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Statistical evaluation of the softening or hardening behaviour of metallic materials

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Abstract

Detailed knowledge of the cyclic behaviour of a material is essential for the design of components which can be plastically deformed by cyclic loads. The design process based on the sole static characteristics could lead, in case of softening behaviour, to a significant underestimation of the plastic strain amplitude and to a consequential overestimation in terms of component life.

The experimental determination of cyclic characteristics is onerous compared to static properties, which can be obtained by a simple tensile test. The objective of this study is to derive a model to evaluate, starting from the knowledge of tensile variables alone, whether the material subject to cyclic loads hardens or softens.

The proposed approach is statistical, based on a sample of about 240 materials. The predictive model results from a multinomial logistic regression, which allows deriving a relationship between independent input variables (tensile variables of the materials) and a dependent output variable, represented by the material belonging to one of the following behaviour categories: hardening, softening, mixed (hardening or softening depending on the load level), stable (small differences between static and cyclic behaviour).

To determine which tensile variables significantly influence the cyclic behaviour, different regressions based on no more than three parameters are compared, to result in a simple and functional calculation tool. Finally, several correlations from the literature are considered, comparing the associated results with those from the derived models. The comparison highlights higher goodness-of-fit for the proposed approach with respect to the state of the art, demonstrating its predictive potential.

Keywords: tensile variables; cyclic variables; logistic regression; probability; design.

1. Introduction

In the design process of a mechanical component, possibilities regarding the material to be employed require timely screening. The phase in which static stresses insisting on the component are quantified is typically followed by the analysis of fatigue loads; occasionally, these activities are not sufficient to guarantee integrity of the component in the long term: in the case of metallic components, sporadic loads higher than static and fatigue limits can modify the

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residual life of the component itself depending on the behaviour of the specific material. The rheological behaviour of metallic alloys subject to repeated stresses in the elasto-plastic field is typically based on the degree of similarity between the cyclic curve and the tensile curve of the material; mainly four conditions can be met, each defining a different category of material:

- Hardening - the material increases its strength if stresses leading to plasticization are applied;
- Softening - a plasticization in the material reduces its mechanical resistance compared to the static case;
- Mixed - the cyclic curve and the tensile curve have an intersection point;
- Stable - there are no significant differences between the tensile and the cyclic curve.

From this classification follows that, if an equal load leading to plasticization is applied, the choice of a softening material in the design phase of a component must be associated with higher safety factors; a sufficient degree of confidence in the category to which the material belongs is fundamental to ease the design process.

The retrieval of information related to the cyclic behaviour of a metallic alloy in the plastic field is typically achieved by testing several specimens, setting the amplitude of the strain for a specimen but varying the amplitude among several specimens (multiple step test); alternatively, it is possible to apply cyclic strain sequences with constant amplitude while periodically increasing such amplitude (single step test), or even to subject the single specimen to repeated blocks of incrementally increasing and decreasing strain (incremental step test) as described by Dowling (2012) and Jones and Hudd (1999). Reviews on typical cyclic curves obtained using these techniques are given in Klesnil and Lukác (1992) and Skelton (1987). A complication in the second method is related to the cyclic behaviour of the material itself, which is occasionally a function of the loading sequence for the single specimen as observed by Belattar et al. (2012); in addition to the variability of the experimental methods, further uncertainties are linked to the test data analysis procedures which can be employed (e.g., refer to Hales et al. (2002), Tomasella et al. (2011) and Zonfrillo and Nappini (2015)). Although differences exist in the results of the three methods, Socie and Morrow (1980) highighted that such differences are negligible in most cases. At any rate, the tests to be performed require more resources than traditional tensile tests; for this reason, cyclic variables are typically retrieved through the use of approximate formulations based on tensile variables. These relations allow obtaining the cyclic strength coefficient (K') and the cyclic hardening exponent (n') contained in the classical relation of the cyclic curve (Eq. 1):

$$\frac{\Delta\epsilon}{2} = \frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma}{2K'}\right)\frac{1}{n'} \tag{1}$$

In Eq. 1, $\Delta\sigma$ represents the amplitude of the cyclic stress, $\Delta\epsilon$ the amplitude of the cyclic strain, while E is the elastic modulus; this equation constitutes the extension to the cyclic field of the Ramberg-Osgood law. Once the two parameters K' and n' are known, it is possible to establish the cyclic behaviour category for the material by direct comparison between the resulting cyclic curve and the tensile curve. At the state of the art, many studies are focused on the quantification of K' and n' for metallic alloys, Zhang et al. (2009), Lopez (2012) and Zonfrillo (2017) among the others; further investigations aimed at identifying the parameters contained in the Manson-Coffin law of Eq. 2, where $2N_f$ is the number of reversal at failure, σ'_f, ϵ'_f, b and c characteristic coefficients:

$$\frac{\Delta\epsilon}{2} = \frac{\sigma_f'}{E}(2N_f)^b + \epsilon_f'(2N_f)^c \tag{2}$$

Given the compatibility relations $b/c=n'$ and $\sigma'_f/\epsilon'^{n'}_f = K'$, both parameters n' and K' can be obtained based on Eq. 2. Although the two equations are closely related, Li et al. (2018) observed that significant differences subsist.

The correlations available from the literature which tie cyclic and tensile variables can, for different reasons, be inadequate to determine the mechanical characteristics of materials. First, the sample from which the correlations are derived may be limited to a few decades of sample materials, as in the case of dated studies (e.g., Zhang et al. (2009)). Conversely, more recent studies like the one by Meggiolaro and Castro (2004) show that the characteristics of the materials are extremely scattered among similar materials (iron alloys, aluminium alloys and titanium alloys): considering a tolerance of ± 10% between the real value of the cyclic variable and the value calculated from the tensile data, a correspondence for about 55% of the sample materials can be observed; this suggests how the correlations between cyclic and tensile variables should be cautiously employed in the design process as suggested by Meggiolaro and Castro (2004) and Li et al. (2016).

