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by single-photon annihilation

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Subtracting photons from arbitrary light fields: experimental test of coherent state invariance by single-photon annihilation

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Abstract. The operator annihilating a single quantum of excitation in a bosonic field is one of the cornerstones for the interpretation and prediction of the behavior of the microscopic quantum world. Here we present a systematic experimental study of the effects of single-photon annihilation on some paradigmatic light states. In particular, by demonstrating the invariance of coherent states by this operation, we provide the first direct verification of their definition as eigenstates of the photon annihilation operator.

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Contents

| | |
|---|-----------|
| 1. Introduction | 2 |
| 2. Experimental | 4 |
| 3. Results and discussion | 6 |
| 3.1. Thermal state | 6 |
| 3.2. Single-photon Fock state | 7 |
| 3.3. Coherent state | 8 |
| 4. Conclusions | 10 |
| Acknowledgments | 10 |
| References | 10 |

1. Introduction

The annihilation operator \hat{a} and its creation counterpart \hat{a}^\dagger operate by decreasing or increasing the number of excitation quanta by one in a bosonic field. For a so-called Fock or number state $|n\rangle$, possessing a well-defined number of quanta n , the annihilation operator reduces this number to $n - 1$, according to

$$\hat{a}|n\rangle = \sqrt{n}|n - 1\rangle. \quad (1)$$

Differently from the conventional deterministic subtraction of objects from an ensemble, the annihilation operator works in a probabilistic way (i.e. the probability of taking away a particle from a state is proportional to the number of particles already there, due to the \sqrt{n} term in equation (1)). So, for a given superposition or mixture of Fock states, the application of \hat{a} not only shifts the quantum number distribution by one unit towards lower values, but it also deeply re-shapes it. Whereas the excitation by a single quantum can turn any classical state into a nonclassical one [1]–[3], the annihilation of a quantum does not produce nonclassicality, but is however able to convert Gaussian states into non-Gaussian ones [4, 5]. Recently, the use of single-photon annihilation as a tool to produce non-Gaussian continuous-variable light states has been proposed and demonstrated (see figure 1) [4, 6, 7]. This is of high interest because non-Gaussian states and operations are now viewed as fundamental ingredients for continuous-variable quantum technologies. In fact, besides enhancing some existing protocols [8]–[11], they are able to perform important communication tasks (like entanglement distillation), which are impossible with Gaussian states and operations only [12, 13].

Even if annihilating quanta with the operator \hat{a} is very different from a classical deterministic removal of a single quantum of excitation, applying \hat{a} is nevertheless often referred to as ‘single-photon subtraction’ in the case of light fields. The fact that \hat{a} operates differently from a simple picture of particle subtraction is also evident from the observation that coherent states do not change under such an operation.

Here, we present a complete experimental investigation of the action of the photon annihilation operator on different kinds of basic quantum light states. As recently observed in the case of a thermal light state [7], we show that single-photon subtraction has rather peculiar effects on the mean number of photons, strictly depending on the statistics of the initial state. In particular, we verify that the final state contains a larger, smaller or equal mean number of photons according to whether the initial state has a super-, sub-, or Poissonian distribution,

