

Review Article

Modelling of Hysteresis in Vibration Control Systems by means of the Bouc-Wen Model

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The review presents developments concerning the modelling of vibration control systems with hysteresis. In particular, the review focuses on applications of the Bouc-Wen model that describes accurate hysteretic behaviour in vibration control devices. The review consists of theoretical aspects of the Bouc-Wen model, identification procedures, and applications in vibration control.

1. Introduction

Vibration control systems are adopted for the suppression or, at least, the attenuation of undesirable vibrations that can affect systems and structures, such as buildings, vehicles, aircraft, and bridges. These systems often show nonlinear behaviour due to variable material properties, changeable geometry, and additional nonlinear devices, resulting in hysteresis phenomena. To predict system responses, many hysteretic models have been developed.

Hysteresis is a nonlinear behaviour encountered in a wide variety of processes in which the relation between input and output variables involves memory effects. The detailed physical modelling of these systems is an arduous task, and the models obtained are often too complex to be used in real-world applications [1]. For this reason, alternative models of systems with hysteresis have been proposed. These models combine some physical understanding of the hysteretic system with some black-box modelling, named “semiphysical” models.

Over the years, various semiphysical models of hysteresis have been proposed. One of the most widely adopted ones is the Bouc-Wen hysteresis model [2, 3]. The best feature of this model is its versatility; that is, through appropriate choices of model parameters, it can represent a wide variety of softening or hardening smoothly varying or nearly bilinear hysteretic behaviours.

The Bouc-Wen model has been extensively used in the current literature to mathematically describe components and devices with hysteretic behaviours. Thus, the objective of this literature review is to provide engineers and researchers with an overview of the work that addresses the modelling of vibration control systems by means of the Bouc-Wen model. This review is divided into three major parts: Section 2 focuses on the mathematical and physical properties of the Bouc-Wen model, whereas Section 3 focuses on identification of the Bouc-Wen model parameters. Section 4 provides results concerning several vibration control case studies in which the effectiveness of the Bouc-Wen model, for the accurate description of hysteresis, is verified.

2. Mathematical Properties of the Bouc-Wen Model

In this section, a description of the mathematical formulation of the Bouc-Wen model is presented.

Consider, as an example, the equation of motion of a single-degree-of-freedom (SDOF) mechanical system:

$$m\ddot{u}(t) + c\dot{u}(t) + F(t) = f(t), \quad (1)$$

where m is the mass, $u(t)$ is the displacement, c is the linear viscous coefficient, $F(t)$ is the restoring force, and $f(t)$ is the

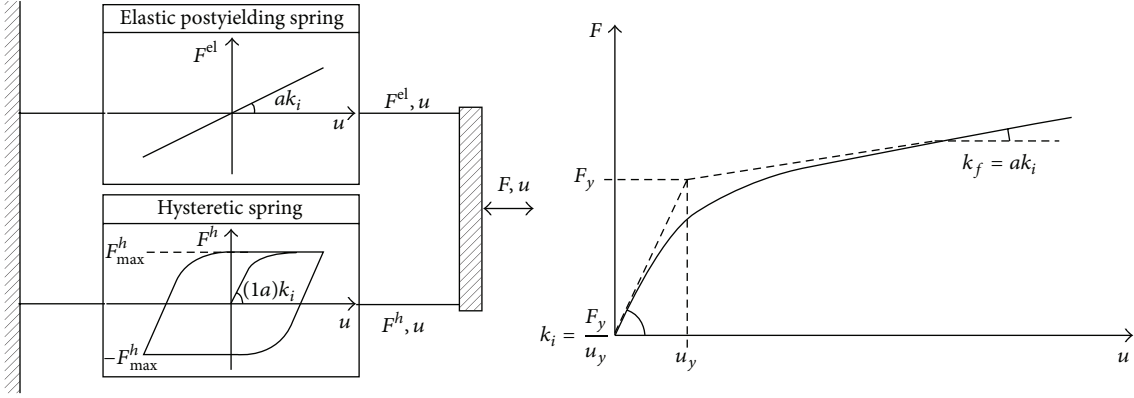


FIGURE 1: Bouc-Wen model [4].

excitation force; the overdot indicates derivative with respect to time.

The restoring force $F(t)$, based on the Bouc-Wen model, is

$$F(t) = a \frac{F_y}{u_y} u(t) + (1-a) F_y z(t), \quad (2)$$

where $a := k_f/k_i$ is the ratio of postyield stiffness k_f to preyield stiffness $k_i := F_y/u_y$, F_y is the yield force, u_y is the yield displacement, and $z(t)$ is a nonobservable dimensionless hysteretic variable that obeys the following nonlinear differential equation with zero initial condition ($z(0) = 0$):

$$\dot{z}(t) = \dot{u}(t) \{A - [\gamma + \beta \operatorname{sgn}(\dot{u}(t) z(t))] |z(t)|^n\}. \quad (3)$$

The coefficients A , β , γ , and n are dimensionless quantities that control the behaviour of the model and $\operatorname{sgn}(\cdot)$ is the signum function. For small values of the positive exponential parameter n the transition from elastic to postelastic branch is smooth, whereas for large values the transition becomes abrupt, approaching that of a bilinear model. The parameters β and γ control the size and shape of the hysteretic loop. The notation varies from paper to paper and very often the places of β and γ are exchanged.

From (2) it follows that the restoring force $F(t)$ can be divided into an elastic and a hysteretic part as follows:

$$F^{\text{el}}(t) = a \frac{F_y}{u_y} u(t) = ak_i u(t) = k_f u(t), \quad (4)$$

$$F^h(t) = (1-a) k_i z(t).$$

Thus, the model can be visualized as two springs connected in parallel [4] (Figure 1).

The parameters of the Bouc-Wen model have the following criteria:

$$\begin{aligned} a &\in [0, 1], \\ k_i &> 0, \\ k_f &> 0, \\ c &> 0, \\ A &> 0, \\ n &> 1, \\ \beta &> 0, \\ \gamma &\in [-\beta, \beta]. \end{aligned} \quad (5)$$

In [23], it has been proved that the parameters of the Bouc-Wen model are functionally redundant, and indeed multiple parameter vectors can produce an identical response under a given excitation. This redundancy can be then removed by fixing parameter A to unity [23].

Constantinou and Adnane [24] suggested the constraint $A/(\beta+\gamma) = 1$ in order to reduce the total number of unknown parameters to six: γ , n , a , F_y , u_y , and c .

In [25], an asymmetrical Bouc-Wen model has been obtained adjusting the velocity as

$$\dot{u}(t) \leftarrow (\dot{u}(t) - \operatorname{sgn}(u(t))), \quad (6)$$

where μ is the scale factor for the adjustment.

