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Procedia Structural Integrity 2 (2016) 2780–2787

Structural Integrity Procedia

www.elsevier.com/locate/procedia

21st European Conference on Fracture, ECF21, 20-24 June 2016, Catania, Italy

Evaluation of crack width in RC ties through a numerical "range" model

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Abstract

Crack phenomenon in reinforced concrete (RC) ties is studied herein by means of a numerical "range model", based on bond between steel and concrete. Since a unique definition of the crack pattern evolution is not possible for these elements, through the definition of this "range", defined by the curves of maximum and minimum crack spacing (and consequently width), all the possible crack patterns of the considered tension member are so included. Comparisons with significant experimental results available in the technical literature prove that the proposed approach can be successfully adopted also for design purposes, since it provides a correct estimate of maximum crack width. The obtained results are compared with Codes provisions (ACI and Model Code 2010) and the effectiveness of different approaches for predicting crack width is analyzed and discussed.

Keywords: RC tie; crack width; crack spacing; range model; bond-slip

1. Introduction

The problem of cracking in reinforced concrete (RC) tensile members has been studied extensively in the past, not only for the analysis of tension zones, such as in composite bridge decks or strut and tie models, but also for comprehending and modeling the behavior of beams in bending. Despite the large number of published studies, the problem of cracking in RC members in tension and flexure has not been definitively solved and a long debate regarding the evaluation of crack width is still ongoing. The lack of a unique formulation unanimously accepted by

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10.1016/j.prostr.2016.06.347

researchers for predicting crack width and spacing is proved by the development of more than twenty formulae, which relate crack width to different most critical parameters; among others, e.g. Broms (1965), Broms and Lutz (1965), Ferry-Borges (1966), Rizkalla and Hwang (1984), Bruggeling (1991). An extensive review on this topic can be found in Borosnyòi and Balàzs (2005).

Except for those relations derived from empirical approaches, most of the formulae are based on two internal mechanisms, namely the diffusion of stresses in the concrete cover and the bond-slip behavior between steel and concrete. Stress diffusion depends on the concrete cover c, whereas bond is often related on the ratio ϕ/ρ, where ϕ is the bar diameter and ρ the reinforcing ratio. Among others, Beeby (2004) questioned the role of bond by affirming the dependence of crack width essentially on the shear deformation of the concrete cover and on the distance from the nearest reinforcing bar. On the other hand, the contribution of bond has been recognized as an important mechanism from the early studies on RC structures (e.g. Watstein and Sees (1945)) to more recent works (e.g. Fantilli et al. (1998), CEB-FIB Bulletin No.10 (2000), Beeby et al. (2005), Chiaia et al. (2009)). A comparison between the results provided by classical one-dimensional models based on bond only and those obtained from two- and three-dimensional models, which also take into account the effect of stress diffusion in concrete blocks, can be found in Bernardi et al. (2014). However, a general agreement on which of these two phenomena has the greatest influence on the cracking behavior of RC elements has not been reached yet and some recent works have once again reopened this aged-old discussion (Pérez Caldentey et al. (2013), Forth and Beeby (2014)). It must be also added that the values of crack spacing and width are disperse, due to the statistical variability of concrete tensile strength and due to the fact that the crack pattern develops progressively as loading increases. For these reasons, average vs. maximum crack width can be considered, even if the maximum values seem to be more interesting when addressing issues of durability, leaching and aesthetics (e.g. Ziari and Kianoush (2009)).

Aim of the present work is first of all to understand if a model essentially based on bond-slip is able to predict crack width and spacing in RC ties; secondly, the obtained results are compared with Codes provisions (Model Code 2010 (2012) and ACI224.2R-92 (1992) in the following MC2010 and ACI224). To this purpose, a one-dimensional numerical "range" model, which assumes plane cross-sections in concrete and a proper bond-slip behavior between steel and concrete, is proposed (so totally neglecting the diffusion mechanism). To take into account the uncertainty of crack pattern evolution, the model provides a range of crack widths and spacing that, according to bond theory, are possible for a given load (Somayaji and Shah (1981), Avalle et al. (1994), Fantilli et al. (1998)). The reliability of the proposed approach is verified through new comparisons with some significant experimental results on RC tension members (Wu and Gilbert (2008), Gijsbers and Hehemann (1977)) available in the technical literature by applying the update MC2010 bond-slip law.