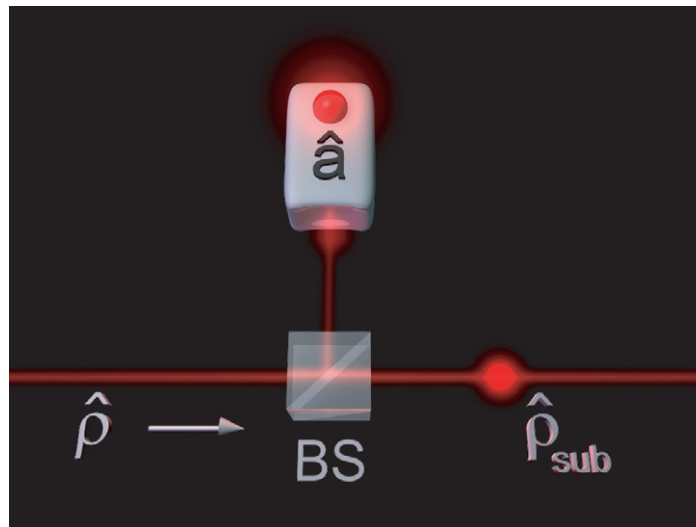


Figure 1. Scheme of the experimental implementation of conditional single-photon subtraction. A high transmissivity beam splitter (BS) splits some light of the incident field (indicated by its density operator $\hat{\rho}$) towards an on/off photodetector. A click in this detector conditionally prepares the photon-subtracted state (denoted by $\hat{\rho}_{\text{sub}}$) and triggers its homodyne detection.

respectively. Furthermore, by demonstrating that coherent states are invariant under the action of the photon annihilation operator, we experimentally verify their definition as eigenstates of \hat{a} : i.e. we directly verify that $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$.

As already shown in the context of photodetection theory [14, 15], one finds that the average number of photons in the photon-subtracted state, \bar{n}_{sub} , is related to the initial one, \bar{n} as $\bar{n}_{\text{sub}} = \bar{n} - 1 + F$, where F is the Fano factor of the initial state, simply given by the ratio between the photon number variance and the mean, which is greater, equal to, or less than unity for super-Poissonian, Poissonian, or sub-Poissonian initial states, respectively. Here, we choose to analyze the effect of single-photon subtraction for three different and paradigmatic light states: coherent, thermal and single-photon Fock ones. For a Fock state, the most sub-Poissonian state available ($F = 0$), the number of photons in the field is precisely known and the subtraction of a single photon is expected to work in a very intuitive way, decreasing such a number by exactly one unit. In the case of a thermal state, a super-Poissonian one with $F = \bar{n} + 1$, the mean photon number doubles to $2\bar{n}$ after photon subtraction, a very counter-intuitive result if one just thinks of it in classical deterministic terms. Note, however, that this is not an entirely quantum effect, since similar results may also be obtained by means of classical statistics when starting from appropriate photon/particle number probability distributions [16]. Finally, as the coherent state is a Poissonian one ($F = 1$), its mean photon number does not change after subtracting a photon by means of the \hat{a} operator. Note that this happens also for phase-diffused coherent states, whose density matrix is diagonal, but still Poissonian weighted, in the Fock basis. However, in the case of a pure coherent state, being an eigenstate of \hat{a} implies that not only its mean photon number and photon probability distribution, but the whole state (represented by its density matrix or Wigner function) is left completely untouched by its application.