In the specific field of application, the use of tensile characteristics of the material to predict its behaviour is relevant, enabling to avoid creation of correlations with the cyclic variables. Once the behaviour has been determined, the designer is able to establish whether this basic information is sufficient to continue in the design process, or whether the data are to be integrated with quantitative analyses; it is possible, for instance, to determine the decrease in the cyclic characteristics of a softening material with respect to the tensile characteristics by appropriate tests, to apply the suitable safety factor.

The objective of the present work is to propose a methodology for the prediction of the hardening, softening, mixed or stable behaviour of the material starting from correlations based on its tensile variables. The correlations are obtained through a multinomial logistic regression on the available data, which allows associating the value of the tensile variables with the probability to belong to a specific behaviour category. The proposed correlations, deduced from a database that collects the tensile characteristics of more than 240 metallic alloys, involve no more than three tensile variables simultaneously: this allows the use of correlations also in the case in which only some tensile characteristics of the material are known, resulting in a simple and functional tool. Finally, comparison with consolidated approaches from literature is carried out, to highlight the superior goodness-of-fit of the proposed correlations and the appropriateness of the developed methodology for design purposes.

2. Materials and methods

The present Section collects the tools necessary to develop the methodology by which the cyclic behaviour of a material can be predicted. The sample materials on which the analysis is based is described in detail first; next, correlations available from literature are described and the results of their application to the sample materials is highlighted: such correlations represent a valid reference for the performance assessment of the proposed methodology. Finally, the relevant statistical concepts are reported for the derivation of multinomial logistic regression models, representing the basis of the methodology.

2.1. Database of materials

The database used to derive correlations between tensile variables and the cyclic behaviour of the material consists of 242 alloys, including iron, aluminium and titanium alloys. The characteristics of these materials are obtained from different bibliographical sources (Zhang et al. (2009), Lopez (2012), Lopez and Fatemi (2012), Boller and Seeger (1987), and Bäumel et al. (1990)). The determination of the behaviour of the single alloy is based on the values of E, K and n for the tensile curve (K and n being the static strength coefficient and static hardening coefficient respectively) and E, K' and n' for the cyclic curve (Eq. 1): considering a range of strain between the two limit values 0.2% (beginning of plasticization) and 2% (life measurable in a few decades or hundreds of cycles), the material softens if the cyclic stress is always lower than the static stress; conversely, the material hardens if the cyclic stress is always higher than the static one. If the cyclic curve is always within a range of ± 5% in respect to the static value, the material is classified as stable; on the other hand, the material has a mixed behaviour if it softens when low strains are applied and hardens when high strains are applied (or vice versa). Figure 1a shows the subdivision of the sample in terms of type of alloy; as can be deduced, the number of iron alloys is preponderant. Figures 1b-c depict the distribution, as a whole and according to the type of alloy respectively, of the cyclic behaviour categories for the sample materials. Figure 1d summarises the number of materials for which a specific tensile variable is available; the study involves nine tensile variables and two cyclic variables, which are not fully available from the literature for the totality of the sample materials: E, yield strength (σ_y), ultimate tensile strength (σ_u), elongation at failure (A_s), area reduction at failure (Ψ), n, K, the real stress and strain at failure (σ_f and ϵ_f respectively), $n'$and K'.

2.2. Correlations available from literature

In the scientific literature, several laws are available to determine the cyclic behaviour of a material, in particular the rules by Smith et al. (1963), Landgraf (1969), Zhang et al. (2009), and Daunys and Šniuolis (2006). In 1963, Smith et al. (1963) are the first to consider a sample of 17 materials, deriving the so-called Manson's rule: the material