Modification of the Bouc-Wen model presented in [26–28] included strength, stiffness, and pinching degradation effects, by means of suitable degradation functions:

$$\begin{aligned} \dot{z}(t) &= \frac{h(z(t))}{\eta(\varepsilon)} \dot{u}(t) \{A(\varepsilon) \\ &\quad - \nu(\varepsilon) [\beta \operatorname{sgn}(\dot{u}(t)) |z(t)|^{n-1} z(t) + \gamma |z(t)|^n]\}, \end{aligned} \quad (7)$$

where ε is the absorbed hysteretic energy and the functions $\nu(\varepsilon)$, $\eta(\varepsilon)$, and $h(z)$ are associated with strength, stiffness, and

pinching degradation effects, respectively. $\nu(\varepsilon)$, $A(\varepsilon)$, and $\eta(\varepsilon)$ are linearly increasing functions defined as

$$\begin{aligned}\nu(\varepsilon) &= \nu_0 + \delta_\nu \varepsilon(t), \\ A(\varepsilon) &= A_0 - \delta_A \varepsilon(t), \\ \eta(\varepsilon) &= \eta_0 + \delta_\eta \varepsilon(t).\end{aligned}\quad (8)$$

The pinching function $h(z)$ is

$$h(z) = 1 - \varsigma_1(\varepsilon) \exp\left(-\frac{(z(t) \operatorname{sgn}(\dot{u}) - qz_u)^2}{(\varsigma_2(\varepsilon))^2}\right), \quad (9)$$

where

$$\begin{aligned}\varsigma_1(\varepsilon) &= (1 - \exp(-p\varepsilon(t)))\varsigma, \\ \varsigma_2(\varepsilon) &= (\psi_0 + \delta_\psi \varepsilon(t))(\lambda + \varsigma_1(\varepsilon)),\end{aligned}\quad (10)$$

and z_u is the ultimate value of z , given by

$$z_u = \sqrt[n]{\frac{1}{\nu(\beta + \gamma)}}. \quad (11)$$

The additional model parameters are $\delta_\nu > 0$, $\delta_A > 0$, $\delta_\eta > 0$, ν_0 , A_0 , η_0 , ψ_0 , δ_ψ , λ , p , and ς . When $\delta_\nu = 0$, $\delta_\eta = 0$, or $h(z) = 1$, respectively, no strength degradation, stiffness degradation, or pinching effect is considered in the model.

It is important to note that a Bouc-Wen model could present a good match with the real data for a specific input, while it could not necessarily keep significant physical properties for different exciting inputs. On the basis of this consideration, the physical and mathematical properties of the Bouc-Wen model are comprehensively discussed in [1] and it has been found that if the system parameters respect the constraints

$$\begin{aligned}n &\geq 1, \\ u_y &> 0, \\ A &> 0, \\ \beta + \gamma &> 0, \\ \beta - \gamma &\geq 0,\end{aligned}\quad (12)$$

then the model is valid independently of the exciting input. When (12) are satisfied, (3) can be expressed in a normalized form.

Defining the parameters

$$\begin{aligned}\rho &= \frac{A}{u_y z_0} > 0, \\ \sigma &= \frac{\beta}{\beta + \gamma} \geq \frac{1}{2}, \\ z_0 &= \sqrt[n]{\frac{A}{\beta + \gamma}},\end{aligned}\quad (13)$$

it follows

$$\begin{aligned}\dot{w}(t) &= \rho \dot{u}(t) \left\{ 1 + |w(t)|^n \sigma \left[1 - \operatorname{sgn}(\dot{u}(t) w(t)) - \frac{1}{\sigma} \right] \right\}\end{aligned}\quad (14)$$

with

$$w(t) = \frac{z(t)}{z_0}. \quad (15)$$

In [29], it is demonstrated that $w(t)$ is bounded in the range $[-1, 1]$.

Substituting (14) and (15) in (2), it follows

$$F(t) = k_f u(t) + k_w w(t), \quad (16)$$

where

$$k_w = (1 - a) F_y z_0 > 0. \quad (17)$$

Consequently, the unknown parameters of the normalized form of the Bouc-Wen model are ρ , σ , n , k_f , and k_w with the following constraints:

$$\begin{aligned}\rho &> 0, \\ \sigma &\geq \frac{1}{2}, \\ n &\geq 1, \\ k_f &> 0, \\ k_w &> 0.\end{aligned}\quad (18)$$

The model parameters can be determined by system identification techniques, using experimental input and output data.

3. Bouc-Wen Model Parameter Identification

The identification of the Bouc-Wen model parameters is performed by adopting an identification algorithm that compares model output signals and those measured, for the same input signals, in order to determine the unknown model parameters. Nonlinearity of the Bouc-Wen model introduces the complexity for parameter identification. Several methods, based on different approaches, have been then proposed. In this section, an overview of different identification algorithms is presented.

An iterative least-squares procedure, based on a modified Gauss-Newton approach, was presented in [5]. Identification was carried out to estimate the parameters of an extended Bouc-Wen model that accounts for strength and stiffness degradation in accordance with [26]. The system identification procedure is shown in Figure 2.

In [30], a vibration control system composed of a magnetorheological damper in series with a magnetorheological elastomer was presented. The Bouc-Wen model was adopted to reproduce the hysteresis of the magnetorheological damper, and the parameters were identified using a least-squares method.

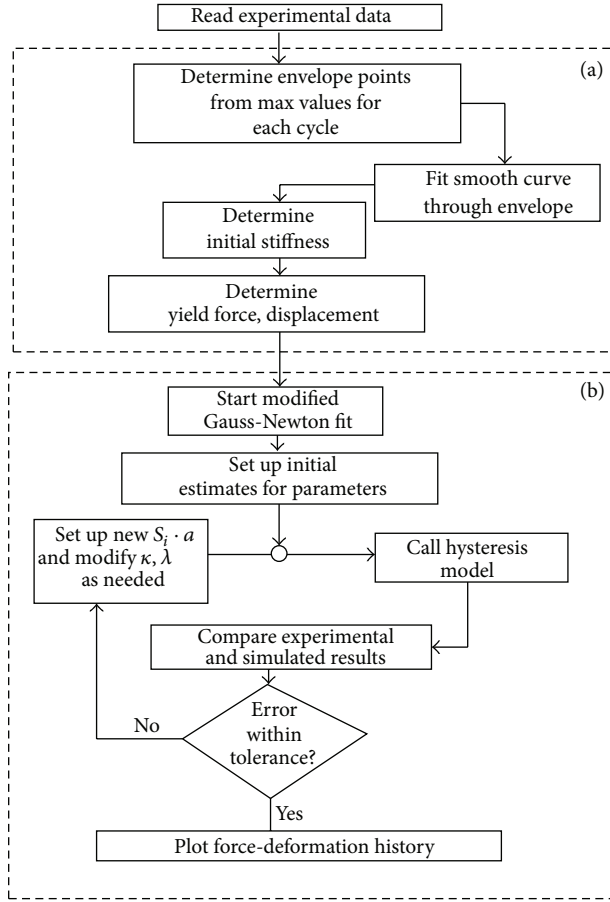


FIGURE 2: Flow diagram for complete identification process [5].