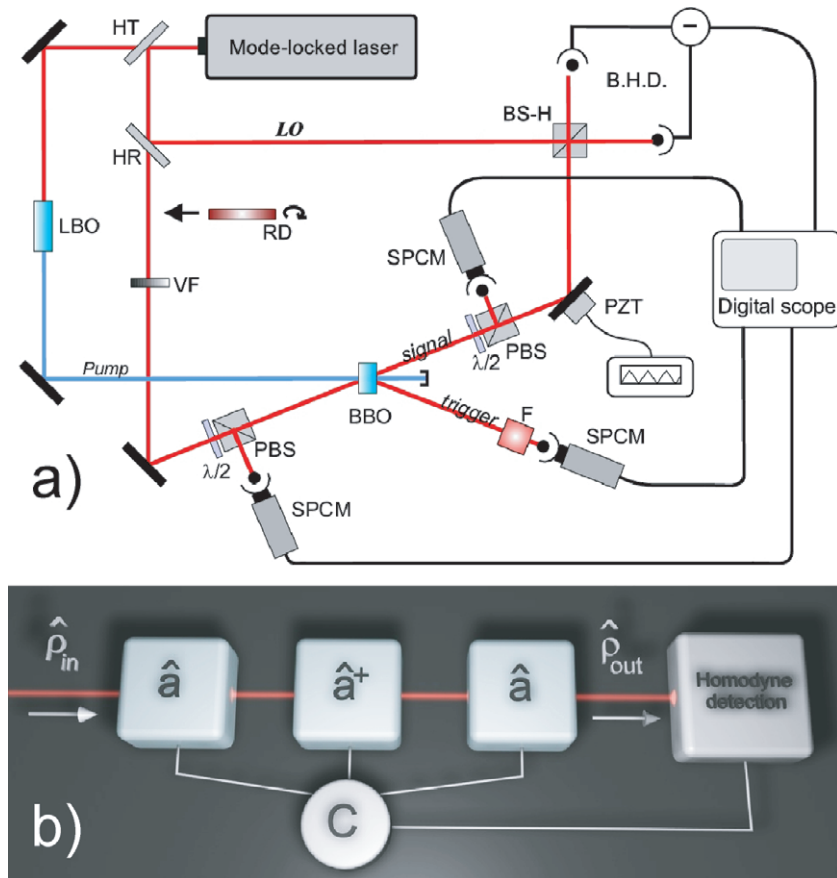


Figure 2. (a) Scheme of the complete experimental setup. Most symbols are defined in the text. LBO is a lithium triborate crystal for frequency doubling of the laser pulses. The BS for photon subtraction is composed of a half wave plate ($\lambda/2$) and a polarizing BS (PBS) for accurate adjustments of its transmissivity. The mode-matching optics are not shown here for clarity. (b) Block diagram of the experimental setup for the implementation of all the measurements described in the text.

2. Experimental

For the experimental verification of such effects we produce, manipulate and analyze light states in well-defined spatiotemporal modes. The temporal mode is defined by the pulsed character of our primary laser source, a mode-locked Ti:Sa laser emitting 1.5 ps pulses at a repetition rate of 82 MHz (see figure 2(a)). Single-photon subtraction is experimentally implemented in a conditional way: a high-transmissivity BS is placed in the path of the light field and the quantum operation succeeds each time that an on/off detector (SPCM, an avalanche photodiode which can discern there being photons from no photons) fires in the reflected mode (see figure 1). It is straightforward to show that such a scheme is a very faithful implementation of the ideal single-mode photon annihilation operator \hat{a} for states with low mean photon numbers and for low BS reflectivity [7]. The resulting states are then analyzed by a balanced homodyne apparatus (BHD) working on a pulse-to-pulse basis triggered by the on/off detectors. The light state is mixed with

a strong reference coherent field (the local oscillator (LO) a portion of the laser output) on a 50–50 beam splitter (BS-H). Particular care has to be taken in order to perform the photon subtraction from exactly the right mode, i.e. from the one probed by homodyne detection. To this purpose, we have to carefully mode-match the subtraction channel to the LO mode; this is done by means of lenses and making use of a continuous wave (CW) laser source for interferometrically precise alignment. The BS outputs are detected by two photodiodes, whose differential signal (proportional to a given field quadratures) is acquired and stored by a digital oscilloscope. A sequence of such measurements taken at different LO phases, proportional to different field quadratures, is then used to completely reconstruct the density matrix of the light state. Quantum state reconstruction is performed after the acquisition of about 10^5 quadrature values triggered by subtraction events, or by double detector firings in the case of subtraction from a Fock state; in such a case the double conditioning serves to first prepare the single-photon Fock state and then subtract a single photon, in the sequence $|0\rangle \rightarrow |1\rangle \rightarrow |0\rangle$. The LO phase is left unlocked for thermal and Fock states, which present no phase dependence, whereas it is actively scanned (by applying a voltage to a piezoelectric transducer (PZT) holding one of the mirrors) for coherent states. The state reconstruction is performed using the maximum likelihood estimation method [17] giving the density matrix that most likely represents the measured homodyne data. We first build a likelihood function contracted for a density matrix truncated to eight elements (only eight and five diagonal ones for the thermal and the Fock states, respectively) with the constraints of Hermiticity, positivity and normalization. Then the function is maximized by an iterative procedure [18, 19] and the errors on the reconstructed density matrix elements are evaluated using the Fisher information [19].