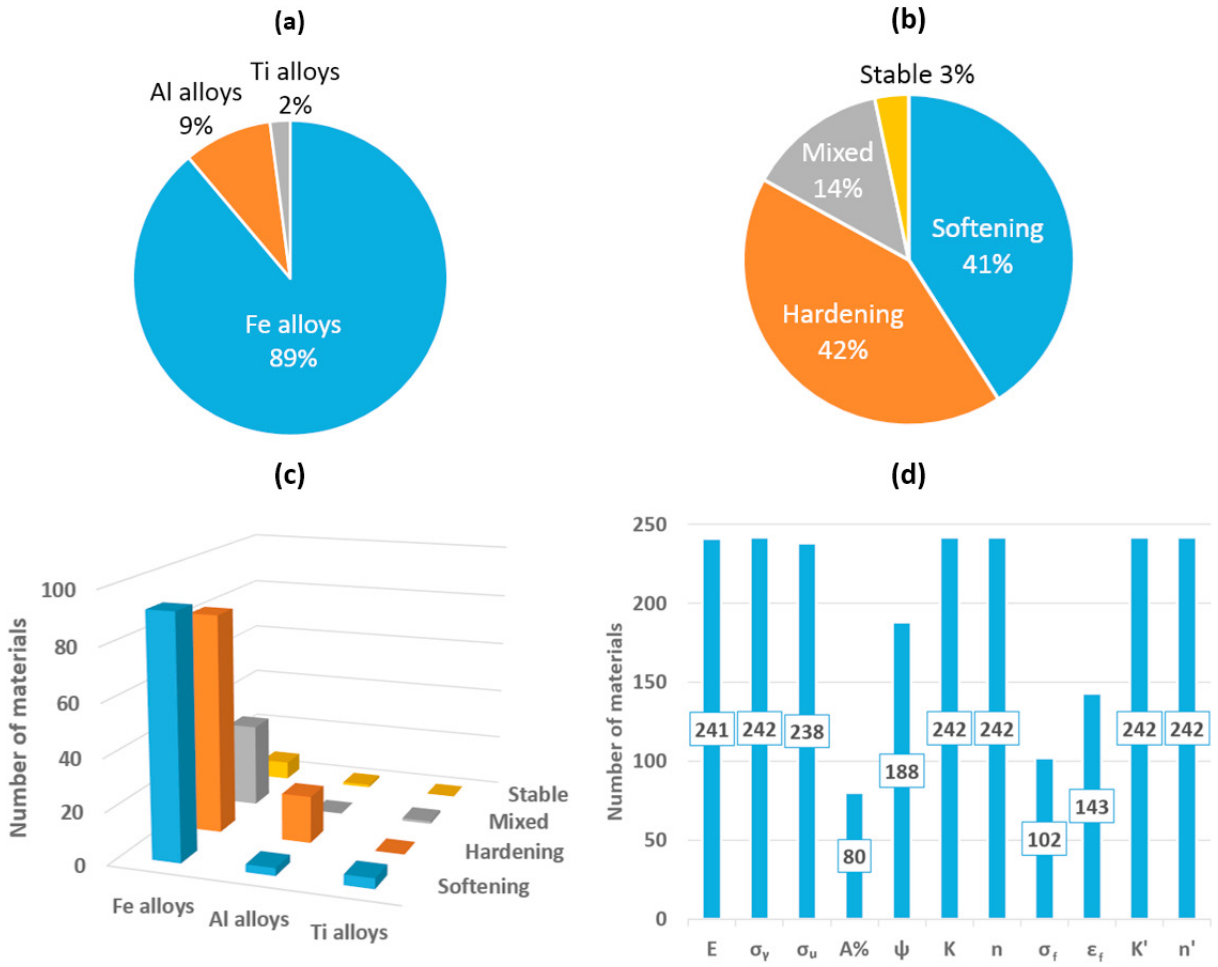

Fig. 1. Distribution of the materials constituting the sample in terms of the type of alloy (a), cyclic behaviour (b), type of alloy and cyclic behaviour (c), availability of tensile variables (d).

hardens in the case $\sigma_u/\sigma_y>1.4$, softens if $\sigma_u/\sigma_y<1.2$ and has undefined behaviour otherwise. Figure 2 shows the predictive capabilities of the Manson's rule applied to the employed database of 242 materials, for which the distribution is reported as a function of the cyclic behaviour and the σ_u/σ_y ratio.

The subsequent rule by Landgraf (1969) provides for the analysis of the hardening exponent n: in case $n<0.1$ a softening behaviour is expected for the material, in case $n>0.2$ a hardening behaviour is predicted while mixed or stable otherwise. Similarly to Figure 2, Figure 3 shows the distribution of the n coefficient among the materials with different cyclic behaviour contained in the database.

For the analysis of the cyclic behaviour, Zhang et al. (2009) propose the use of a fracture ductility parameter $\alpha=\Psi\epsilon_f$, whose value is proportional to the material's ability to plastically deform: based on a sample of 40 alloys of iron, aluminium and titanium, the analysis shows that if $\alpha<2\%$ or $20\%<\alpha<65\%$ the material softens; otherwise, the alloy hardens. From the relation between ϵ_f and Ψ, it also turns out that $\alpha=\epsilon_f\,(1-exp(-\epsilon_f))= -\Psi ln(1-\Psi)$; given the three previous equations which are formally identical, it is possible to use them alternatively if a parameter among the constituents is not known. Therefore, a single formulation for the parameter of Zhang α is employed, whose value is $\Psi\,\epsilon_f$ if both parameters Ψ and ϵ_f are known for the material, $\epsilon_f(1-exp\,(\epsilon_f))$ if Ψ is not known and $-\Psi ln(1-\Psi)$ in case ϵ_f is not known. Figure 4 shows the α parameter value for the materials contained in the available database; the α datum is available for 194 of the 242 materials.

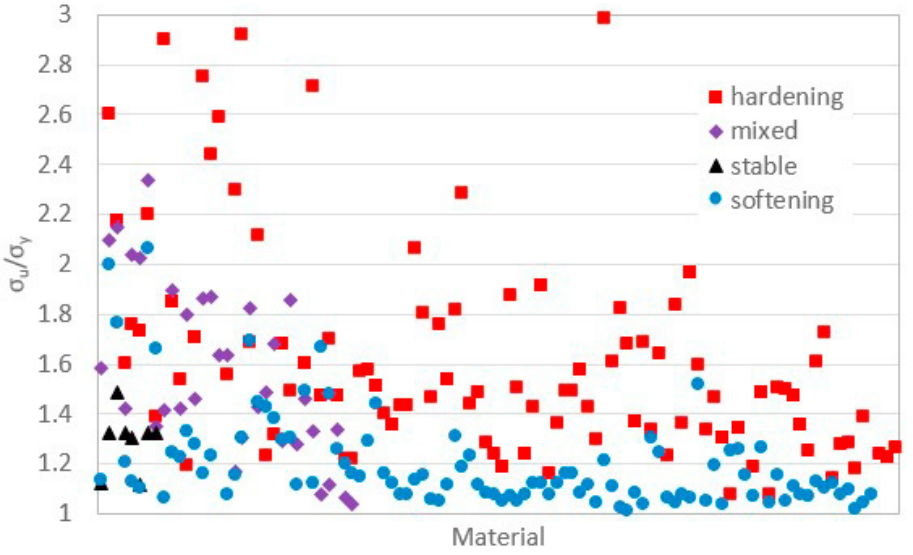

Fig. 2. Distribution of σ_u/σ_y among the sample materials (Manson's rule).

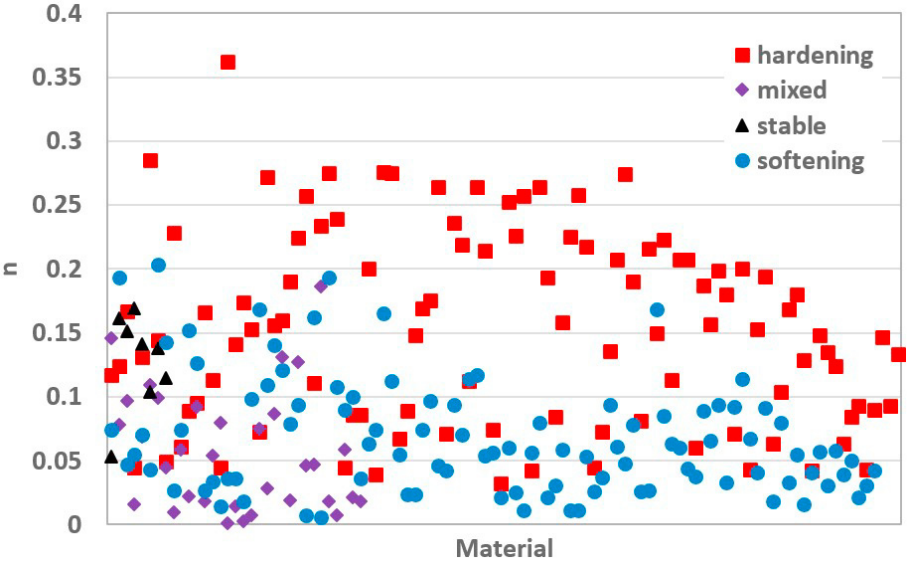

Fig. 3. Distribution of n among the sample materials (Landgraf's rule).