On-line parameter identification of the Bouc-Wen model has also drawn a lot of attention to researchers. Studies in [31, 32] presented an on-line identification method using a least-squares adaptive law. Another study in [33] employed an adaptive on-line identification methodology with a variable trace method to adjust the adaptation gain matrix. In [34], a linear parameterized estimator was established for the on-line estimation of the hysteretic Bouc-Wen model with unknown coefficients (including the parameter n). In [35], an adaptive on-line identification algorithm was proposed for parametric and nonparametric identification of structural models and was applied to a generalized Bouc-Wen model. The proposed identification methodology was a recursive least-squares algorithm that required only acceleration measurements.

In addition to the least-squares regression method, a genetic based identification algorithm was proposed in [36]. The reproduction procedure adopts the roulette wheel selection and the method of crossover and uniform mutation [37].

To account for asymmetric behaviour, a modified Bouc-Wen model [6] was adopted for modelling a PZT actuator. A modified particle swarm optimization algorithm [38] was proposed in order to identify and optimize the model parameters (see Figure 3). The fitness function in Figure 3 is evaluated as the root-mean-square error between the actual measured displacement and the model output.

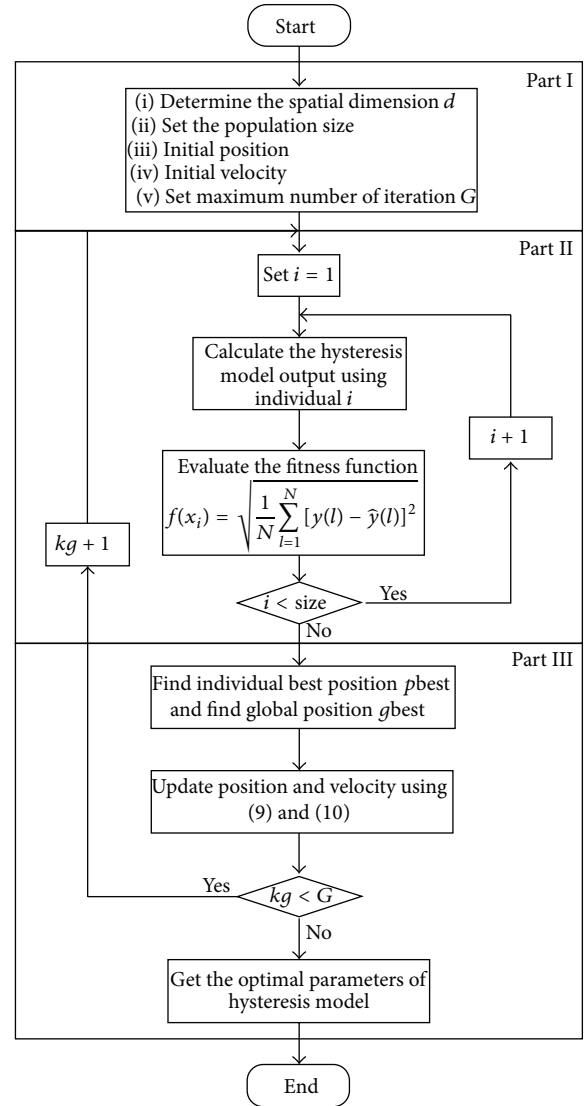


FIGURE 3: Flowchart of the identification hysteretic model parameters [6].

The particle swarm optimization was also adopted in [39] to identify the parameters of Bouc-Wen model. In this study, the identified model was used to describe the dynamics of a large-scale magnetorheological damper for seismic hazard mitigation.

Gauss-Newton iterations were used as a method of estimating the parameters of hysteretic system with slip on the basis of input-output data [40]. Reference [41] proposed a frequency domain parametric identification method of nonlinear hysteretic isolators. In [42, 43], an identification method for the normalized Bouc-Wen model was developed. Using the analytical description of the hysteresis loop developed in [29], an algorithm was proposed along with its analytical proof. It consisted in exciting the Bouc-Wen model with two periodic signals with a loading-unloading shape (wave periodic) which gives rise asymptotically to a hysteretic periodic response. The obtained two limit cycles

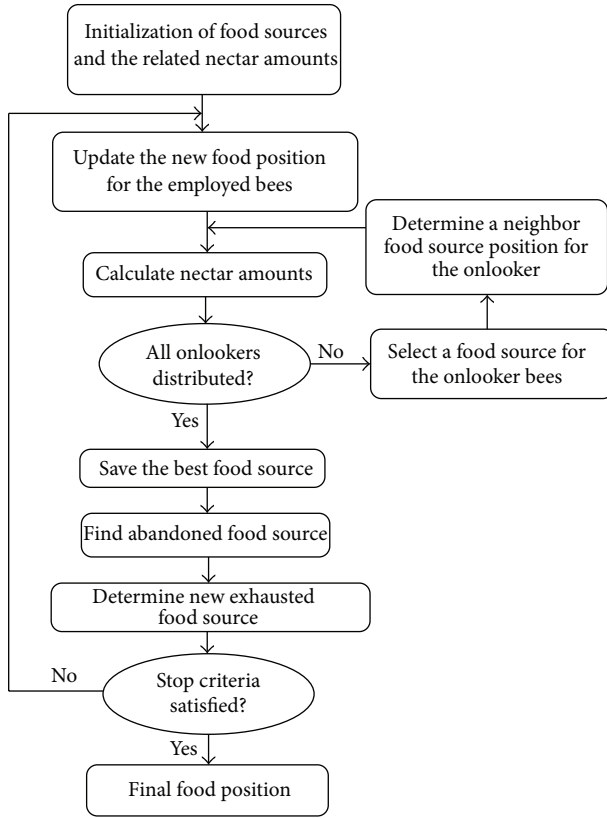


FIGURE 4: Flowchart of the parameter identification algorithm presented in [7].

were then used as an input to exactly determine the unknown parameters.

The limit cycle approach was also adopted in [10] to identify the parameters of the Bouc-Wen model adopted to reproduce the hysteresis of a wire-cable vibration isolator. In [44], the limit cycle approach was adopted to identify the parameters of a large-scale magnetorheological fluid damper model.

Another optimization method based on artificial bee colony algorithm [7, 45] was developed to determine the optimum parameters of Bouc-Wen hysteretic systems. The proposed flowchart is shown in Figure 4.

In [46, 47], a constrained nonlinear optimization was exploited in parametric identification. In [48], the Bouc-Wen nonlinear hysteresis term was approximated by a power series expansion of suitable basis functions, and then the coefficients of the functions were determined using the standard least-squares method.

Bouc-Wen model parameters could be also identified with procedures based on nonlinear filtering, using, for example, the extended Kalman filter (EKF) or the unscented Kalman filter (UKF). Yang and Ma [49] proposed a constrained EKF with a global weighted iteration strategy, which was effective in estimating all the parameters of the Bouc-Wen model of hysteresis. Zhang et al. [50] also applied the EKF for the identification of hysteretic systems that exhibit

degradation and pinching; all the parameters of the BW-model were identified without problem.

In [51–53], the UKF was used for the identification of nondegrading and degrading hysteretic systems. The identification results show that the UKF is well suited for the identification of complicated nonlinear systems and that this methodology can yield accurate estimates of the parameters of the Bouc-Wen models. The results also show that the UKF outperforms the EKF with regard to computational efficiency and robustness to measurement noise levels.