Coherent states are simply obtained by splitting off (by means of a high reflectivity (HR)) BS, a small portion of the main laser output and properly attenuating it by means of neutral density filters (VF). Thermal states are produced by inserting a rotating ground glass disk (RD) in the coherent laser beam. The scattered light forms a random spatial distribution of speckles whose average size is larger than the core diameter of a single-mode fiber used to collect it. When the ground glass disk rotates, light exits the fiber in a clean collimated spatial mode with random amplitude and phase fluctuations yielding the photon distribution typical of a thermal source [20]. Indeed, if the rate of subtraction events is smaller than the inverse coherence time of the state (mainly dictated by the rotation speed of the disk), then photon subtraction is performed on a sequence of completely uncorrelated (both in amplitude and phase) coherent states, i.e. a thermal state by definition. The generation of single-photon Fock states, on the other hand, is performed in a conditional way based on the spontaneous parametric down-conversion (SPDC) in a nonlinear crystal. The frequency-doubled laser pulse train serves as a pump for a type-I BBO crystal, which generates pairs of SPDC photons at the same wavelength of the laser source in two distinct spatial modes called signal and idler. The firing of an on/off photodetector placed in the idler (trigger) path after narrow spectral and spatial filters (F) is used to conditionally prepare a single photon in a well-defined spatiotemporal mode of the signal channel (further details are given in [2, 21]). The LO field is then accurately matched to this mode in order to maximize the efficiency of homodyne detection.

It is interesting to note that all the measurements described below are based on simple variations upon the scheme implemented in our previous experiment for demonstrating the noncommutativity of creation and annihilation operators [7]. A simplified block diagram of the experiment is depicted in figure 2(b) and may help to visualize the involved operations. A module for single-photon creation (implemented by conditional parametric down-conversion

and denoted by \hat{a}^\dagger) is placed between two modules for single-photon annihilation (implemented by means of low-reflectivity BSs and denoted by \hat{a}). In previous experiments alternated sequences of the operators were obtained by conditioning upon double clicks from the first/second or second/third modules with a thermal state as the input. In this case, we use the triggers from one or both subtraction modules for single or double photon subtraction from the thermal state. The subtraction from the coherent state is simply obtained by removing the ground glass disk. On the other hand, the subtraction of a single photon from the Fock state is conditioned upon a double click from the second/third modules when seeding is blocked and nothing (the vacuum field) is fed as the input.

3. Results and discussion

In the following, we only consider the experimentally reconstructed data about the investigated states, without making particular assumptions on their character and without correcting for the various inefficiencies involved in the state preparation and detection. However, since we are mainly interested in a comparison of the different states before and after photon subtraction, many of these systematic effects disappear in the analysis.

A coherent and a thermal state with reconstructed mean photon numbers of $\bar{n}_{\text{coh}} = 0.63 \pm 0.02$ (corresponding to $|\alpha| \approx 0.8$), and $\bar{n}_{\text{th}} = 0.36 \pm 0.05$, respectively, have been used. With a BS reflectivity set to about 1–2%, subtraction rates between 50 and 200 kHz are obtained.

3.1. Thermal state

The reconstructed density matrix elements and the corresponding Wigner functions for the thermal state and for its photon-subtracted version are reported in figures 3(a) and (b). A reshaping and broadening of the photon number distribution is evident and the final reconstructed mean photon number $\bar{n}_{\text{th,sub}} = 0.69 \pm 0.07$ is in very good agreement with the expected $\bar{n}_{\text{th,sub}} = 2\bar{n}_{\text{th}}$ within the experimental errors. A flattening of the Wigner function is also visible in figure 3(b), and the departure from a Gaussian state can be estimated by calculating the kurtosis of the measured quadrature distributions. It results in $K = 2.999$ for the thermal state (to be compared to 3 for an ideal Gaussian distribution), whereas it reduces to $K = 2.795$ for the subtracted thermal state.