Based on 47 materials, Daunys and Šniuolis (2006) demonstrate how the cyclic behaviour of the alloy can be inferred from the parameters Ψ and σ_u/σ_y: if $\sigma_u/\sigma_y>1.8$ the material hardens, if $\sigma_u/\sigma_y<1.4$ and $\Psi<0.7$ the material softens, when $\sigma_u/\sigma_y<1.4$ and $\Psi>0.7$ the material behaviour is stable; for $1.4<\sigma_u/\sigma_y<1.8$ a transition zone can be highlighted in which, independently of Ψ, both softening, hardening or stabilization occur. To graphically explain the relations provided by Daunys, Figure 5 illustrates the distribution on the $\Psi,\sigma_u/\sigma_y$ plane of the materials contained in the analysed database.

Table 1 summarises, in an intelligible and complementary way to Figure 2-5, the results of the four proposed rules application to the available database of materials. As for the softening behaviour, Landgraf's rule allows obtaining the best results, with a high number of correct predictions (81%) and reduced errors (1%); for the remaining 18% of the cases, a specific behaviour is not predicted. Considering only the hardening materials, the best performances are associated with Manson's rule. For what regards mixed and stable materials, Zhang's rule is inapplicable as it

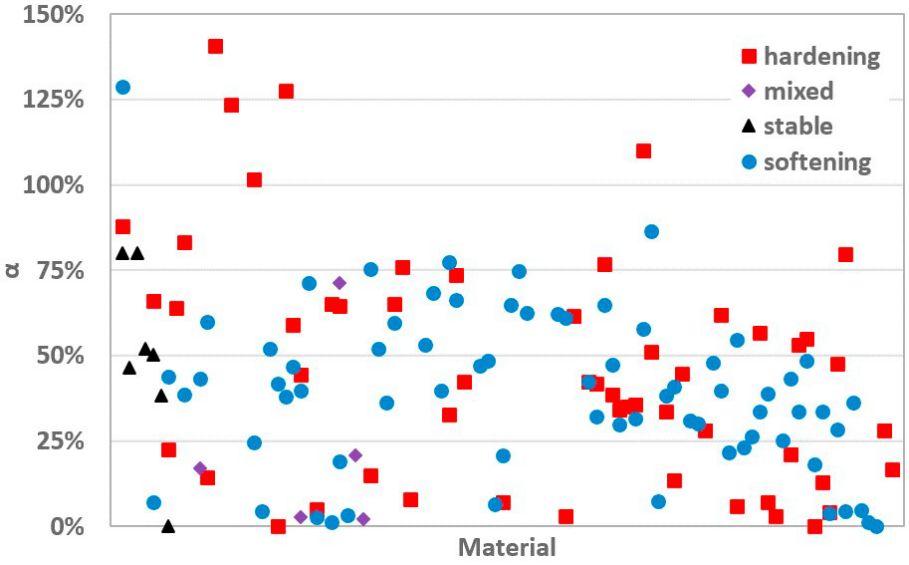

Fig. 4. Distribution of α among the sample materials (Zhang's rule).

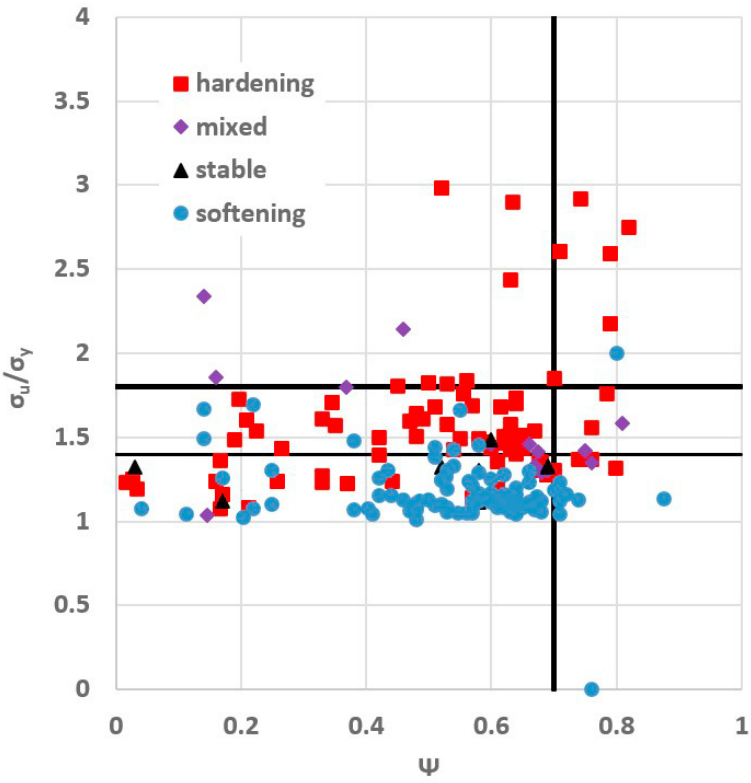

Fig. 5. Distribution of σ_u/σ_y and Ψ among the sample materials (Daunys's rule).

is limited to predicting softening or hardening behaviour alone; the whole set of considered rules are characterized by limited predictive ability, with Manson's rule being able to sufficiently limit the number of errors (38%). Overall, Manson's rule is characterized by the greatest number of correct predictions (55%) and a minimum number of errors (21%).