2. Numerical "range" model

As known, RC tension ties are characterized by a basically uniform state of stress along their length and consequently the cracking process starts at the weakest spot, whose position is uncertain, while subsequent cracks will occur in locations that depend on this initial random event. The uncertainty related to the initial crack pattern and its evolution is herein tackled through a simple procedure, by considering two limit configurations, which bound all possible crack patterns, within a well-defined field or "range" (Avalle et al. (1994), Fantilli et al. (1998)). These two limit configurations respectively correspond to the case of a tension tie block in incipient cracking condition and immediately after the opening of the crack. In the first case, it is assumed that the block length – and consequently the maximum crack spacing s_r – is equal to $l_{max} = 2l_t$, being l_t the transmission length, and that the concrete tensile stress reaches the tensile strength f_{ct} in the middle of the block (Fig. 1a). In the second limit configuration, the block previously defined is assumed to crack just in the midspan, so forming two separate blocks of length $l_{min} = l_t$ corresponding to the minimum crack spacing s_r; in this case, the concrete stress in the halfway section of each new block is lower than f_{ct} (Fig. 1a). It can be reminded that, for a given load N, the transmission length l_t is the length that is necessary to transfer bond stresses attaining perfect bond. According to the bond-slip model, a crack spacing larger than $2l_t$ is impossible since it would imply concrete stresses higher than f_{ct}, whereas a crack spacing shorter than l_t would descend from cracking of a block whose maximum concrete stress is lower than f_{ct}. These two conditions, limiting the range of all possible crack configurations, can then be traced by assuming that the block length $l_{max} = 2l_t$ varies continuously as a function of the applied axial load N.

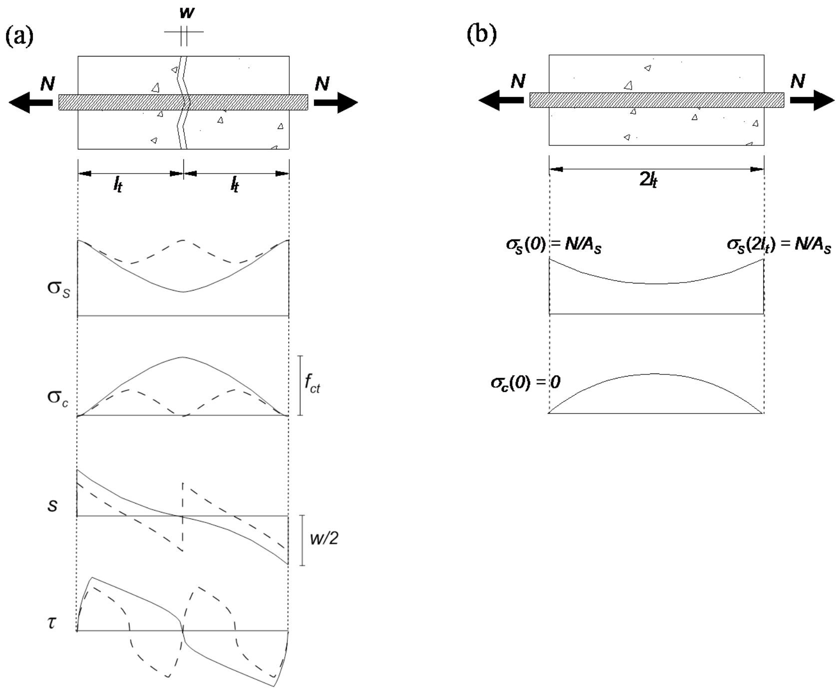


Fig. 1. (a) Sketch of the two limit configurations considered in the range model, respectively corresponding to the case of a tension tie block in incipient cracking condition (continuous line) and after the opening of the crack (dashed line); (b) boundary conditions at the ends of the tensile block of length $2l_t$.

2.1. Kinematics, equilibrium and compatibility equations

The basic assumptions of the adopted numerical model (see also Bernardi et al. (2014)) are sketched in Figure 2. As can be seen, the model is based on the presence of the slip s and the bond stresses τ at the interface between the reinforcing bar and the surrounding concrete. The slip s is defined as the difference between the displacement of two points that were originally in contact, respectively belonging to reinforcement and concrete (that means $s = u_s - u_c$). The corresponding bond stress τ is defined as a function of the current slip through a suitable bond-slip relation. With reference to the free body diagram of an element with infinitesimal length, the compatibility condition (Eq. 1, Fig. 2b), as well as the axial equilibrium of the reinforcing bar (Eq. 2, Fig. 2c) and of the whole cross-section (Eq. 3, Fig. 2c) can be written as:

$$\frac{ds(x)}{dx} = \varepsilon_s(x) - \varepsilon_c(x) = \frac{\sigma_s(x)}{E_s} - \frac{\sigma_c(x)}{E_c} \tag{1}$$

$$\frac{d\sigma_s(x)}{dx} = \frac{n_b\ \pi\ \phi}{A_s}\tau(s) \tag{2}$$

$$\frac{d\sigma_c(x)}{dx} = -\frac{A_s}{A_c}\ \frac{d\sigma_s(x)}{dx} = -\frac{n_b\ \pi\ \phi}{A_c}\ \tau(s) \tag{3}$$

where $\varepsilon_c(x)$ and $\varepsilon_s(x)$ are the strains in concrete and in the reinforcing bar, respectively, while $\sigma_c(x)$ and $\sigma_s(x)$ are the corresponding stresses. Moreover, A_c, A_s and E_c, E_s are the area and the Young modulus of concrete and steel rebar, while n_b and ϕ represent the number and diameter of reinforcing bars.

