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# Fatigue life analysis of edge-notches with damage

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**Abstract**

In the present paper, the fracture mechanics-based computational model is proposed for estimating the strength of double-edge notched configuration under cyclic loading. The propagation of a quarter-elliptical corner crack located at one of two semi-circular edge-notches is analytically investigated by means of the fatigue life to failure and crack path. Also, through experimental observations available in the literature the reliability of obtained theoretical results is discussed.


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*Keywords:* Fatigue strength; quarter-elliptical corner crack; two edge-notches; crack path.

## 1. Introduction

The stress-concentration phenomenon that appears at critical zones where geometrical discontinuities exist may endanger the service performance of mechanical systems, due to the formation of either the through or part-through (quarter-elliptical and semi-elliptical) crack-like damages. Therefore, to prevent such a harmful effect caused by the abrupt disturbance of the stress field, the fatigue behaviour of damaged notches (holes, cutouts, grooves) should be analyzed employing the fracture mechanics-based computational models.

Under cyclic loading, the failure of open and pin loaded holes with quarter-elliptical corner cracks or semi-elliptical cracks has been theoretically examined by Grandt *et al*. (1982) using the Paris' crack growth concept and

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the finite element technique. Then, Guo (1993) was suggested the analytical concept for assessing the stability of quarter-elliptical corner crack at the pin-loaded hole. The same fatigue damage has been investigated by Mikheevskiy *et al*. (2012) and Boljanović *et al.* (2017) taking into account the crack growth concept proposed by Noroozi *et al.* (2007) together with the weight function method, and through the crack growth law suggested by Zhan *et al.* (2014) and the *J*-integral method, respectively. Further, for the fatigue analysis of quarter-elliptical corner crack located at the single-edge notched plate Zhao and Wu (1988), and Newman (1995) were applied the weight function method and the crack closure concept, respectively.

In the present paper, the failure of safety critical double-edge notched configuration with quarter-elliptical corner crack was analyzed under cyclic loading. As a matter of fact, the analytical failure model was developed to evaluate the residual life and crack path, and then, the influence of relevant geometric parameters on the fatigue strength was discussed for damaged notches.

## 2. Crack propagation at edge-notch

The fatigue performance of quarter-elliptical corner crack at one of two semi-circular edge-notches is here examined through two critical directions on the crack front (Fig. 1) employing the crack growth concept proposed by Huang and Moan (2017), as follows:

$$\frac{da}{dN} = C_A (M\,\Delta K_A)^{m_A}, \; \frac{db}{dN} = C_B (M\,\Delta K_B)^{m_B} \tag{1}$$

where $a$, $b$ and $da/dN$, $db/dN$ are crack length and crack growth rate in depth and surface direction, respectively, $C_A$, $m_A$, $C_B$, $m_B$ represent material parameters, whereas $\Delta K_A$, $\Delta K_B$ are the stress intensity factors, respectively, for two positions examined at the crack front.

Further, to describe the interaction between environment and service loadings, the fracture-mechanics parameter $M$ was taken into consideration (with respect to stress ratio $R$), as follows:

$$M = \begin{cases} (1-R)^{-\beta_1} & -5 \le R < 0 \\ (1-R)^{-\beta} & 0 \le R < 0.5 \\ (1.05 - 1.4R + 0.6R^2)^{-\beta} & 0.5 \le R < 1 \end{cases} \tag{2}$$

where $\beta$ and $\beta_1$ are crack growth exponents related to the environmental conditions.

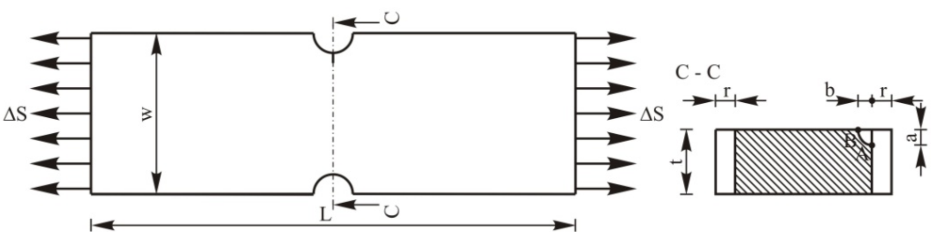


Fig. 1. Double-edge notched plate with quarter-elliptical corner crack.