Table 1. Performances of the rules available from literature in terms of behaviour prediction for the materials constituting the sample.

Behaviour	Prediction	Manson	Zhang	Landgraf	Daunys
Softening	Number of materials	98	88	99	98
	Correct predictions	67.3%	68.2%	80.8%	71.4%
	Incorrect predictions	12.2%	31.8%	1.0%	17.3%
	No prediction	20.4%	0.0%	18.2%	11.2%
Hardening	Number of materials	99	81	102	99
	Correct predictions	65.7%	49.4%	28.4%	23.2%
	Incorrect predictions	8.1%	50.6%	31.4%	33.3%
	No prediction	26.3%	0.0%	40.2%	43.0%
Mixed	Number of materials	33	8	8	8
	Correct predictions	0.0%	0.0%	0.0%	0.0%
	Incorrect predictions	81.8%	100.0%	12.5%	87.5%
	No prediction	18.2%	0.0%	87.5%	12.5%
Stable	Number of materials	8	8	8	8
	Correct predictions	0.0%	0.0%	0.0%	0.0%
	Incorrect predictions	37.5%	100.0%	12.5%	87.5%
	No prediction	62.5%	0.0%	87.5%	12.5%
Total	Number of materials	238	194	242	238
	Correct predictions	55.0%	51.5%	45.0%	39.1%
	Incorrect predictions	21.0%	48.5%	25.6%	32.8%
	No prediction	23.9%	0.0%	29.3%	28.2%

2.3. Multinomial logistic regression

Logistic regression (or logit) is a regression method which is applied when the values of the output variable are Boolean rather than continuous; the logit field of application ranges from the analysis of defect detection capabilities in production processes (probability of detection curves, as described in Guo et al. (2006)) to road safety - Vangi et al. (2019). The output is approximated by a function of m independent variables x_j (also called features); the analytical expression of the function (a sigmoid if two features are employed) is represented by Eq. 3:

$$p = \frac{1}{1 + exp(a_0 + \sum_{j=1}^{m} a_j x_j)} \tag{3}$$

The $m+1$ coefficients of best approximation a_0 and a_j ($j=1...m$) are determined by error minimization methods. In the particular case in which the output is not represented by values 0 and 1 alone but by a categorical variable, multinomial logistic regression is referred to: the formulation in Eq. 3 is still valid for the prediction of the i-th level of the categorical variable; nevertheless, assuming $i=1...l$, the probability to obtain the i-th output is to be expressed relative to the probability pb of obtaining a reference level (called baseline):

$$ln(\frac{p_i}{p_b}) = \frac{1}{1 + exp(b_0 + \sum_{i=1}^{1} b_i x_i)} \tag{4}$$

From Eq. 4 directly follows that $\sum_{i=1}^{l} p_i/p_b = \sum_{i=1}^{l} exp(y_i)$; based on the relation $p_b + \sum_{i=1}^{l} p_i = 1$, the probability to get the i-th level as an output and the baseline are respectively reported in Eqs. 5-6:

$$p_b = \frac{1}{1 + \sum_{i=1}^{1} exp(y_i)} \tag{5}$$

Table 2. Regression coefficients for several remarkable logit models; the "softening" behaviour is considered as the baseline.

Features	Coefficients	Softening	Hardening	Mixed	Stable
$f_1=ln(\epsilon_f)$ $f_2=ln(\alpha)$ $f_3= ln(\sigma_y)$	b_0	-	28.28089	10.16496	9.182184
	b_1	-	-0.13768	-2.09979	0.339301
	b_2	-	0.024766	-0.11952	0.040093
	b_3	-	-4.44346	-2.09634	-1.73681
$f_1=ln(K)$ $f_2=ln(n)$ $f_3= ln(\sigma_u)$	b_0	-	33.1706	18.03498	9.947907
	b_1	-	-7.45408	-8.79049	-0.69825
	b_2	-	4.083877	1.219599	1.690081
	b_3	-	4.276269	6.659731	-0.47914
$f_1= \epsilon_f$ $f_2=K$ $f_3=n$	b_0	-	-0.18601	2.087147	-4.25493
	b_1	-	-0.01749	-2.42881	-0.02598
	b_2	-	-0.00284	-0.0023	-0.00036
	b_3	-	32.07813	-8.41461	24.27729
$f_1=\sigma_u$ $f_2= \sigma_u/\sigma_y$	b_0	-	-4.269	-5.00546	-3.95593
	b_1	-	-0.00336	-0.00305	-0.00108
	b_2	-	5.228917	4.873493	2.031187
$f_1=ln(K)$ $f_2=ln(n)$	b_0	-	33.68488	24.0729	11.16145
	b_1	-	-3.79222	-3.55744	-1.20294
	b_2	-	3.016824	0.308149	2.049434

$$p_b = \frac{\sum_{i=1}^{1} exp(y_i)}{1 + \sum_{i=1}^{1} exp(y_i)} \tag{6}$$

Different methods are available for the calculation of the coefficients constituting y_i function, mainly based on the statistical characteristics of the input data; the maximum a posteriori estimation and the iterative least-squares method (LSM) are mentioned, respectively extensions of the maximum likelihood and LSM to the specific problem.

3. Results

To develop the models for material behavior prediction, it is possible to combine the nine tensile variables described in Section 2.1 with five variables from the literature (Section 2.2): σ_u/σ_y, ϵ_f/Ψ, $\epsilon_f \cdot \Psi$, ϵ_f(1-exp (ϵ_f)), -$\Psi ln(1-\Psi)$; the resulting 14 features can be considered at the same time to obtain extremely complicated logistic regression models. In order to limit the complexity of these models, in the present work the probability for a material to fall into a specific category of cyclic behaviour is set to depend at most on three features. This results in 14 possible regression models based on a single feature, 91 on two features and 364 on three. It is also possible to develop further 469 models by considering the natural logarithm of each feature, allowing deleting the exponential component from Eqs. 5-6; therefore, the total of the considered tested models amounts to 938. For the development of multinomial logistic regression models, the softening behaviour was considered as the baseline; the derived coefficients for the specific regression model would be the same if another category is considered as the baseline. Table 2 shows the regression coefficients for several remarkable models obtained, and the features from which these models are derived (Eq. 4).