4. Vibration Control System Modelling Using the Bouc-Wen Model

The main scope of the vibration control is the suppression or, at least, the attenuation of undesirable vibrations that can affect systems and structures, such as buildings, vehicles, aircraft, and bridges. Vibration control is typically realized using passive, semiactive, or active [54–58] systems, and considerable hysteretic behaviours can be found in each of these.

Passive vibration control, such as the passive base isolation, is one solution that has proven effective for enhancing structural performance against seismic events. The main concept behind passive base isolation is to increase the structure flexibility, thus avoiding potentially dangerous seismic ground motions [59–62]. Base isolation bearings have been installed in many buildings for seismic protection [63]; however, large base displacements resulting from the increased flexibility of the passive isolation system can potentially exceed the prescribed limit of structural designs under severe seismic excitations [64–66].

Semiactive vibration control consists of a passive isolation system combined with a controllable semiactive device [67–71]. Semiactive vibration control is one control technique that consumes less power to change the features of the isolation system, but no mechanical energy is introduced into the structural system. Differently from passive control techniques, semiactive control systems have higher variability due to the different capabilities of energy dissipation from the control devices when the power levels are changed, such as variable stiffness and/or damping values.

Active vibration control is another control technique which uses the energy generated from the active control devices, supplied by means of an external power source, to improve the vibrational system performance. The mitigation of the vibration phenomena is based on the employment of suitable actuators that transmit mechanical energy to the structure and, in recent years, a great number of studies have been accomplished on active vibration control of flexible structures using piezoelectric actuators. Among the several kinds of actuation systems, piezoelectric ceramic materials have received much diffusion in the active vibration control because of their mechanical simplicity, small volume, light weight, large useful bandwidth, efficient conversion between electrical energy and mechanical energy, and easy integration with various metallic and composite structures [72].



FIGURE 5: Isolator prototype [8].

4.1. Passive Systems. The Bouc-Wen model has been widely adopted for the numerical modelling of passive hysteretic control devices. In this section, an overview of various components for different applications is presented.

Passive seismic isolators aim to reduce the earthquake input energy to the structure to keep linear structural vibration. Many devices are strongly nonlinear, showing different hysteretic behaviours. In this context, the Bouc-Wen model has been used extensively for its intrinsic ability in describing the behaviour of a wide range of real-world passive seismic isolators.

In [8], a prototype seismic isolator, composed of a wire rope spring and a ball transfer unit, was proposed as shown in Figure 5. The nonlinear behaviour of the restoring force was represented by the Bouc-Wen model.

The Bouc-Wen model was used in [73] to model the dissipated energy by the wire rope isolators for seismic protection of equipment in buildings. In [9, 10], modified Bouc-Wen models were adopted to numerically reproduce hardening behaviours [9] (see Figure 6) or asymmetrical hysteresis cycles [10] (see Figure 7) of wire rope isolators.

A combined energy dissipation system was presented in [11]. As shown in Figure 8, a lead rubber damper (LRD) and its parallel connection with an oil damper (OD) were used in the braces of a structural frame. The restoring force characteristic of the LRD was then simulated by the Bouc-Wen hysteretic model.

In [12], the Bouc-Wen model was employed to mathematically represent two different elastomeric seismic isolators (see Figure 9), characterized by different characteristics because of the elastomeric layers and reinforcements (in the following indicated with “IUT_a” and “IUT_b,” resp.). The experimental and simulated hysteresis cycles for IUT_a and IUT_b were demonstrated in Figures 10(a) and 10(b), respectively.

In [74], the Bouc-Wen model was utilized to mathematically model a frictional behaviour of Teflon sliding bearings for a base isolation application. The study in [13] facilitated the Bouc-Wen model to describe the behaviour of hysteretic dampers that interconnect two adjacent structures subjected to seismic excitation as shown in Figure 11.

Effectiveness of dissipative passive devices used as connections between two structures was investigated [75]. An analytical model for the response of a reinforced concrete

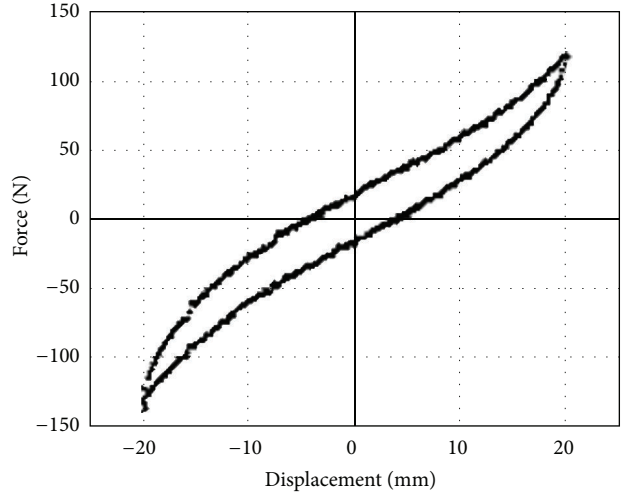


FIGURE 6: Hysteresis cycle with hardening [9].

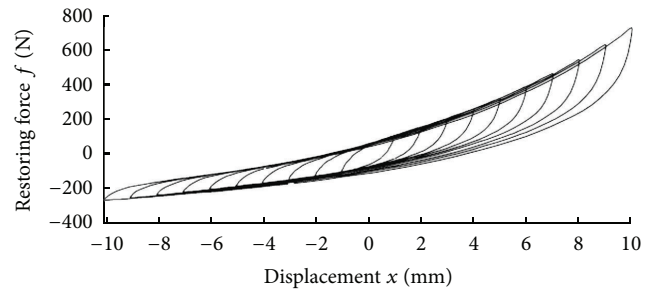


FIGURE 7: Asymmetrical cycles for tension-compression loading of a wire rope isolator [10].

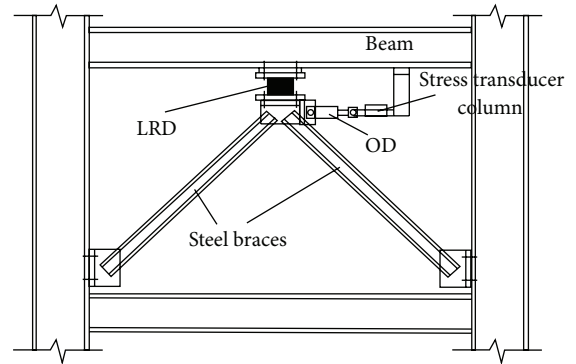


FIGURE 8: The configuration of the combined energy dissipation system in a frame [11].

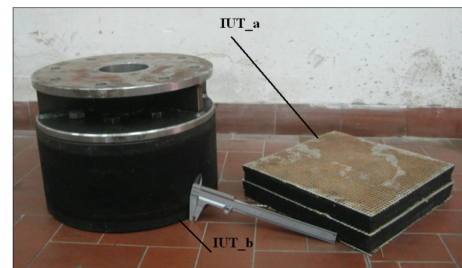


FIGURE 9: Seismic isolators presented in [12].