Here, we extend our tests by also implementing a sequence of two single-photon subtractions (double clicks from the first and third modules in figure 2(b)). Due to the low success rate of this operator sequence (about 170 Hz in this case), this can only be tested for the thermal state input which does not require additional conditioning events (as for Fock states) nor phase stability (as for coherent states). Also in this case, the results are quite intriguing (see figure 3(c)): the kurtosis of the measured quadrature distribution is now $K = 2.577$ and the resulting photon number distribution gets even broader. The reconstructed mean photon number becomes $\bar{n}_{\text{th,subsub}} = 1.03 \pm 0.05$, as expected by the theoretical value $\bar{n}_{\text{th,subsub}} = 3\bar{n}_{\text{th}}$: annihilating two photons from a thermal state results in a tripling of its mean photon number.

The apparently contradictory increase of the mean photon number by photon subtraction has already been observed [7] and can be explained from the fact that we are not dealing with the simple extraction of one particle from an initial state. Instead, we are dealing with a probabilistic annihilation of photons whose probability is proportional to the number of photons in the initial field.

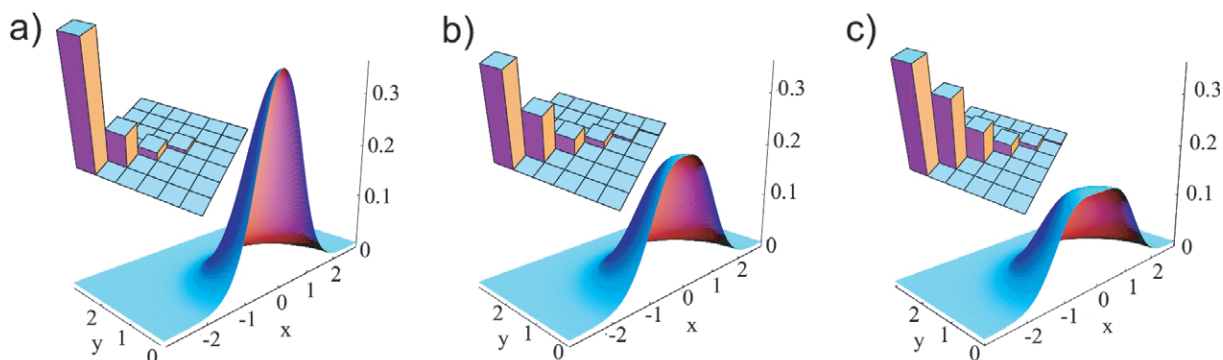


Figure 3. Experimentally reconstructed diagonal density matrix elements and Wigner functions for: (a) thermal state; (b) photon-subtracted thermal state; and (c) thermal state after a double photon subtraction.

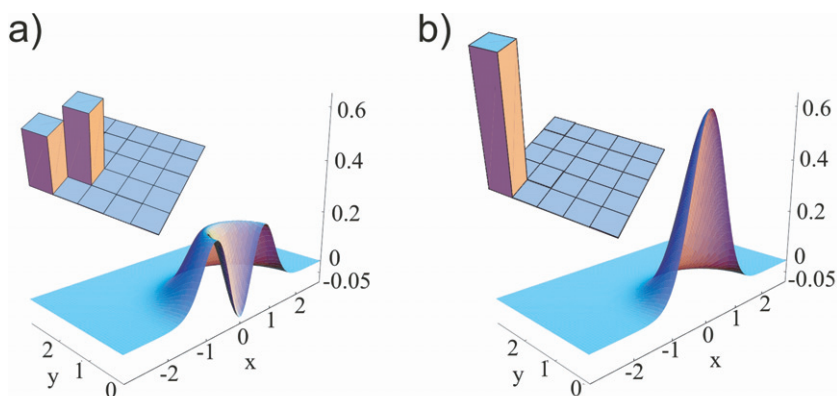


Figure 4. Experimentally reconstructed diagonal density matrix elements and Wigner functions for: (a) single-photon Fock state and (b) photon-subtracted Fock state.