The example of logistic regression model in Figure 6, for which σ_u and σ_y are considered, evidences that an output is always derived, regardless of congruence between the value of the features and physical reality (σ_u must be higher than or equal to σ_y for any material). Additionally, Figure 6 highlights that the logistic regression model obtained does not in itself provide a prediction on the actual behaviour of the material, but the probability that the behaviour of the material is softening, hardening, mixed or stable based on the value of its features. Therefore, it is necessary to propose a criterion which, starting from the derived probabilistic data, allows deducing the actual behaviour of the material.

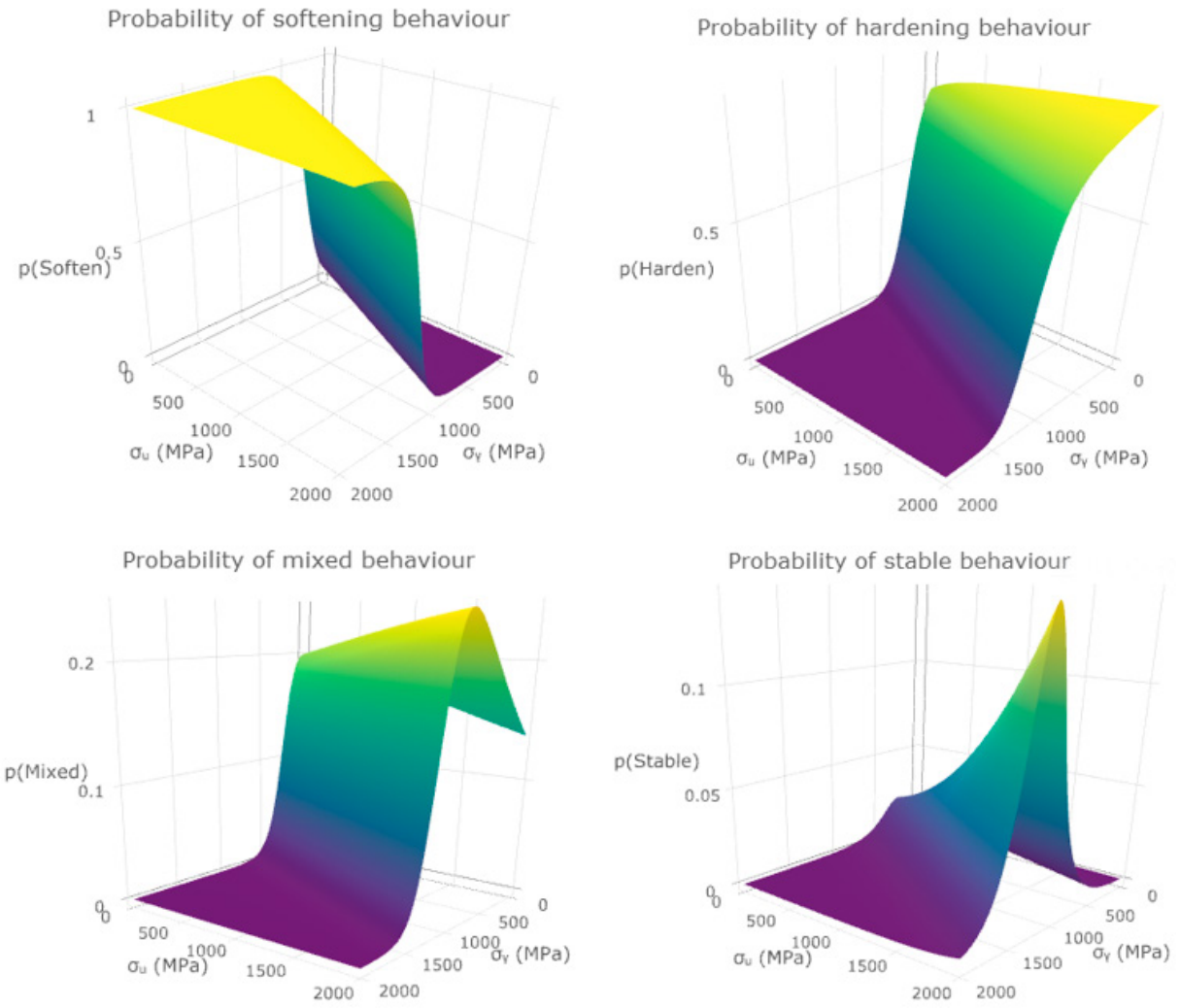

Fig. 6. Probability for a material to belong to each behaviour category based on its features (σ_u and σ_y in the example)

For the prediction of the actual behaviour of the material, a criterion is proposed based on a threshold value T: given the values of the tensile variables of a specific material, the behaviour of the material is the one associated with the maximum probability among those that exceed T; if none of the probabilities exceed T, no specific behaviour is associated with the material. The choice of a specific threshold T has a double effect on the predictive capabilities of the methodology: as T increases, it is possible to predict the rheological behaviour of a smaller quantity of sample materials (i.e., the behaviour of an ever-increasing number of materials is not predicted); on the other hand, a high value of T allows obtaining a high number of correct predictions on the total of the materials for which the prediction is not ambiguous. Given the fundamental importance of the T parameter, a variability has been considered for T between 0.4 and 0.8 with a step of 0.1 for each logistic regression model derived: in correspondence with each value of T, it is necessary to highlight how many materials are correctly classified by the model and how many are incorrectly classified. To this end, the concepts of sensitivity S and precision R are introduced:

$$S = \frac{TP}{TP + FN} \tag{7}$$

$$R = \frac{TP}{TP + FP} \tag{8}$$

Table 3. Number of materials on which the remarkable models of Table 3 are based and related F_1 scores for each behaviour category.