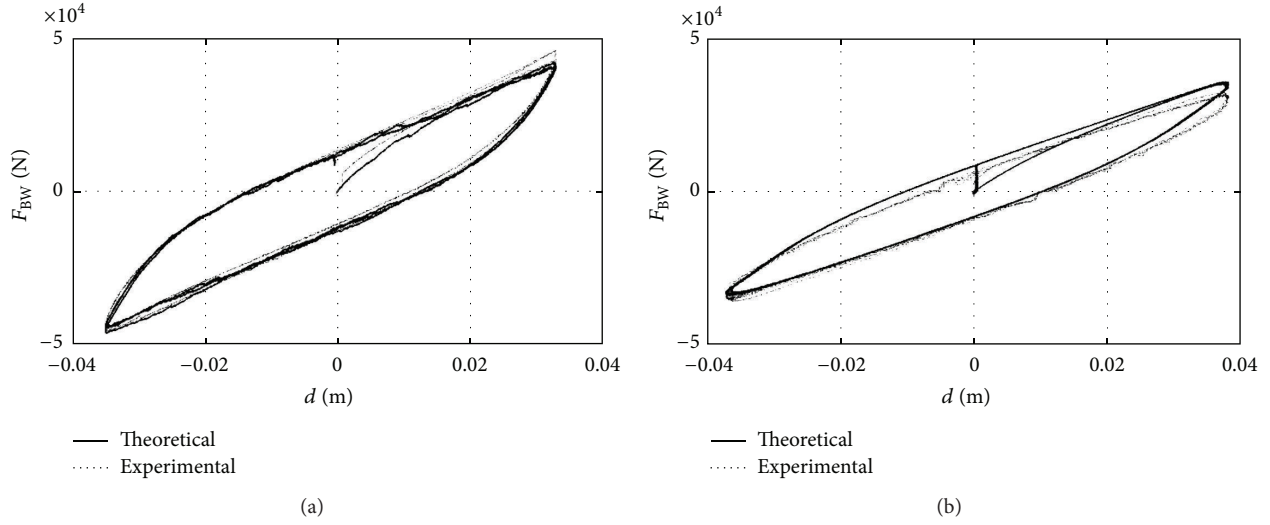


FIGURE 10: Experimental and simulated hysteretic cycles: (a) IUT_a; (b) IUT_b [12].

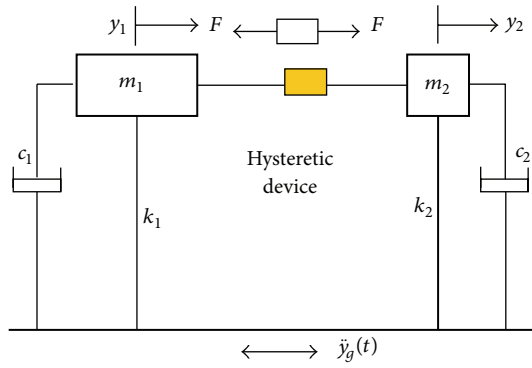


FIGURE 11: Schematic of the hysteretic device that interconnects two adjacent structures [13].

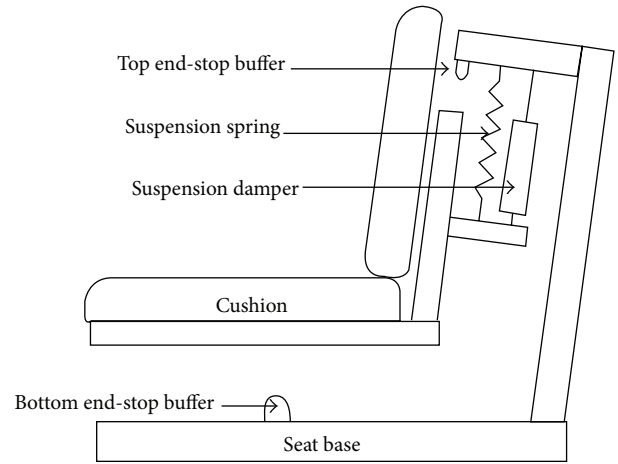


FIGURE 12: Schematic of the industrial truck seat [14].

panel with friction energy dampers was presented. The modelling of the hysteretic device was developed using an extended Bouc-Wen model. In [14], the study verified that the Bouc-Wen model well predicted the responses of suspension seats of the off-road machines to transient inputs as shown in Figure 12. The Bouc-Wen model used in [14] is shown schematically in Figure 13. The Bouc-Wen coefficients were obtained by minimizing the difference between the predicted and measured acceleration of a load supported in the seat. The measured hysteresis force-deflection cycles for the bottom buffers are shown in Figure 14. The study in [14] concluded that the Bouc-Wen model can provide a useful simulation of an existing seat.

4.2. Semiactive Systems. The magnetorheological (MR) fluids consist of suspensions of micron-sized ferrous particles immersed in a carrier fluid; their rheological behaviour can be changed by means of an applied magnetic field.

Researchers have used the controllable variation in yield stress to develop various smart devices [76–79]. In recent years, magnetorheological dampers have been widely studied as a controllable engineering component because of their continuously controllable mechanical properties and rapid response [80]. As shown in Figure 15, MR dampers can operate under three different fluid working modes [15]: shear [81, 82], flow [83–85], and squeeze [86–89]. The shear mode occurs when one wall of a gap translates or rotates relative to the other wall. In the shear mode, the fluid is sheared parallel to the walls. The flow mode occurs when two walls of a gap are fixed, as in a valve system, and the fluid flows through the gap and along the longitudinal axis of the walls. The squeeze mode occurs when the walls move toward each other, squeezing out the fluid. The fluid in the squeeze mode flows orthogonally to the direction of wall motion. According to the motion of the

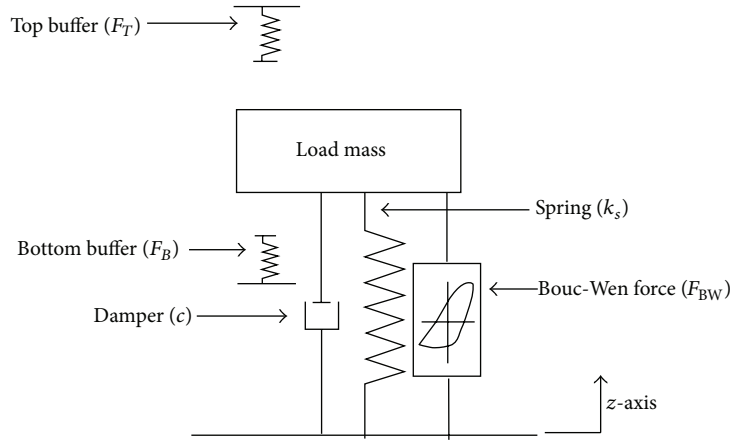


FIGURE 13: Schematic of the Bouc-Wen model [14].

