# Power Allocation for Goodput Optimization in BICM-OFDM systems 

I. Stupia $\dagger$, L. Vandendorpe $\ddagger$, J. Louveaux $\ddagger$, F. Giannetti $\dagger$, V. Lottici $\dagger$, N.A. D’Andrea $\dagger$<br>$\dagger$ Department of Information Engineering, University of Pisa,<br>Via G. Caruso, 16 - I-56122 Pisa, Italy<br>$\ddagger$ Communications and Remote Sensing Lab, Université Catholique de Louvain, Place du Levant, 2 - B-1348 Louvain La Neuve, Belgium<br>$\{$ ivan.stupia, filippo.giannetti, vincenzo.lottici, aldo.dandrea \}@iet.unipi.it, $\{l u c . v a n d e n d o r p e$, jerome.louveaux $\}$ @uclouvain.be


#### Abstract

This paper deals with the power allocation problem for coded multicarrier transmission. Specifically, we focus on a bit interleaved coded modulation (BICM) packet transmission implemented with Orthogonal Frequency Division Multiplexing (OFDM) and in the presence of automatic repeat request (ARQ) protocol. Capitalizing on the binary-input output-symmetric (BIOS) nature of the BICM channel it is provided a simple upper-bound of the rate of information bits received without any error, the so called goodput. Based on this theoretical characterization, we develop a power allocation strategy among the different subcarriers so that the system goodput performance metric is maximized. The effectiveness of the proposed method is numerically testified for BICM-OFDM transmission in the context of the typical WLAN scenario.


## I. Introduction

At the high data rates typically required to support multimedia services, harsh multipath propagation conditions are typically experienced in both urban outdoor and indoor terrestrial scenarios, thereby making the design of an efficient and reliable transmission scheme a particularly demanding task. A viable answer to this need consists in a cross-layer design approach where a mix of up-to-date efficient techniques for modulation, channel coding and link adaptation are properly combined. In the context of high data rate transmissions over wireless frequency selective channels, one of the most efficient modulation formats is represented by multicarrier (MC) techniques. In the form of orthogonal frequency division multiplexing (OFDM), MC schemes have been embedded in several standards such as Wi-Fi WLAN IEEE $802.11 \mathrm{a} / \mathrm{g} / \mathrm{n}$, Wi-Max broadband wireless access IEEE 802.16, and digital audio and video broadcasting (DAB and DVB). In order to further increase the system robustness against the troubles arising from the wireless propagation channels, an efficient modulation scheme has to be combined, however, with an as much as powerful channel coding technique. This is the case of BICM, which was proposed in 1992 by Zehavi as a pragmatic coding scheme for bandwidth-efficient communications [1]. This is based on the insertion of a bit-interleaver between the channel encoder and the modulator in order to increase the diversity order. Later, a theoretical foundation for BICM was given by Caire, Taricco and Biglieri [2]. Different methods to evaluate the performances of BICM systems have been proposed. Most
of them are based on the union bound and the expurgation technique proposed in [2]. Recently, in [3], thanks to the binary-input output-symmetric (BIOS) nature of the channel, a simple yet accurate computation of the Pair-Wise Error Probability (PEP) based on the saddlepoint approximation has been provided. In [10] this approach has been extended to the MIMO-BICM systems. The superior code diversity and the design flexibility of BICM have motivated many common wireless applications. For instance IEEE 802.11 WLAN is an application of BICM which is implemented with OFDM.

The goal of a cross-layer design is fully achieved on condition that efficient link adaptation schemes be properly employed as well. The idea is to optimize the overall system performance under the constraint of fixed radio resources. To this end, the water-filling policy [11] has gained a considerable interest to power and bit resource allocation across the subcarriers of a OFDM-based system. In [4], through the extension of the conventional BICM-OFDM Pair-Wise Error Probability (PEP) analysis, a bit and power allocation algorithm has been proposed to improve the total bit-rate. However, in some WLAN-based applications, only error-free packets are kept by the receiver, while the others are retransmitted through an automatic repeat request (ARQ) retransmission mechanism. Therefore, an optimized allocation strategy has necessarily to maximize over the available resources the number of transmitted bits in the error-free packet by unit of time, or goodput for short. Based on the above baseline and differently from the works published so far in the literature, the aim of this paper is to present a power allocation strategy that aims at improving the goodput achievable in a packet-based BICMOFDM sytems. The theoretical foundation of the allocation algorithms consists in a simplified PEP analysis. After delineating in Sect. II the BICM-OFDM model, Sect. III will be devoted to the evaluation of a simple upper bound of the goodput metric. Simulation results will be provided in Sect. IV for a typical WLAN scenario, followed some concluding remarks in Sect. V.