Also, in the failure-resistance assessment of fatigue damages, by integrating above crack growth rates from initial $a_0$, $b_0$ to final $a_f$, $b_f$ crack lengths the number of loading cycles $N$ can be calculated, i.e.:

$$N = \int_{a_0}^{a_f} \frac{da}{C_A (M\,\Delta K_A)^{m_A}}, \quad N = \int_{b_0}^{b_f} \frac{db}{C_B (M\,\Delta K_B)^{m_B}} \tag{3}$$

Additionally, it is worth to mention that the residual strength was simultaneously assessed for two relevant positions at the crack front through the software program developed here, using the Euler's numerical integration method.

## 3. Fatigue behaviour of quarter-elliptical crack using the stress intensity factor

Disturbed stress field of the damaged edge-notch configuration (Fig. 1) was theoretically here analyzed by means of the stress intensity factor (Wu *et al*., 1998; Carpinteri, 1994), as follows:

$$\Delta K = F_{qec}\,\Delta S \sqrt{\frac{\pi a}{Q}} \tag{4}$$

where $\Delta S$ is the applied stress range, $a$ represents crack length in depth direction (Fig. 1), $Q$ and $\Delta K$ are the ellipse shape factor and stress intensity factor range, respectively. Further, $F_{qec}$ denotes a corrective function depending on the crack geometry and loading conditions, which can be expressed as follows:

$$F_{qec} = \left( M_1 + M_2 \left(\frac{a}{t}\right)^2 + M_3 \left(\frac{a}{t}\right)^4 \right) g_1\, g_2\, g_3\, g_4\, g_5\, f_\phi\, f_w \tag{5}$$

Note that, the relevant crack shape effects (related to crack size, front face) are taken into account through suitable correction factors ($M_1$, $M_2$, $M_3$), and in the case of $a/b \leq 1$, according to Wu *et al*. (1998), they are given by:

$$M_1 = 1.13 - \frac{0.09\,a}{b}, \quad M_2 = -0.54 + \frac{0.89}{0.2 + \frac{a}{b}}, \quad M_3 = 0.5 - \frac{1}{0.65 + \frac{a}{b}} \tag{6}$$

where $b$ is the crack length in surface direction. Also, the effects of a notch radius, angle location and thickness were examined by means the correction factors $g_1$, $g_2$, $g_3$, $g_4$, $g_5$ and $f_\phi$, respectively, as is reported by Wu *et al*. (1998).

Furthermore, in the fatigue strength assessments the influence of elliptical crack shape and the finite-width effect associated with crack length has been taken into consideration through the ellipse shape factor $Q$ and the correction factor $f_w$, defined as follows:

$$Q = 1 + 1.464 \left(\frac{a}{b}\right)^{1.464} \tag{7}$$

$$f_w = 1 + 4.6\gamma + 3.46\gamma^2 - 7.55\gamma^3 + 5.82\gamma^4 \quad \text{with} \quad \gamma = \frac{b}{w - 2r} \tag{8}$$

where $w$ and $r$ are the width of the double-edge notched plate/specimen and radius of the notch, respectively.

## 4. Implementation of the proposed analytical failure model in residual strength evaluations

Now the failure-resistance of the semi-circular edge-notches with quarter-elliptical corner crack was designed employing the proposed fracture mechanics-based computational model. Damage tolerance analysis herein examined takes into account the residual life, the crack path as well as the influence of relevant geometric parameters on the notch fatigue strength.

### *4.1. Fatigue life calculation*

In the first Section, the failure of double-edge notched configuration with corner crack (Fig. 1), made of 2024 T3 aluminium alloy, was investigated in terms of the residual life. Thus, the applied cyclic loading with constant amplitude ($R$ = 0.1) was characterized by two different levels of maximum stress ($S_{max}$= 68.97 MPa and 91.73 MPa), whereas an initial quarter-elliptical corner crack lengths were equal to $a_0 = b_0$ = 1.35 mm and 1.45 mm, respectively. Further, the following geometrical and material parameters were assumed: $t$ = 6.35 mm, $k_T$ = 3.20, $w$ = 38.1 mm, $\beta$=0.7, $C_B$ = 5 $10^{-11}$, $m_A$= $m_B$ = 2.73, respectively.