3.2. Single-photon Fock state

The experimental results in the case of a Fock state are presented in figure 4. Here, due to the finite experimental efficiency, some residual vacuum contribution (about 40%) appears in the photon number distribution of the prepared single photon; however, the corresponding Wigner function clearly shows a nonclassical negativity. Due to the low parametric gain coefficient, essentially no higher-order terms are present. After photon subtraction the state is thus totally converted into the vacuum state, without other significant contributions. Note that in this case one can safely relax the constraint of very low reflectivity for the subtraction BS. Indeed, a low reflectivity ensures that no operations other than a single-photon subtraction can occur (see [16]), but these are impossible in this case where no multi-photon terms are present in the initial state. Therefore, a higher reflectivity (22%) is used to partially compensate for the low state production rates (only about 30 Hz) due to the double conditioning.

The result of photon subtraction from a Fock state is far from surprising: in this case, it is intuitively clear that sending a one photon state through a BS yields the vacuum in one arm if a click is registered in the other. The fact that a single photon split between two paths can only be

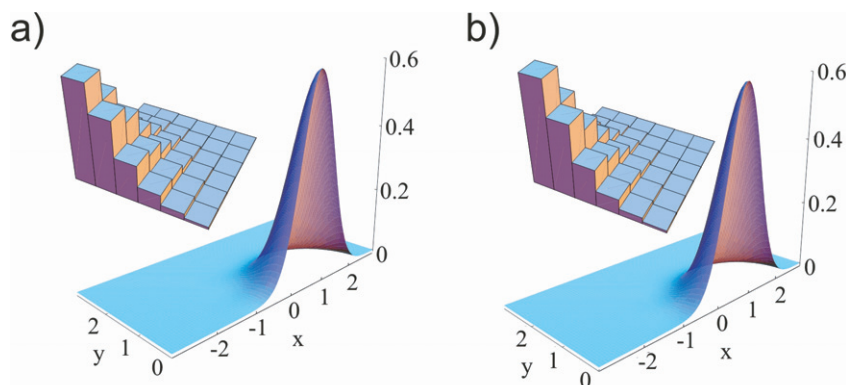


Figure 5. Experimentally reconstructed density matrix elements and Wigner functions for: (a) coherent state and (b) photon-subtracted coherent state.

detected in one of them is indeed a rather standard way to characterize single-photon sources in anticoincidence experiments. Such a direct and complete tomographic characterization of this phenomenon had nonetheless never been performed before.

3.3. Coherent state

Finally, the experimentally reconstructed density matrix elements and Wigner functions for a coherent state and for its photon-subtracted version are presented in figure 5. It is evident that the operation of single-photon subtraction has no measurable effect on the state. Both the reconstructed density matrix elements and the Wigner functions are unchanged after splitting one photon off the incoming field, within the experimental errors. The expected effect of attenuation due to the nonnull reflectivity of the BS in the nonideal experimental implementation of \hat{a} is within these errors. The fidelity between the initial coherent state and the photon-subtracted one has been calculated and is found to be $F = 0.98$. A coherent state is thus experimentally proven to be an eigenstate of the \hat{a} operator.

However, it is particularly interesting to make some more considerations about photon subtraction in this particular case. In reality, conditioning upon the reflection of a single photon by the BS should not work at all for a coherent state. It is well known that coherent states are split by a BS in such a way that no correlation (neither quantum nor classical) is left between the outgoing modes. Therefore, whatever one does in one output mode has absolutely no effect on the other and conditioning is useless.

We experimentally test this by also performing measurements on the coherent state just passing through the BS, but without using the conditioning signal from the on/off detector. The resulting state is once again the same as the original one (within the experimental uncertainties) and the fidelity between the final states obtained with or without conditioning is larger than 0.99.