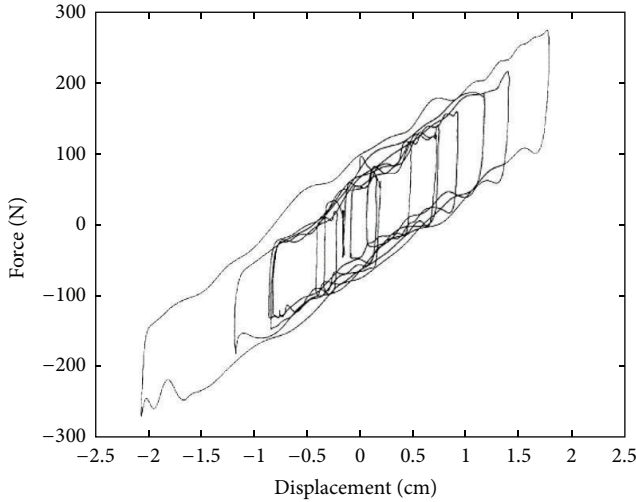


FIGURE 14: Force-displacement diagram—industrial truck seat exposed to random excitation [14].

constitutive elements, MR dampers can be categorized as linear MR dampers [16] (see Figure 16) and rotary MR dampers.

The modelling of MR dampers represents an important role to accurately describe their behaviours. A validated dynamic model allows executing several fundamental steps such as performance prediction, design, numerical simulations, and control synthesis. To this end, it is important to highlight the hysteretic phenomenon that occurs in these devices. Indeed, as it can be observed from experimental data in Figure 17, a hysteretic loop appears in the force-velocity diagram.

Several approaches have been established to model the MR dampers, and among these, the parametric approach drew considerable attention to researchers. The parametric models were developed using the schematization of the device as a combination of different physical elements, and

a typical example was constituted by the Bouc-Wen hysteresis operator-based dynamic model [20, 90–93].

The Bouc-Wen model has been extensively applied to simulate the hysteresis loops since it possesses the force-displacement and force-velocity behaviour of the MR dampers. In the following, the most widely adopted approaches will be presented. Spencer Jr. et al. [15, 17] employed the Bouc-Wen hysteretic operator to represent the hysteretic behaviour of MR dampers, and the schematic of the proposed simple Bouc-Wen model for MR dampers is shown in Figure 18 [15, 17]. The damping force in this system is given by

$$F = c_0 \dot{x} + k_0 (x - x_0) + \alpha z, \quad (19)$$

where c_0 and k_0 are the viscous damping and stiffness, respectively; x_0 represents an initial displacement due to the presence of an accumulator; z is the evolutionary variable governed by (3). By adjusting the parameter values α , β , γ , and n , the force-velocity relationship is characterized.

Later, Spencer Jr. et al. [15, 17] proposed a modified Bouc-Wen model to predict the behaviour of MR dampers over a wide range of inputs as shown in Figure 19. The model is given by the following equations:

$$F = c_1 \dot{y} + k_1 (x - x_0), \quad (20)$$

where

$$\dot{y} = \frac{1}{c_0 + c_1} [\alpha z + c_0 \dot{x} + k_0 (x - y)], \quad (21)$$

$$\dot{z} = -\gamma |\dot{x} - \dot{y}| |z|^{n-1} z - \beta (\dot{x} - \dot{y}) |z|^n + A (\dot{x} - \dot{y}),$$

and k_1 is the accumulator stiffness; c_0 and c_1 are the viscous damping observed at large and low velocities, respectively; k_0 is the stiffness at large velocities; x_0 is the term that accounts for the presence of an accumulator. The scale and shape of the hysteresis loop can be adjusted by γ , β , A , and n .

To model the behaviour of the shear mode MR damper (see Figure 20(a)), a Bouc-Wen hysteresis operator-based

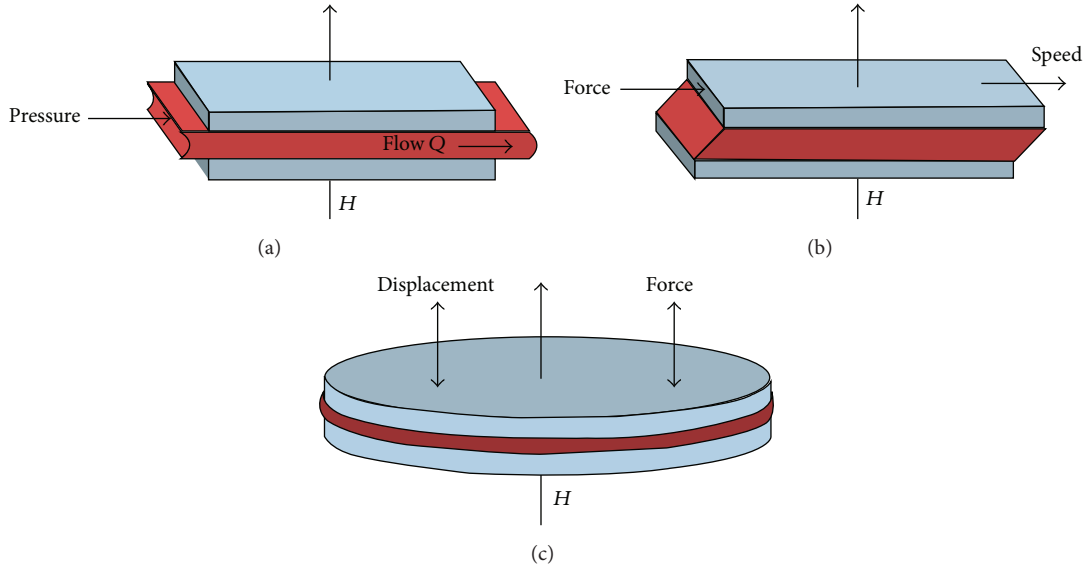


FIGURE 15: MR fluid operational modes [15]: (a) the flow mode, (b) the direct shear mode, and (c) the squeeze mode.

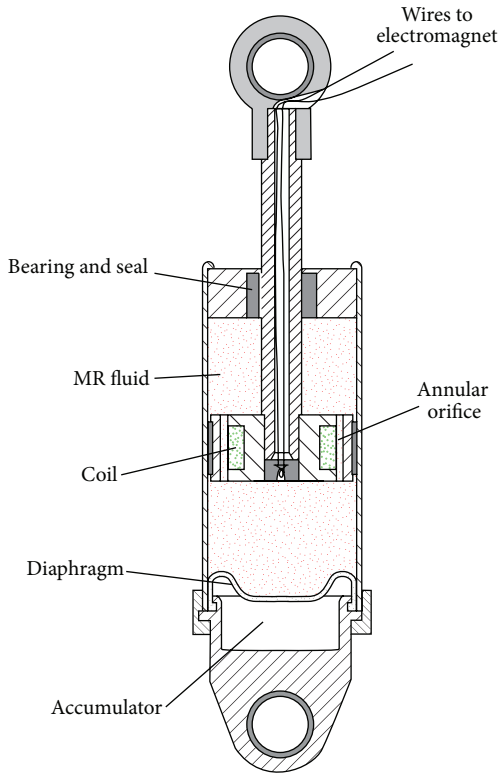


FIGURE 16: Typical scheme of a linear MR damper [16].