In the fracture mechanics-based analysis presented, the stress intensity factor and fatigue life were calculated by applying Eqs. (4)-(8) and Eqs. (2)-(3), respectively, as is discussed in Sections above. The estimated number of loading cycles (in the case of maximum stress equal to $S_{max}$= 68.97 MPa and 91.73 MPa) is shown in Figs. 2a and b, respectively, for the crack growth in surface direction. Then, such assessments were compared with those experimentally tested by Everett *et al*. (2001), as is shown in Table 1. Examining Table 1 and Fig. 2 it can infer that appropriate fatigue results, related to the crack length and the number of loading cycles, are in a quite good agreement.

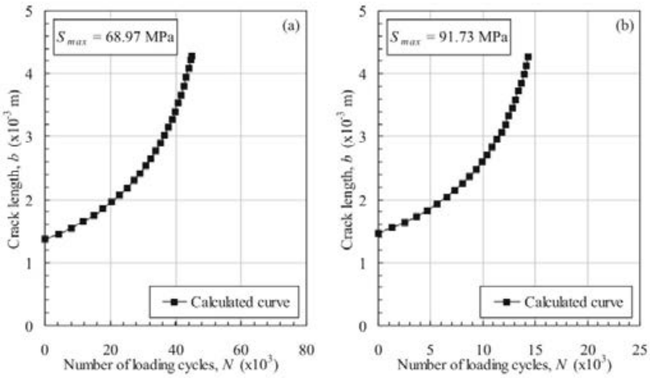


Fig. 2. Fatigue analysis of quarter-elliptical corner crack ($R$ = 0.1): (a) $b$ versus $N$, $S_{max}$= 68.97 MPa; (b) $b$ versus $N$, $S_{max}$= 91.73 MPa. Calculated curves are the present results.

Table 1. Number of loading cycles and corresponding crack lengths in surface direction (experimental data are reported by Everett *et al*. (2001) and the present results are the calculated results).

| $S_{max}$= 68.97 MPa | | | $S_{max}$= 91.73 MPa | | |
|---|---|---|---|---|---|
| $b$ (mm) | $N^{cal.}$ (cycl.) | $N^{exp.}$ (cycl.) | $b$ (mm) | $N^{cal.}$ (cycl.) | $N^{exp.}$ (cycl.) |
| 1.896 | 19155 | 21009 | 1.961 | 5647 | 5416 |
| 2.291 | 27215 | 28386 | 2.931 | 11346 | 10000 |
| 3.644 | 41671 | 43544 | 3.597 | 13130 | 13861 |
| 4.171 | 44244 | 48004 | 4.391 | 14437 | 16944 |

### 4.2. Influence of width, thickness and crack shape on the failure-resistance of the double-edge notched plate

The fatigue behaviour of quarter-elliptical corner crack (Fig. 1) was herein analyzed by taking into account the effects of width and thickness. For two notched plate configurations ($t$ = 7 mm, $r$ = 7.5 mm and $w$ = 50 mm, $r$ = 8 mm), whose material parameters are the same as those examined in Section above, the residual strength was evaluated in terms of the life to failure. Such calculations were performed in the case of three different plate widths ($w$ = 45 mm, 54 mm, 64.8 mm) and for three values of thicknesses ($t$ = 6.54 mm, 8.50 mm, 11.05 mm), by assuming the following initial crack lengths and applied maximum forces: $a_0$ = 1.5 mm, $b_0$ = 1.8 mm, $P_{max}$= 25000 N and $a_0$=$b_0$ = 2 mm, $P_{max}$= 30000 N, respectively. Then, the failure stability of edge-notches ($t$ = 7 mm, $r$ = 7.5 mm, $w$=45 mm, $a_0$ = 1.5 mm, $P_{max}$= 25000 N) was also estimated through Egs. (2)-(8) for three different crack lengths in surface direction ($b_0$ = 1.5 mm, 1.8 mm, 2.16 mm). The calculated number of loading cycles (as a function of crack length) in which the effects of width and thickness are examined, and those related to the crack shape effect are shown in Fig. 3a, b and Fig. 3c, d, for depth and surface direction, respectively.