Notations: Matrices are in upper case bold while column vectors are in lower case bold with an underscore. $(\cdot)^{T}$ is used to denote the transpose and $\mathbf{D}(\cdot)$ is the diagonalization operator which converts an $N$-dimensional vector into an $N \times$
$N$ diagonal matrix.

## II. System Description and Error Performance Analysis

## A. System Model



Fig. 1. System Model.
With reference to the block diagram depicted in fig. 1 a packet of $N_{b}$ information bits is transmitted through a frame of $L$ OFDM blocks. The information bits are encoded using a convolutional code with code rate $R$ and free distance $d_{\text {free }}$. The total number of coded binary symbols related to a packet of $N_{b}$ information bits is

$$
\begin{equation*}
N_{c}=\frac{N_{b}}{R} \tag{1}
\end{equation*}
$$

The total bandwidth is split into $N$ subcarriers and the number of bits allocated to the $n$th subcarrier is $m_{n}$ (which is independent from the OFDM block). The $N$-dimensional adaptive modulator is concatenated with the encoder through a bit-interleaver $(\pi)$ which randomly maps the coded bit $b_{k}$ into the bit $i\left(i=1, \cdots, m_{n}\right)$ of the subcarrier $n(n=1, \cdots, N)$ of the OFDM block $l(l=1, \cdots, L)$, denoted as $c_{l, n, i}$, with probability

$$
\begin{equation*}
\operatorname{Pr}\left\{b_{k} \rightarrow c_{l, n, i}\right\} \triangleq p(k, l, n, i)=\frac{1}{N_{c}} \tag{2}
\end{equation*}
$$

The interleaved bits are Gray mapped into the symbols of a $2^{m_{n}}$-QAM signal set $\chi^{(n)}$, which are transmitted with power $p_{n} \cdot S\left(0 \leq p_{n} \leq 1\right)$ where $S$ is the maximum value of the average power which can be allocated to every subcarrier. To simplify the analysis, the QAM symbols $\left\{x_{n}\right\}_{n=1}^{N}$ are normalized such that $\mathrm{E}_{x_{n}}\left[\left|x_{n}\right|^{2}\right]=1$ where $\mathrm{E}_{x}[\cdot]$ is the statistical expectation over the variable $x$. The power constraint is given by

$$
\begin{equation*}
\frac{1}{N} \sum_{n=1}^{N} p_{n} \leq 1 \tag{3}
\end{equation*}
$$

The elements of the data block are frequency mapped to the $N$ available subcarriers using an IFFT unit. A conventional cyclic prefix is appended at the beginning of each IFFT output block to maintain the subcarriers orthogonal with each others and avoid interference between successive symbols. The OFDM obtained signal experiences a frequency selective
fading channel. We will assume the channel stationarity during the whole packet duration.

At the receiver side the samples are collected into blocks of size $N+N_{c p}$. After removal of the cyclic prefix, they are transformed by an FFT unit of size $N$. Then, we end up with the expression of the output of the $n$th channel in the $l$ th OFDM block:

$$
\begin{equation*}
z_{n}(l)=\sqrt{S p_{n}} H_{n} x_{n}(l)+w_{n}(l) \tag{4}
\end{equation*}
$$

where $w_{n}(l)$ is a zero-mean unit-variance complex Gaussian random variable. For the sake of simplicity it is possible to use the following vectorial notation, as well

$$
\begin{equation*}
\underline{\mathbf{z}}(l)=\mathbf{A} \underline{\mathbf{x}}(l)+\underline{\mathbf{w}}(l) \tag{5}
\end{equation*}
$$

where $\mathbf{A}$ is a complex $N$-dimensional diagonal matrix defined as

$$
\begin{equation*}
\mathbf{A}=\mathbf{D}\left(\sqrt{S p_{1}} H_{1}, \sqrt{S p_{2}} H_{2}, \cdots, \sqrt{S p_{N}} H_{N}\right) \tag{6}
\end{equation*}
$$

In the following, in order to improve the readability of the paper the time index $l$ will be omitted. The symbol and the noise vector are $\underline{\mathbf{x}}=\left(x_{1}, x_{2}, \cdots, x_{N}\right)^{T}$, and $\underline{\mathbf{w}}=$ $\left(w_{1}, w_{2}, \cdots, w_{N}\right)^{T}$, respectively.