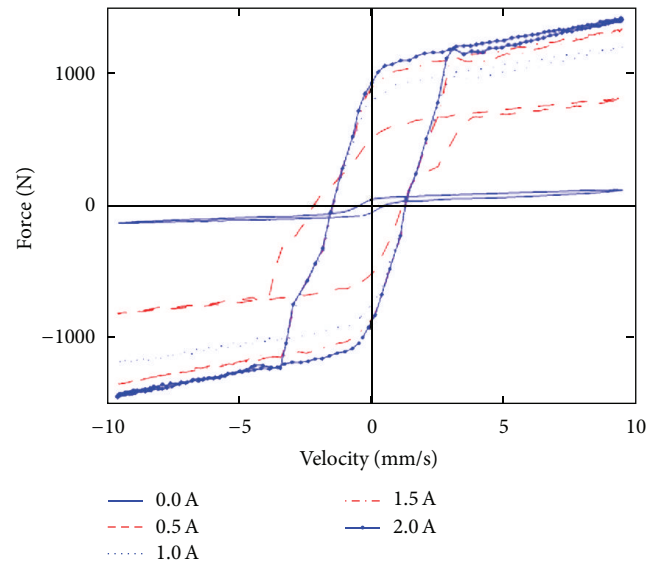


FIGURE 17: The damping force versus velocity [16].

where z is the evolutionary variable given by (3). The Bouc-Wen model in Figure 21 was developed for large-scale MR dampers [20], and the damper force is given by

$$F = m\ddot{x} + c_0(\dot{x})\dot{x} + k_0x + \alpha z + f_0. \quad (23)$$

In (23), m is the equivalent mass; k_0 is the accumulator stiffness; f_0 is the damper friction force; z is the evolutionary variable governed by (3). $c_0(\dot{x})$ is given by

$$c_0(\dot{x}) = a_1 e^{-(a_2|\dot{x}|)^p}, \quad (24)$$

where a_1 , a_2 , and p are positive constants.

dynamic model, as shown in Figure 20(b), was proposed [18, 19]. The equation governing the damping force is given by

$$F = c_0\dot{x} + \alpha z, \quad (22)$$

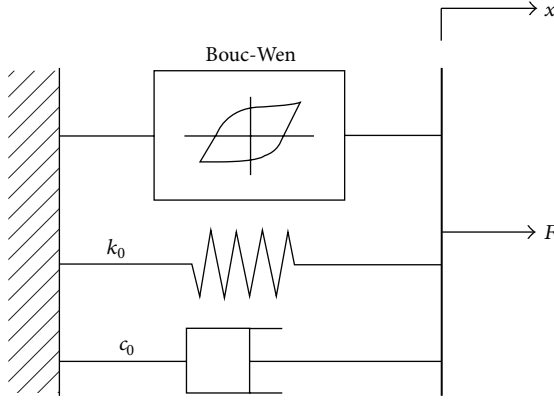


FIGURE 18: Simple Bouc-Wen model for MR dampers [15, 17].

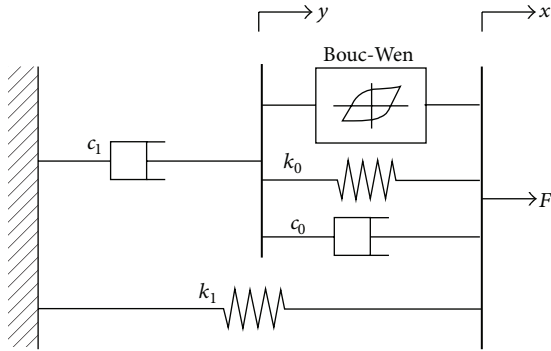


FIGURE 19: Modified Bouc-Wen model for MR dampers [15, 17].

4.3. Active Systems. Piezoelectric actuators (PEAs) represent a functional tool in the field of the active vibration control and, in the recent years, extensive researches have been conducted for modelling and control. PEAs utilize the converse piezoelectric effect of piezoelectric materials to generate displacement and force. Indeed, a piece of piezoelectric material will be mechanically strained if subject to an electric field (by placing it into the electric field or applying voltages to its surfaces).

The hysteresis nonlinearity constitutes one of the principal key issues to be solved in PEAs and its modelling is of fundamental importance for their control. In PEAs, hysteresis exists in both the electric field- (voltage-) polarization relationship and the electric field- (voltage-) strain (deformation or displacement) relationship (Figure 22), with the latter being mostly of concern in micro- and nanopositioning systems, and it is caused by the nonlinearities in the converse piezoelectric effect of the unit cells and the switching and movement of domain walls [21, 94].

The hysteresis trajectory of a PEA can be treated as being composed of three types of components: (1) the major loop which is the hysteresis loop that spans the whole input (voltage) range, (2) the minor loops which are the hysteresis

loops that only span portions of the input range, and (3) the initial ascending curve. As hysteresis is the major nonlinearity of PEAs and possesses detrimental effects on the positioning accuracy and stability margins of feedback control systems [95], compensation of hysteresis has always been a major concern in modelling and control of PEAs.

In [96], a PEA was modelled by means of the Bouc-Wen model and a PID control was applied. The dynamic relationship governing the actuator is given by

$$m\ddot{x} + c\dot{x} + k_a x = k_b k_x u + k_b k_w w, \quad (25)$$

where w is the evolutionary variable governed by (14); m is the equivalent mass; c is the damping coefficient; k_a and k_b are elastic constants; x is the actuator displacement; k_x and k_w are constant gains; n , ρ , and σ are the Bouc-Wen model parameters.

A Bouc-Wen based approach was introduced in [22] to compensate the hysteresis of piezoelectric actuators (see Figure 23) via a feedforward control. Indeed, feedback controllers for small systems, such as micro/nanoactuators, are strongly limited by the difficulty integrating the sensor. According to the multiplicative-inverse structure, the proposed compensator scheme was adapted to hysteresis with an advantage that no more computation was required for the compensator.

The hysteresis compensation is established using the following relationship:

$$U = \frac{1}{d_p} (y_r + H(U)) \quad (26)$$

U being the applied electrical voltage; d_p the piezoelectric coefficient; y_r the target displacement of the actuator; $H(U)$ a nonlinear operator due to the Bouc-Wen model. Equation (26) is used as a compensator that employs y_r as an input and U as an output with respect to the scheme in Figure 24.

To linearize the hysteresis behaviour of stack piezoelectric ceramic actuators, the feedforward linearization method, based on the Bouc-Wen model, and the hybrid linearization method, combining the feedforward method and PI feedback loop, were proposed and explored in [97]. The rapid control prototypes of the linearization controllers were established and tested, and the results showed that both the feedforward and hybrid linearization methods can linearize the hysteresis behaviour. Meanwhile, PEAs can exhibit an asymmetric hysteretic behaviour so that a modified Bouc-Wen model was proposed in [6] by introducing an input bias and an asymmetric factor into the standard Bouc-Wen hysteresis model.