Since conditioning has no effect in the case of a coherent state, one might be tempted to affirm that no real operation has been performed, and that the observed state invariance is just a natural consequence of this. However, one should rather put it in another way: by demonstrating that the conditioning does not affect the final states we have further proved that coherent states are eigenstates of \hat{a} , since it is exactly this that makes the BS outputs uncorrelated. This can be demonstrated explicitly: a BS mixes two input fields to bring about two output fields which

are usually correlated to each other. This is true not only when two output fields are quantum mechanically correlated from nonclassical input fields, but also when they are classically correlated from, for example, two thermal input fields of different temperatures [22]. However, in some rare cases, the output fields are factorizable and they are totally independent to each other. In the following, we demonstrate that when one input field is the vacuum, factorizable output fields are possible only when the other field is in a coherent state, due to bosonic algebra.

Let us consider two input modes a and b of a BS with reflectivity r and transmissivity t . We denote bosonic annihilation \hat{a} (\hat{b}) and creation \hat{a}^\dagger (\hat{b}^\dagger) operators for mode a (b). The BS operator, which is normally described as $\hat{B} = \exp\left\{\frac{\theta}{2}(\hat{a}^\dagger\hat{b} - \hat{a}\hat{b}^\dagger)\right\}$ with $r = \sin\frac{\theta}{2}$ and $t = \cos\frac{\theta}{2}$, can also be written in the following form [23]:

$$\hat{B}(r) = \exp\left(-\frac{r}{t}\hat{a}\hat{b}^\dagger\right) t^{\hat{a}^\dagger\hat{a}} t^{-\hat{b}^\dagger\hat{b}} \exp\left(\frac{r}{t}\hat{a}^\dagger\hat{b}\right). \quad (2)$$

One of the input modes, say b , is assumed to be the vacuum and the other input mode to be an arbitrary state $|\psi\rangle_a$. Using the Taylor expansion of the exponential function in equation (2) and the fact that $\hat{b}|0\rangle_b = 0$, it is obtained that

$$\hat{B}(r)|\psi\rangle_a|0\rangle_b = e^{-(r/t)\hat{a}\hat{b}^\dagger}|\psi'\rangle_a|0\rangle_b, \quad (3)$$

where $|\psi'\rangle = t^{\hat{a}^\dagger\hat{a}}|\psi\rangle \equiv \sum_n C(n)|n\rangle$. Again using the Taylor expansion of the exponential operator in equation (3) and the bosonic operator algebra, it is found that

$$\hat{B}(r)|\psi\rangle_a|0\rangle_b = \sum_{m=0}^{\infty} \left(-\frac{r}{t}\right)^m \frac{1}{\sqrt{m!}}|m\rangle_b \otimes \underbrace{\left(\sum_{n=m}^{\infty} C(n) \sqrt{\frac{n!}{(n-m)!}}|n-m\rangle_a\right)}_{F(m)}. \quad (4)$$

For the two output modes a and b to be factorizable, all $F(m)$'s have to be proportional to each other for every m , in other words, the following relation has to hold with proportionality factors a_p ($p = 0, 1, 2, \dots$):

$$a_0 F(0) = a_1 F(1) = a_2 F(2) = a_3 F(3) = \dots \quad (5)$$

From this, the following simple relation is found between the proportionality factors:

$$a_0 = \frac{a_1^2}{a_2} = \frac{a_2^3}{a_3} = \frac{a_3^4}{a_4} = \dots, \quad (6)$$

which is solved to give the relation: $a_p = a_0^{p(k-1)+1}$. Substituting this into equation (5) and using the definition of $F(m)$, we find that $C(n-1) = a_0^{k-1} \sqrt{n} C(n)$. With the use of the normalization condition for $|\psi\rangle$, the weight function is found to be

$$C(n) = \frac{(t\alpha)^n}{\sqrt{n!}} e^{-|\alpha|^2/2}, \quad (7)$$

where $\alpha = 1/ta_0^{k-1}$. This is a Poissonian weight, showing that input state $|\psi\rangle$ must be a coherent state for the output fields to be factorizable when the other input is vacuum. Our proof in the case of a pure state input can be straightforwardly extended to all mixed state inputs, for the other input field to the BS in the vacuum. Thus, the only input field which gives the factorizable output fields by a BS is the coherent state when the other input field is the vacuum.