## B. PEP evaluation

Let $\underline{\mathbf{b}}$ and $\underline{\hat{\mathbf{b}}}$ denote two distinct codewords originating from the same state and merging after $d$ trellis steps. The aim of this sub-section is to evaluate the probability of the pairwise error event $\operatorname{Pr}(\underline{\mathbf{b}} \rightarrow \underline{\hat{\mathbf{b}}} \mid \mathbf{A})$. In case of ideal channel state information, the BICM log-likelihood metric for the $k$ th coded binary symbol at the decoder input can expressed as

$$
\begin{equation*}
\Lambda_{k}=\log \frac{\sum_{x_{k} \in \chi_{\hat{b}}^{\left(i_{k}, n_{k}\right)}} p\left(z_{k} \mid x_{k}, \mathbf{A}\right)}{\sum_{x_{k} \in \chi_{b}^{\left(i_{k}, n_{k}\right)}} p\left(z_{k} \mid x_{k}, \mathbf{A}\right)} \tag{7}
\end{equation*}
$$

where $p\left(z_{k} \mid \cdot\right)$ is the conditional probability density function of the random variable $z_{k}$, while $n_{k}$ and $i_{k}$ represent the subcarrier and the bit position into the symbol related to the $k$ th coded bit, respectively. Here, $\chi_{b}^{(i, n)}$ represents the subset of all the M-QAM symbols $x \in \chi^{(n)}$ whose $i$ th bit is equal to $b$. Let us denote the diagonal elements of the matrix $\mathbf{A}$ by $A_{n}$. So, the log-likelihood metric $\Lambda_{k}$ can be rewritten as

$$
\begin{equation*}
\Lambda_{k}=\log \frac{\sum_{x_{k} \in \chi_{\hat{b}}^{\left(i_{k}, n_{k}\right)}} \exp \left(-\left|z_{k}-A_{n_{k}} x_{k}\right|^{2}\right)}{\sum_{x_{k} \in \chi_{b}^{\left(i_{k}, n_{k}\right)}} \exp \left(-\left|z_{k}-A_{n_{k}} x_{k}\right|^{2}\right)} \tag{8}
\end{equation*}
$$

Under the assumption of ideal interleaving, the BICM subcarriers behave as a memoryless BIOS channels and the PEP can be computed as the tail probability [3]

$$
\begin{equation*}
\operatorname{Pr}(\underline{\mathbf{b}} \rightarrow \underline{\hat{\mathbf{b}}} \mid \mathbf{A})=\operatorname{Pr}\left(\sum_{k=1}^{d} \Lambda_{k}>0\right) \tag{9}
\end{equation*}
$$

Unfortunately, the computation of (9) by the probability density function of $\sum_{k=1}^{d} \Lambda_{k}$ is too involved. However, we can
evaluate the PEP by the moment generating function, defined as

$$
\begin{equation*}
M_{\Lambda}(s)=\mathrm{E}_{\Lambda}[\exp \{s \Lambda\}] \tag{10}
\end{equation*}
$$

Since the log-likelihood ratios are i.i.d. random variables

$$
\begin{align*}
\operatorname{Pr}(\underline{\mathbf{b}} \rightarrow \underline{\hat{\mathbf{b}}} \mid \mathbf{A}) & =\frac{1}{2 \pi j} \int_{\sigma-j \infty}^{\sigma+j \infty} M_{\sum_{k=1}^{d} \Lambda_{k}}(s) \frac{\mathrm{d} s}{s} \\
& =\frac{1}{2 \pi j} \int_{\sigma-j \infty}^{\sigma+j \infty}\left[M_{\Lambda}(s)\right]^{d} \frac{\mathrm{~d} s}{s} \tag{11}
\end{align*}
$$