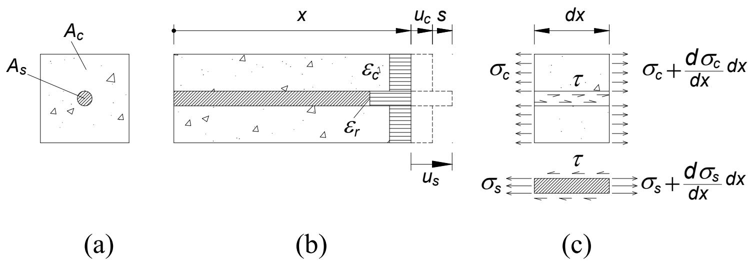


Fig. 2. Basic assumptions of the adopted model: (a) cross-section, (b) kinematics and (c) equilibrium conditions.

2.2. Constitutive and interface laws

The steel bar is assumed to have a linear elastic behavior during the analysis, which is limited to serviceability conditions. Similarly, the behavior of concrete in tension is assumed to be linear elastic until the attainment of its tensile strength f_{ct}, when a transversal crack forms. Differently from the approach adopted for the analysis of tension ties in Bernardi et al. (2014), the presence of cohesive tensile stresses across crack surfaces is neglected. Because of its simplifying hypotheses, the model proposed herein cannot indeed take into account the real development of cracking process as loading increases – which obviously exerts an influence on cohesion stresses and their evolution – as well as the possible presence of unloading due to the appearance of new cracks. Anyway, since the primary purpose of this paper is to provide a reliable estimate of maximum crack width, the solution obtained without considering the contribution of cohesion lays on the safe side.

The proposed model, which adopts the bond-slip relation suggested in MC2010, allows also to consider – or not – the presence of a bond deterioration zone near transverse cracks, due to splitting and crushing of concrete around the bar beside the crack surface. In more details, the influence of transverse cracking has been taken into account by properly reducing the bond stresses for those parts of the reinforcement placed at a distance $x_\lambda \leq 2\phi$ from a free surface, through the introduction of a damage factor $\lambda = 0.5\, x_\lambda / \phi \leq 1$, as suggested in MC2010.

2.3. Numerical solution procedure

Expressions (1), (2), and (3) form a system of differential equations that can be solved numerically, through a procedure based on the collocation method, implementing the three stage Lobatto formula. Automatic mesh selection and error control are based on the residual of the continuous solution (Shampine et al. (2003)). According to Figure 1b, with reference to the first limit configuration, corresponding to the case of a tension tie block of length $l_{max} = 2l_t$, the following boundary conditions at the two ends of the member (being N the applied load) can be written:

$$\sigma_s(0) = \frac{N}{A_s}\ ;\ \sigma_c(0) = 0\ ;\ \sigma_s(2l_t) = \frac{N}{A_s}\ . \tag{4}$$

Besides, the incipient cracking condition requires $\sigma_c(l_t) = f_{ct}$ at the middle section of the block, where $s = 0$ occurs. From a mathematical point of view, the problem represents a particular boundary value problem, whose solution requires the determination of the unknown length l_t. This length is here evaluated with a trial and error procedure based on bisection, secant and inverse quadratic interpolation methods.

The second limit configuration is determined assuming that the previous block cracks just at midspan, so forming two separate blocks of length $l_{min} = l_t$. In this case, the boundary conditions at the two ends of each member are still the same of Equations (4), even if they are now referred to the interval $[0, l_t]$. In the middle section of the new block $s = 0$ occurs and the stress $\sigma_c(l_t/2)$ is unknown, but l_t is now defined. The problem can be then solved through the same procedure previously described. Figure 1a qualitatively shows the two limit configurations considered in the range model, in terms of stress in the reinforcing bar and in concrete, as well as in terms of slip and bond stress.

As can be observed, the solution is symmetrical on each considered block length and the crack width w is evaluated as twice the slip occurring in correspondence of block ends ($w = 2s$).

3. Comparison between numerical predictions, experimental results and Code provisions

The above described numerical model has been applied to simulate two experimental programs carried out on RC tension members, whose results are available in technical literature (Wu and Gilbert (2008), Gijsbers and Hehemann (1977)). Their choice has been related to the availability of several experimental data monitored during test execution, such as the development of crack pattern and the measurement of the corresponding maximum and minimum crack widths, as well as specimen elongation.