4. Conclusions

In conclusion, we have realized a faithful experimental implementation of the photon annihilation operator \hat{a} and we have applied it to a few paradigmatic quantum states in order to verify its peculiar effects. We have clearly shown the predicted re-shaping of the photon number distributions with a change in the mean photon number depending on the statistics of the original state. Moreover, by demonstrating that this operation has no effect on a coherent state, we have provided the first direct verification of the definition of coherent states as eigenstates of \hat{a} . Subtracting one photon from a quantum light state has already been identified as a simple way to turn Gaussian states into non-Gaussian ones, which may be extremely useful for future communication and computing tasks. This work, besides its implications for fundamental quantum physics, demonstrates that an accurate mastering of these basic quantum operations is now available.

Acknowledgments

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References

- [1] Zavatta A, Viciani S and Bellini M 2004 *Science* **306** 660
- [2] Zavatta A, Viciani S and Bellini M 2005 *Phys. Rev. A* **72** 023820
- [3] Zavatta A, Parigi V and Bellini M 2007 *Phys. Rev. A* **75** 052106
- [4] Wenger J, Tualle-Brouiri R and Grangier P 2004 *Phys. Rev. Lett.* **92** 153601
- [5] Kim M S, Park E, Knight P L and Jeong H 2005 *Phys. Rev. A* **71** 043805
- [6] Ourjoumtsev A, Tualle-Brouiri R, Laurat J and Grangier P 2006 *Science* **312** 83
- [7] Parigi V, Zavatta A, Kim M S and Bellini M 2007 *Science* **317** 1890
- [8] Opatrný T, Kurizki G and Welsch D-G 2000 *Phys. Rev. A* **61** 032302
- [9] Cochrane P T, Ralph T C and Milburn G J 2002 *Phys. Rev. A* **65** 062306
- [10] Olivares S, Paris M G A and Bonifacio R 2003 *Phys. Rev. A* **67** 032314
- [11] Cerf N J, Krüger O, Navez P, Werner R F and Wolf M M 2005 *Phys. Rev. Lett.* **95** 070501
- [12] Eisert J, Scheel S and Plenio M B 2002 *Phys. Rev. Lett.* **89** 137903
- [13] Giedke G and Ignacio Cirac J 2002 *Phys. Rev. A* **66** 032316
- [14] Ueda M, Imoto N and Ogawa T 1990 *Phys. Rev. A* **41** 3891
- [15] Mizrahi S S and Dodonov V V 2002 *J. Phys. A: Math. Gen.* **35** 8847
- [16] Kim M S 2008 *J. Phys. B: At. Mol. Opt. Phys.* **41** 133001
- [17] Banaszek K, D'Ariano G M, Paris M G A and Sacchi M F 1999 *Phys. Rev. A* **61** 010304
- [18] Lvovsky A I 2004 *J. Opt. B: Quantum Semiclass. Opt.* **6** S556
- [19] Hradil Z, Mogleitsev D and Rehacek J 2006 *Phys. Rev. Lett.* **96** 230401
- [20] Arecchi F T 1965 *Phys. Rev. Lett.* **15** 912
- [21] Zavatta A, Viciani S and Bellini M 2004 *Phys. Rev. A* **70** 053821
- [22] Kim M S, Son W, Bužek V and Knight P L 2002 *Phys. Rev. A* **65** 032323
- [23] Barnett S M and Radmore P M 1997 *Methods in Theoretical Quantum Optics* (Oxford: Oxford University Press)