticity. The importance of an accurate mathematical modeling of the unrecoverable-plastic behavior in the rational analysis of the creep response of Hot Mix Asphalt for road flexible pavements, has been demonstrated. The one dimensional formulation of the constitutive model, expressed in numerical terms, represents the main part of the calibration procedure. The simplex algorithm is used for the parameter's optimization procedure. The model was calibrated and validated based on experimental creep recovery tests, performed at different stress levels and loading times on a high performance hot mix asphalt concrete for road pavements. However the proposed identification approach can also calibrate and validate the analyzed visco-elastoplastic model for other bituminous mixtures.

The different hardening laws produce a differentiation of the values of the constitutive parameters that cannot be neglected, depending on whether those plastic or those viscoelastic are considered, so demonstrating the complexity arising from a constitutive modeling of the creep response of a bituminous mixture. The experimental-numerical validation procedure verified that the proposed model gives an acceptable numerical interpretation of the most significant aspects of the creep recovery response of the hot mix asphalt concrete considered, both qualitatively and quantitatively, in terms of maximum and permanent strains.

The originality of the proposed approach is based on the introduction of plasticity into a physically congruent visco-elastic constitutive model. The new formulation proposed in this paper allows the possibility of predicting numerically the permanent deformation response of a flexible pavement under a static load at high service temperatures. Hence, it can enhance the thickness design of asphalt pavements. The model formulation in this paper must be further verified with respect to repeated load axial tests to better appreciate the potential significant contribution of the mixed type hardening law, compared to the more simple isotropic hardening rule.

Aiming to achieve a more general constitutive formulation, the present model should be further elaborated to comprise the Drucker–Prager yield conditions in order to model different response in tension and compression.

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