where
$M_{\Lambda}(s)=\mathrm{E}_{z, k}\left[\left(\frac{\sum_{\tilde{x} \in \chi_{\tilde{b}}^{\left(i_{k}, n_{k}\right)}} \exp \left\{-\left|z-A_{n_{k}} \tilde{x}\right|^{2}\right\}}{\sum_{\tilde{x} \in \chi_{b}^{\left(i_{k}, n_{k}\right)}} \exp \left\{-\left|z-A_{n_{k}} \tilde{x}\right|^{2}\right\}}\right)^{s}\right]$.
Substituting the expression of the subcarrier output (4) and grouping in a vector $\underline{\mathbf{v}}=\left(x, i_{k}, n_{k}, w_{k}\right)$ all the random elements in the log-likelihood metric, we have

$$
\begin{align*}
M_{\Lambda}(s) & = \\
\mathrm{E}_{\underline{\mathbf{v}}} & {\left[\left(\frac{\sum_{\tilde{x} \in \chi_{\hat{b}}^{\left(i_{k}, n_{k}\right)}} \exp \left\{-\left|A_{n_{k}}(x-\tilde{x})+w_{k}\right|^{2}\right\}}{\sum_{\tilde{x} \in \chi_{b}^{\left(i_{k}, n_{k}\right)}} \exp \left\{-\left|A_{n_{k}}(x-\tilde{x})+w_{k}\right|^{2}\right\}}\right)^{s}\right] . } \tag{13}
\end{align*}
$$

Note that at sufficiently high SNRs the numerator in (13) is dominated by the term corresponding to the nearest neighbor (in the sense of Euclidean distance) in the complementary subset $\chi_{\hat{b}}^{\left(i_{k}, n_{k}\right)}$. As a consequence, we can apply the Dominated Convergence Theorem ( [3], [10]) obtaining

$$
\begin{equation*}
M_{\Lambda}(s)=\mathrm{E}_{i_{k}, n_{k}}\left[\exp \left\{-\left|A_{k}\right|^{2} d^{2}(x, \tilde{x})\left(s-s^{2}\right)\right\}\right] \tag{14}
\end{equation*}
$$

where $\tilde{x}$ is the nearest neighbor of $x$ in the complementary subset. Approximating the distance between $x$ and $\tilde{x}$ as the minimum Euclidean distance between the symbols in the QAM set associated with the subcarrier $n_{k}$, we get

$$
\begin{equation*}
M_{\Lambda}(s)=\mathrm{E}_{n_{k}}\left[\exp \left\{-\left|A_{k}\right|^{2} d_{m i n, n_{k}}^{2}\left(s-s^{2}\right)\right\}\right] \tag{15}
\end{equation*}
$$

Finally, averaging over the subcarriers we end up with

$$
\begin{equation*}
M_{\Lambda}(s)=\sum_{n=1}^{N} \operatorname{Pr}(n) \exp \left\{-\left|A_{n}\right|^{2} d_{m i n, n}^{2}\left(s-s^{2}\right)\right\} \tag{16}
\end{equation*}
$$

where

$$
\begin{equation*}
\operatorname{Pr}(n)=\frac{m_{n}}{\sum_{j=1}^{N} m_{j}} \tag{17}
\end{equation*}
$$

is the probability that a codeword bit is sent through the $n$th subcarrier in the case of ideal interleaving. In the remainder of the paper two different methods, the Bhattacharyya Bound and the Saddlepoint approximation will be used to estimate the PEP.

1) Bhattacharyya Bound: A simple method to evaluate the PEP is to upperbound it as

$$
\begin{equation*}
\operatorname{Pr}(\underline{\mathbf{b}} \rightarrow \underline{\hat{\mathbf{b}}} \mid \mathbf{A}) \leq\left[M_{\Lambda}(\hat{s})\right]^{d} \tag{18}
\end{equation*}
$$

where $\hat{s}$ is the saddlepoint. For BIOS channels $\hat{s}=1 / 2$ and the Bhattacharyya Bound coincides with the Chernoff Bound [3].
2) Saddlepoint Approximation: A very simple yet accurate method to compute the integral (11) is the saddlepoint approximation [3], [10]. This approximation is

$$
\begin{equation*}
\operatorname{Pr}(\underline{\mathbf{b}} \rightarrow \underline{\hat{\mathbf{b}}} \mid \mathbf{A})=\frac{\left[M_{\Lambda}(\hat{s})\right]^{d}}{\sqrt{2 \pi d \kappa_{\Lambda}^{\prime \prime}(\hat{s}) \hat{s}}} \tag{19}
\end{equation*}
$$

where $\kappa_{\Lambda}^{\prime \prime}(\cdot)$ is the second derivative of the cumulant generating function $\kappa_{\Lambda}(s) \triangleq \log M_{\Lambda}(s)$.