5. Conclusion

This review reported the literature related to the utilization of the Bouc-Wen model for modelling hysterical behaviours of several vibration control systems. The review was organized into three sections that address specific issues: mathematical property of the Bouc-Wen model, identification of the model parameters, and applications of the Bouc-Wen model for

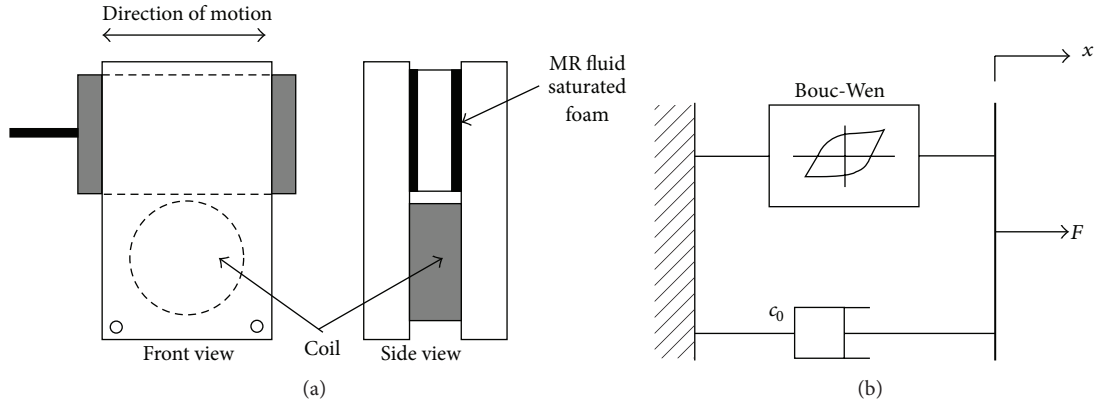


FIGURE 20: (a) Schematic diagram of shear mode MR damper; (b) mechanical model of the parallel plate MR damper [15, 18, 19].

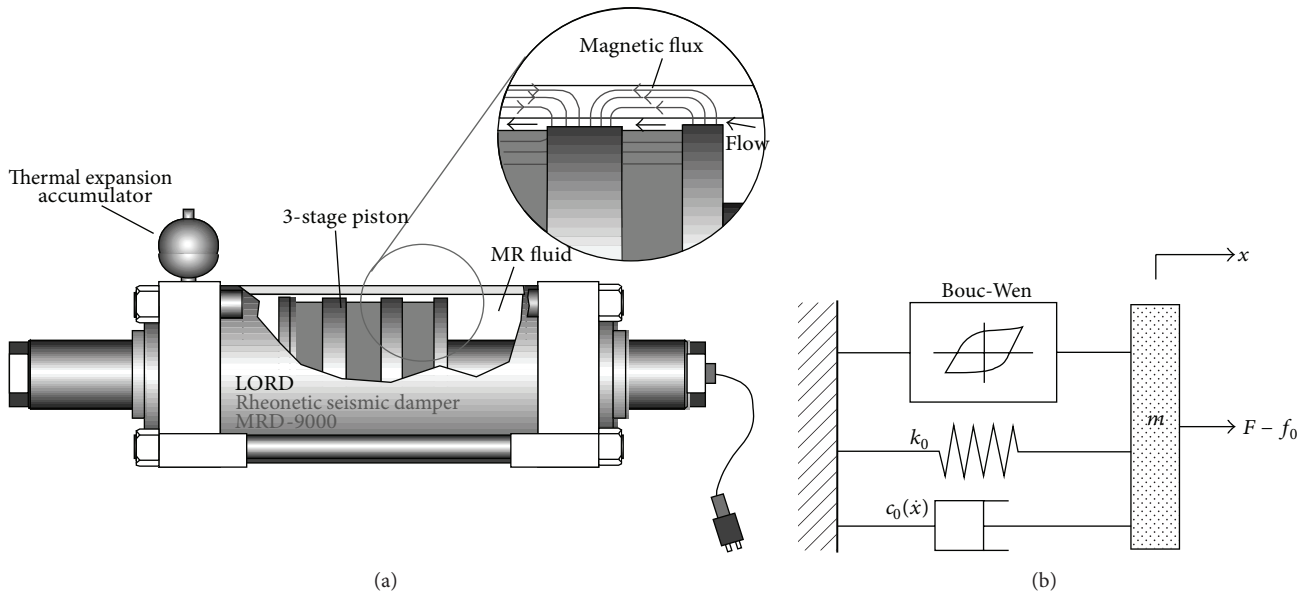


FIGURE 21: (a) Schematic of large-scale magnetorheological fluid damper; (b) Bouc-Wen model for large-scale MR dampers [15, 20].

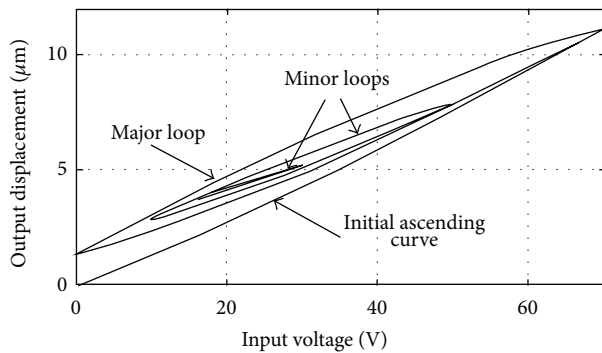


FIGURE 22: Hysteresis of a PEA [21].

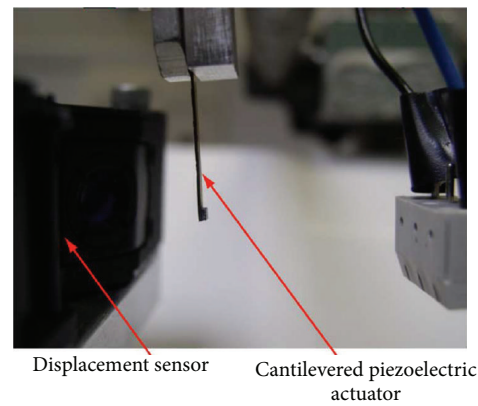


FIGURE 23: Photography of the piezoelectric actuator [22].

the dynamical description of different types of vibration control systems. Section 2 described the theoretical basis of the Bouc-Wen model, its first formulation, and its successive

modifications. Section 3 presented different approaches used for the identification of the Bouc-Wen model parameters.

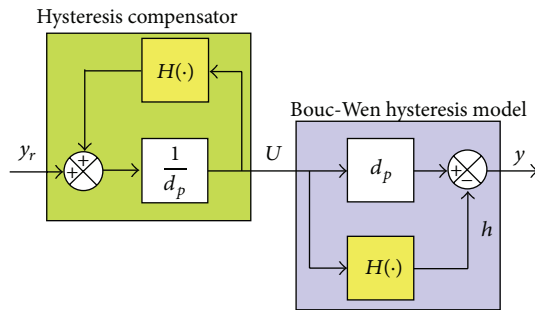


FIGURE 24: Bouc-Wen based compensator [22].

Section 4 provided several applications of the Bouc-Wen model for the modelling of devices extensively used in vibration control. Each section has presented what, from the authors' point of view, are the main contributions for the specific issue.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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