Features		Softening	Hardening	Mixed	Stable	Total
$f_1=ln(\epsilon_f)$	Number of materials	72	59	5	7	143
$f_2=ln(\alpha)$	F_1 score	93.1%	84.7%	0.0%	14.3%	82.5%
$f_3=ln(\sigma_y)$	Correct predictions	93.1%	84.7%	0.0%	14.3%	82.5%
$f_1=ln(K)$	Number of materials	98	99	33	8	238
$f_2=ln(n)$	F_1 score	91.8%	82.8%	43.1%	0.0%	78.2%
$f_3= ln(\sigma_u)$	Correct predictions	91.8%	82.8%	42.4%	0.0%	78.2%
$f_1= \epsilon_f$	Number of materials	72	59	5	7	143
$f_2=K$	F_1 score	94.4%	85.5%	0.0%	0.0%	82.5%
$f_3=n$	Correct predictions	94.4%	84.7%	0.0%	0.0%	82.5%
$f_1=\sigma_u$	Number of materials	98	99	33	8	238
$f_2= \sigma_u/\sigma_y$	F_1 score	90.3%	86.9%	0.0%	0.0%	73.1%
	Correct predictions	89.8%	86.9%	0.0%	0.0%	73.1%
$f_1=ln(K)$	Number of materials	99	102	33	8	242
$f_2=ln(n)$	F_1 score	88.8%	80.8%	36.9%	0.0%	74.8%
	Correct predictions	87.9%	80.4%	36.4%	0.0%	74.8%

where *TP* represents, for a specific category of material, the number of true positive predictions, *FN* the number of false negative predictions and *FP* the number of false positive predictions; these individual concepts are generally summarised in the so-called "confusion matrix". In the specific problem, *S* represents the ratio between the frequency of threshold exceeding which leasd to correct classification of the material and the total number of materials with such behaviour within the sample; *R* instead represents the ratio between the frequency of threshold exceeding which leads to correct classification of the material and the number of times such behaviour is predicted. Finally, the so-called F_1 score (representing the harmonic mean of *S* and *R*) allows assessing the actual predictive ability for the specific combination of model and threshold considered. Table 3 shows, for the models highlighted in Table 2, the number of materials on which these models are based and the associated F_1 scores for each material category.

4. Discussion

First, models have been excluded for which the combination of features is not available for at least 100 materials. The highest goodness-of-fit results from an imposed value of *T* equal to 0.4, for all derived logit models; as shown in Table 3, considering logarithmic values of the features often leads to better predictive abilities. Starting from F_1 scores, an inappropriateness of all the derived models in predicting both stable and mixed behaviour is also highlighted. Based on the data reported in Table 3, the model for which ϵ_f, *K* and *n* are the considered features can be pointed out as the best one in terms of softening behaviour prediction (94% correctly predicted). Although this model can be extremely useful if applied during the design phase, it is clear that considering different features as $ln(K)$, $ln(n)$ and $ln(\sigma_u)$ results in a model with comparable goodness-of-fit, but more robust as a consequence of the higher number of materials used for the calculation (143 against 238). The predictive ability on the entire sample for this latter model is lower, also due to a lower predictive ability in terms of hardening behaviour. To highlight the predictive abilities of both models, Fig. 7 shows the distribution of the sample materials based on the values of ϵ_f, *K*, *n* (Fig.7a) and $ln(K)$, $ln(n)$, $ln(\sigma_u)$ (Fig. 7b), as a function of the materials' behaviour; green-coloured points represent correct predictions of material behaviour, red-coloured points represent incorrect predictions.

Considering the whole set of materials, the aforementioned combination of $ln(K)$, $ln(n)$, $ln(\sigma_u)$ turns out to be the absolute best, if features are excluded whose data are not available for almost all materials collected in the database. Other combinations of three features in which at least one of the two "plastic" parameters of the Ramberg-Osgood relation (*K*, *n*) is present perform similarly, as well as the combination of *K* and *n* which correctly predicts the behaviour of 75% of the sample. Nevertheless, the values of *K* and *n* are often absent in databases collecting material properties, available from web sources or technical literature. Excluding these parameters, the best performances are exhibited by combinations between σ_u and σ_y and in particular by those based on σ_u and σ_u/σ_y. A prediction based on this logit