## III. Power Allocation

This section describes the power allocation strategy to improve the system goodput performance.

## A. Goodput expression

Let us note that the PEP doesn't depend on the sequences $\underline{\mathbf{b}}$ and $\underline{\mathbf{b}}$ but only on the Hamming distance $d$. So, let us write

$$
\begin{equation*}
\operatorname{Pr}(\underline{\mathbf{b}} \rightarrow \underline{\hat{\mathbf{b}}} \mid \mathbf{A})=\operatorname{Pr}(d \mid \mathbf{A}) \tag{20}
\end{equation*}
$$

Then, using the union bound, the packet error rate (PER) $P_{u}(\mathbf{A})$ is bounded by

$$
\begin{equation*}
P_{u}(\mathbf{A}) \leq \sum_{d=d_{\text {free }}}^{N_{c}} \omega(d) \operatorname{Pr}(d \mid \mathbf{A}) \tag{21}
\end{equation*}
$$

where $\omega(d)$ is the weight of all error events at Hamming distance $d$ and $d_{\text {free }}$ is the minimum distance between two codewords. Due to the complexity of the problem, in order to provide a manageable expression of the goodput, we will use the Battacharyya bound. Substituting the PEP upper bound (18) in (21) we get

$$
\begin{align*}
& P_{u}(\mathbf{A}) \leq \sum_{d=d_{\text {free }}}^{N_{c}} \omega(d)\left[M_{\Lambda}(\hat{s})\right]^{d} \\
& \quad=\sum_{d=d_{\text {free }}}^{N_{c}} \omega(d)\left[\sum_{n=1}^{N} \operatorname{Pr}(n) \exp \left\{\frac{-\left|A_{n}\right|^{2} d_{\text {min }, n}^{2}}{4}\right\}\right]^{d} . \tag{22}
\end{align*}
$$

The goodput is defined as the number of data bits delivered in error-free packets per unit of time. Assuming the transmission and re-transmission of the packets controlled by the selectiverepeat (SR) ARQ scheme described in [7] and taking the OFDM symbol period as unit of time we can express the goodput as:

$$
\begin{equation*}
G P=R \sum_{n=1}^{N} m_{n} \cdot\left(1-P_{u}(\mathbf{A})\right) \tag{23}
\end{equation*}
$$

## B. Power allocation strategy

Given the goodput expression, let us note that the only term in (23) that depends on power is the PER. So, to maximize the goodput it is sufficient to find the set of powers $\left\{p_{i}^{*}\right\}_{i=1}^{N}$ that minimizes the PEP for each value of the Hamming distance $d$. This subsection outlines the structure of an iterative procedure to minimize the PER upper bound given in (22). First of all, we take the derivative of the PEP natural logarithm respect to the power $p_{i}$ allocated on the generic $i$ th subcarrier:

$$
\begin{array}{r}
\frac{\partial}{\partial p_{i}} \log \operatorname{Pr}(d \mid \mathbf{A})=-d D_{i}(0) \exp \left\{\frac{-S\left|H_{i}\right|^{2} p_{i} d_{\min , i}^{2}}{4}\right\} \\
\frac{1}{\sum_{n=1}^{N} \operatorname{Pr}(n) \exp \left\{\frac{-S\left|H_{n}\right|^{2} p_{n} d_{\min , n}^{2}}{4}\right\}} \tag{24}
\end{array}
$$

where

$$
\begin{equation*}
D_{i}(0)=\operatorname{Pr}(i) \frac{S\left|H_{i}\right|^{2} d_{\min , i}^{2}}{4} \tag{25}
\end{equation*}
$$

Starting with all the channels switched off, i.e. $p_{i}=0$, the derivative (24) is given by

$$
\begin{equation*}
\frac{\partial}{\partial p_{i}} \log \operatorname{Pr}(d \mid \mathbf{A})=-d \cdot D_{i}(0) \quad \forall i \tag{26}
\end{equation*}
$$

The power allocation algorithm can be summarized as follows:

- initialize $p_{i}=0, \forall i$. Sort the absolute values of the derivative (26) in the decreasing order and denote the corresponding subcarrier indexes by the vector $\left(i_{1}, i_{2}, \cdots, i_{N}\right)$. Then, increment the power $p_{i_{1}}$ until

$$
\begin{equation*}
\left.\frac{\partial}{\partial p_{i_{1}}} \log \operatorname{Pr}(d \mid \mathbf{A})\right|_{p_{i_{1}}=p_{i_{1}}(1)}=\left.\frac{\partial}{\partial p_{i_{2}}} \log \operatorname{Pr}(d \mid \mathbf{A})\right|_{p_{i_{2}}=0}, \tag{27}
\end{equation*}
$$

when $p_{i_{1}}(1)$ indicates the power increment for the first step. From (27) we get

$$
\begin{equation*}
p_{i_{1}}(1)=\gamma_{i_{1}} \log \left(D_{i_{1}}(0) / D_{i_{2}}(0)\right) \tag{28}
\end{equation*}
$$

where $\gamma_{i}$ is defined as

$$
\begin{equation*}
\gamma_{i}=\frac{4}{S\left|H_{i}\right|^{2} d_{\min , i}^{2}} \tag{29}
\end{equation*}
$$

- In the second step, increment the power of both subcarriers $i_{1}$ and $i_{2}$, imposing that

$$
\begin{gather*}
\left.\frac{\partial}{\partial p_{i_{1}}} \log \operatorname{Pr}(d \mid \mathbf{A})\right|_{p_{i}=p_{i_{1}}(1)+p_{i_{1}}(2)}= \\
\left.\frac{\partial}{\partial p_{i_{2}}} \log \operatorname{Pr}(d \mid \mathbf{A})\right|_{p_{i_{2}}=p_{i_{2}}(2)} . \tag{30}
\end{gather*}
$$

Substituting (24) in (30) and after some manipulation, we obtain

$$
\begin{equation*}
\left|H_{i_{1}}\right|^{2} d_{m i n, i_{1}}^{2} p_{i_{1}}(2)=\left|H_{i_{2}}\right|^{2} d_{\min , i_{2}}^{2} p_{i_{2}}(2) \tag{31}
\end{equation*}
$$

We can make some remarks about this result: equation (31) means that the power allocated at the second step yields uniform SNR increment. In other words, the SNR on either subcarrier has to be incremented by the same
quantity. This result is obtained for each step and for each subcarrier.

- The power gain of $i_{1}$ and $i_{2}$ is incremented until their derivative is the same as the derivative on $i_{3}$. From this condition we can get the power allocation at the second step, and so on.
Generalizing at the $k$ th step the power expression and using the power constraint

$$
\begin{align*}
& \frac{1}{N}\left[p_{i_{1}}(1)+p_{i_{1}}(2)+\cdots+p_{i_{1}}(k)+p_{i_{2}}(2)+\cdots\right. \\
&\left.\cdots+p_{i_{k-1}}(k)+p_{i_{k}}(k)\right]=1 \tag{32}
\end{align*}
$$

we obtain the optimum power allocation policy given by

$$
\begin{equation*}
p_{i}^{*}=\frac{N-\sum_{n=1}^{N^{\prime}} \gamma_{n}\left[\log D_{n}(0)-\log D_{i}(0)\right]}{N^{\prime} \bar{\gamma} / \gamma_{i}} \tag{33}
\end{equation*}
$$

where $N^{\prime}$ is the number of subcarriers with a non-null power allocation and

$$
\begin{equation*}
\bar{\gamma}=\frac{1}{N^{\prime}} \sum_{n=1}^{N^{\prime}} \gamma_{n} \tag{34}
\end{equation*}
$$

is the average of $\gamma_{n}$ over the useful subcarriers. Given the power allocation policy, we can get the expression of the goodput as function of the set $\left\{\gamma_{n}\right\}_{n=1}^{N^{\prime}}$. Substituting the optimal power expression (33) into the saddlepoint approximation (19) and after some calculation we get:

$$
\begin{equation*}
\operatorname{Pr}(d \mid \mathbf{A})=\frac{\exp \left\{-d \frac{N}{N^{\prime} \bar{\gamma}}\right\}}{\sqrt{\frac{8 \pi d N}{N^{\prime} \bar{\gamma}}}}\left[\frac{\prod_{n=1}^{N^{\prime}}\left(\frac{\operatorname{Pr}(n)}{\gamma_{n}}\right)^{\frac{\alpha_{n}}{N^{\prime}}}}{\left(\frac{1}{N^{\prime} \bar{\gamma}}\right)}\right]^{d} \tag{35}
\end{equation*}
$$