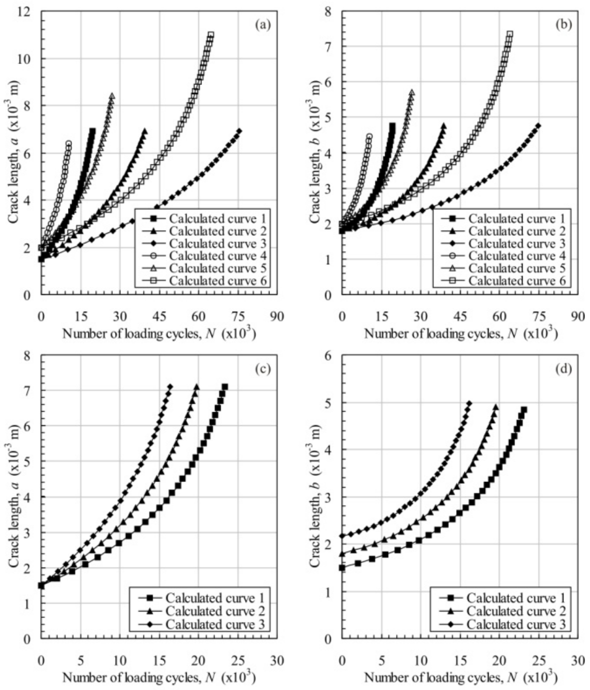


Fig. 3. Fatigue analysis of quarter-elliptical corner crack ($R$ = 0.1): (a) $a$ versus $N$; (b) $b$ versus $N$ (1 – $w$ = 45 mm, 2 – $w$ = 54 mm, 3 – $w$ = 64.8 mm, 4 – $t$ = 6.54 mm, 5 – $t$ = 8.50 mm, 6 – $t$ = 11.05 mm); (c) $a$ versus $N$; (d) $b$ versus $N$ (1 – $b_0$ = 1.5 mm, 2 – $b_0$ = 1.8 mm, 3 – $b_0$ = 2.16 mm).

### 4.3. Simulation of quarter-elliptical corner crack under cyclic loading

The failure-resistance of fatigue damage located at one notch of the double-edge notched plate (Fig. 1) was assessed through the crack growth path. Such quarter-elliptical crack-like flaw is characterized by the following initial lengths: $a_0$ = 1.5 mm, $b_0$ = 1.9 mm in depth and surface direction, respectively. The plate ($w$ = 55 mm, $t$ = 8.7 mm, $r$ = 8 mm), subjected to cyclic loading ($S_{max}$= 58.52 MPa, $R$ = 0.1), is made of 2024 T3 aluminium alloy and the same material parameters are used as those mentioned in Section 4.1.

In the notch performance analysis examined, through Eqs. (1)-(2) and Eqs. (4)-(8) the crack growth rate together with stress intensity factor were evaluated for appropriate depth crack lengths. Further, in each step of crack extension to calculate the crack length in surface direction the stress intensity factors from the preceding step are employed. By adopting that the vertical axis and the horizontal axis correlate to the plate front face and to the semi-circular notch-hole, respectively, the evaluated crack paths are presented in Fig. 4a and b for ten different crack depth lengths.

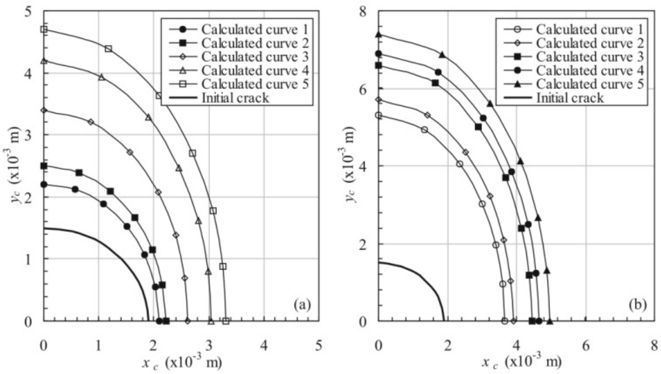


Fig. 4. Crack path evaluations ($a_0$=1.5 mm, $b_0$=1.9 mm): (a) 1 – $a$ = 2.2 mm, 2 – $a$ = 2.5 mm, 3 – $a$ = 3.4 mm, , 4 – $a$ = 4.2 mm, 5 – $a$ = 4.7 mm; (b) 1 – $a$ = 5.3 mm, 2 – $a$ = 5.7 mm, 3 – $a$ = 6.6 mm, , 4 – $a$ = 6.9 mm, 5 – $a$ = 7.4 mm.

## 5. Conclusions

In the present paper, the fracture mechanics-based analysis of double-edge notched configuration with quarter-elliptical corner crack was carried out through developed computational model. By means of Huang-Moan crack growth concept the environmental and service loading conditions were taken into account for evaluating the fatigue life and crack path. Also, an analytical approach here proposed for the stress intensity factor provides a quick and reliably estimates the final crack lengths and corresponding number of loading cycles to failure in the case of safety-critical edge-notches.

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