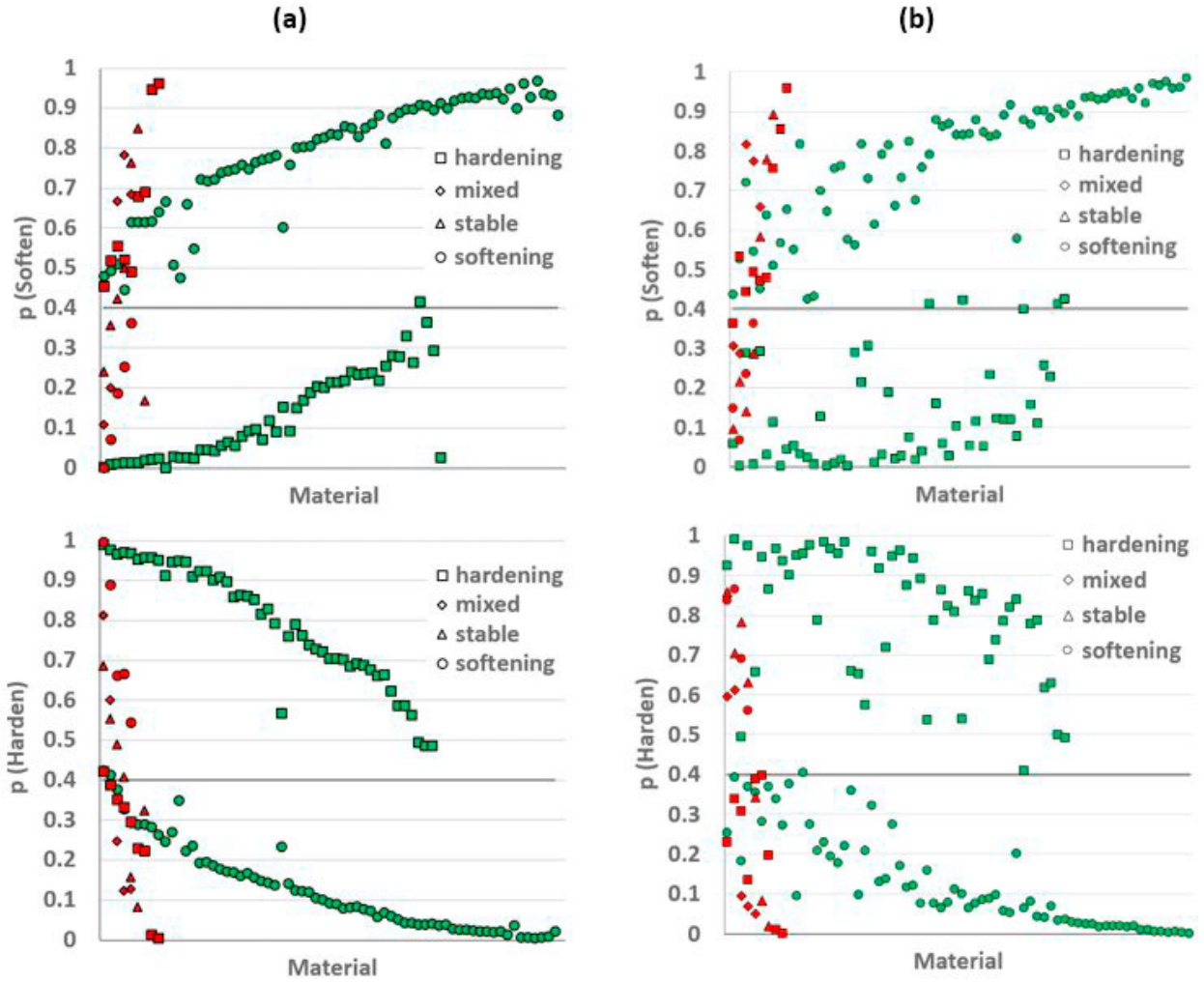

Fig. 7. Distribution of the sample materials as a function of their behaviour, based on ϵ_f, K, n and $ln(K)$, $ln(n)$, $ln(\sigma_u)$ characteristics – (a) and (b) respectively; green-coloured points represent correct predictions of material behaviour, red coloured points represent incorrect predictions.

model can be directly compared with Manson's rule: from the comparison between the results collected in Table 1 and Table 3, an F_1 score higher than or equal to the percentage of correct predictions associated with Manson's rule is always observed, for all the categories of rheological behaviour; therefore, the use of logistic regression allows developing a model, based on the same features, with higher predictive abilities. Combinations including features related to the ductility of materials have been tested on fewer materials, as the value for several materials is not available for the employed database. The predictive abilities of the best combinations are slightly higher than those previously mentioned. In particular, the α feature (Zhang parameter) plays a decisive role. Its logarithm is present in one of the remarkable models reported in Table 2. It is worth noting that this feature must be always combined with one or two load parameters to obtain reliable logit models: considering α alone, the performances decrease considerably.

From a comparison with the data in Table 1, it is possible to highlight that the use of the various models proposed entails a significant increase in the predictive abilities with respect to the state of the art. Let us consider the model based on the $ln(K)$, $ln(n)$ and $ln(\sigma_u)$ features, for which data are available for almost the entire sample: analysing the prediction of softening behavior only, this model has a prediction rate higher than the one associated with Landgraf's rule (81%); the proposed model allows limiting errors also in terms of prediction for the remaining behaviours. Ultimately, the methodology integrates the state of the art focused on the prediction of the rheological behaviour of materials, expanding the data set on which the analyses are based; through this study the bijectivity between tensile

characteristics and cyclic behaviour of materials is mainly confirmed, as well as corroborated by the use of logistic regression; although quantitative aspects inherent to the cyclic variables are not covered, the highlighted methodology provides important suggestions on the most statistically relevant parameters to be considered for such quantification.

5. Conclusions

The present work illustrated a methodology to predict the softening, hardening, mixed and stable behaviour of iron, aluminium and titanium alloys. Compared to the literature in the specific field of application, an element of absolute novelty is the use of logistic regression models: these models allow associating the tensile and cyclic characteristics of one of the 240 sample alloys with the probability to belong to each of the four rheological behaviour categories; if a threshold value is exceeded by the maximum of those probabilities, the prediction on the actual behaviour of the material is finally provided. Considering the possible difficulties in retrieving the tensile and cyclic characteristics of materials, the coefficients of several logistic regression models based on different material features are given; this allows to expand the field of application of the methodology, according to the specific needs of the user.

To evaluate the performance of the methodology with respect to the state of the art, different rules from literature proposed over the years by Manson, Zhang, Landgraf and Daunys applied to the 240 alloys were considered. The accuracy of the results obtained through the use of the proposed methodology is significantly higher than the one for all the available rules from the literature: in particular, for several remarkable models a correct behaviour prediction is found for 82% of the sample, while the application of the rules from literature is associated with 55% of correct predictions; limiting the interest to the softening behaviour only, the correct predictions amount to 94% against 81% for the rules from literature.

An additional benefit deriving from the use of the methodology in the prediction process of the material rheological behaviour is to be found in the possibility of optimising its performance considering the appropriate input variables: the work focused exploratorily on logistic regression models in which the features coincide with single tensile or cyclic variables; however, different combinations of simple variables can be employed, for example by using exponential functions or products similarly to Zhang's approach: complex features can be generated, whose inclusion in the logistic regression models allows reaching a superior goodness-of-fit.

The excellent predictive abilities of the proposed methodology in terms of softening behaviour of material are particularly relevant for the designer: if a material with predicted non-softening behaviour is selected during the design phase of a mechanical component, it would be possible to avoid reconstruction of the cyclic curve by tests. Even if performing such tests on the materials selected in the design phase is generally advisable, the methodology represents a fundamental progress towards a considerable saving of resources; on the basis of the promising results achieved, the integration of additional materials, tensile variables associated with the sample and threshold-based criteria will allow strengthening the proposed models and the methodology as a whole.

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