where $\alpha_{n} \triangleq \frac{\gamma_{n}}{\bar{\gamma}}$. Let us note that

$$
\begin{equation*}
\overline{\operatorname{Pr}(d \mid \mathbf{A})}=\exp \left\{-d \frac{N}{N^{\prime} \bar{\gamma}}\right\} / \sqrt{\frac{8 \pi d N}{N^{\prime} \bar{\gamma}}} \tag{36}
\end{equation*}
$$

is the PEP obtained for an equivalent system where the useful subcarriers have the same power gain equal to $P_{T} / N^{\prime}$ and the signal over each subcarrier experiences a channel such that $S\left|H_{n}\right|^{2} d_{\min , n}^{2}=4 / \bar{\gamma}$. Since $d_{\min , n}^{2}=\frac{6}{2^{m_{n}-1}}$, we have

$$
\begin{equation*}
\frac{\left|H_{n}\right|^{2}}{2^{m_{n}}-1}=\frac{2}{3 \bar{\gamma} S} \tag{37}
\end{equation*}
$$

Eventually, using jointly equations (23),(21),(35) and (36), we can express the goodput as

$$
\begin{align*}
& G P=R \sum_{n=1}^{N} m_{n} \\
& \quad\left(1-\sum_{d=d_{\text {free }}}^{N_{c}} \omega(d) \overline{\operatorname{Pr}(d \mid \mathbf{A})}\left[\frac{\prod_{n=1}^{N^{\prime}}\left(\frac{\operatorname{Pr}(n)}{\gamma_{n}}\right)^{\frac{\alpha_{n}}{N^{\prime}}}}{\left(\frac{1}{N^{\prime} \bar{\gamma}}\right)}\right]^{d}\right) \tag{38}
\end{align*}
$$

The performance of the algorithm outlined above can be further enhanced by integrating it with a bit-loading strategy which maximizes the goodput expressed in (38) with respect to bit allocation.


Fig. 2. PER Vs Average Subcarrier Power.

## IV. Simulation results

The effectiveness of the proposed power allocation policy in improving the goodput performance of a BICM-OFDM system in the presence of frequency selective channel is verified through numerical simulation. The performance of the proposed power allocation is also compared to the non adaptive scheme and the classical "water-filling" power allocation. In the sequel, we will focus on a BICM-OFDM scheme with $N=64, N_{c p}=16$ and the signaling interval 50 ns . The system operates with a $1 / 2$ code rate and QPSK modulation. The signal experiences a 6 -tap multipath channel, wherein each path is modeled as a Rayleigh channel independent from others. We have assumed a data packet length of 1 kbits. Let us now observe the system performance improvement that can be achieved when the proposed power allocation policy is applied. The performance improvement is first shown in terms of packet error rate. The results depicted in Fig. 2 refer to: i) PER in case of uniform power distribution (squares); ii) water-filling power allocation (triangles); iii) Goodput-Oriented Power Allocation policy (GOPA) (circles). The proposed algorithm achieves a maximum gain in terms of average power allocated per subcarrier of 2.5 dB with respect to the uniform power allocation and almost 1 dB with respect to the water-filling. The test shown in Fig. 3 deals with the goodput performance. The GOPA algorithm is shown to achieve a maximum gain of $16 \mathrm{~b} i \mathrm{t} / \mathrm{OFDM}$ symbol with respect to the uniform power allocation and 4bit/OFDM symbol with respect to the water-filling.

## V. Concluding remarks

A theoretical goodput analysis for the BICM-OFDM system under frequency selective fading channel has been presented. Based on this, we derived a power allocation policy to maximize the goodput performance. The proposed approach has been verified by computer simulations under typical operating conditions. Numerical results have shown the effectiveness of the proposed strategy. The simulations show a goodput improvement with respect to the non adaptive case and to the "classical water filling policy. Moreover, given the optimal


Fig. 3. Goodput Vs Average Subcarrier Power.
power, the expression of the goodput, based on the PEP saddlepoint approximation, has been derived.

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