3.1. Description of experimental tests

The attention has been initially focused on two RC ties tested by Wu and Gilbert (2008). These specimens, respectively named STN12 and STN16, were 1100 mm long, had a square cross-section with 100 mm side and were reinforced with a central steel bar (with 12 mm or 16 mm diameter). In addition to these, another RC tie tested by Gijsbers and Hehemann (1977) and denoted as GH12 in the following, has been also numerically analyzed. The selected specimen, 600 mm long, was characterized by a square cross-section with 72 mm side and a rebar diameter equal to 12 mm, so having a reinforcement ratio ρ similar to that of STN16 sample. Table 1 summarizes the experimental mechanical properties of concrete and steel. All the tests were performed under displacement control, by applying a monotonically increasing axial elongation at the ends of the steel bar.

Table 1. Concrete and steel mechanical properties.

Sample	Concrete			Steel	
	f_c [MPa]	f_{ct} [MPa]	E_c [MPa]	f_{ys} [MPa]	E_s [MPa]
STN12-STN16	21.56	2.00	22400	540	200000
GH12	29.60	2.15	28000	430	205600

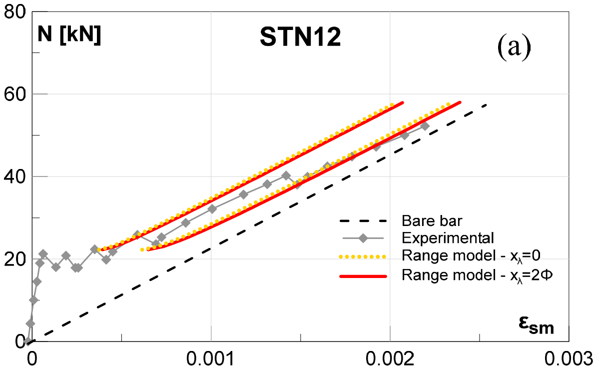
As already mentioned, bond behavior between concrete and steel has been taken into account by adopting the bond-slip law suggested in MC2010, where the parameters corresponding to pull-out failure and good bond conditions have been considered. Since no information was available about the bar deformation properties, parameter s_3, representing the clear distance between the ribs, has been here assumed equal to 10 mm, as suggested in Harajli and Mabsout (2002).

3.2. Numerical results vs experimental data and Code provisions

In this Section, the available experimental results are compared with numerical predictions. Numerical analyses have been repeated twice, by considering or not the bond deterioration near crack surfaces. The curves labeled "*Range model – $x_\lambda = 0$*" (plotted with a dotted line) in the graphs of Figures 3 and 4, refer to the numerical prediction concerning the maximum and minimum crack spacing configurations, without considering the presence of a bond deterioration zone. On the contrary, the curves labeled "*Range model – $x_\lambda = 2\phi$*" (plotted with a continuous line) delimit the range of possible crack configurations when a damage length equal to 2ϕ is assumed.

The results obtained from the "range model" have been first compared to the experimental data in terms of applied axial load N vs. average steel strain ε_{sm}, so as to verify the ability of the proposed procedure to correctly catch the global behavior of the analyzed tension members (Fig. 3). Since this paper mainly focuses on crack width prediction, for brevity only the results obtained for specimens STN12 and GH12 in terms of member deformability are reported in Figure 3. Similar results are found also for STN16 member.

As can be seen, the influence of transverse cracking does not appear to be significant in terms of deformability, since the two obtained ranges (with $x_\lambda = 0$ or $x_\lambda = 2\phi$) are almost superimposed. It can be observed that numerical curves bound the experimental results also when the tension stiffening contribution is remarkable (i.e. STN12 sample).



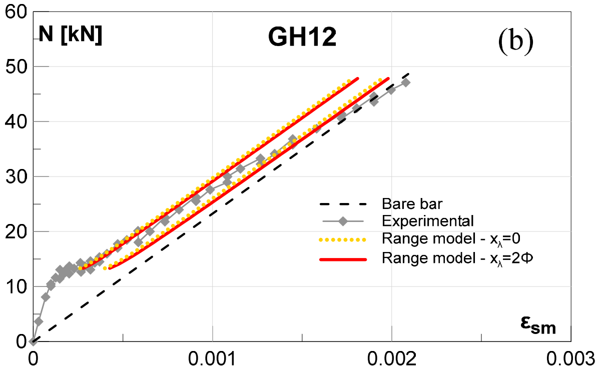


Fig. 3. Comparison between numerical and experimental results for specimens (a) STN12, tested by Wu and Gilbert (2008), and (b) GH12, tested by Gijsbers and Hehemann (1977) in terms of applied load N vs. average steel strain ε_{sm}.

Relatively to cracking behavior, Figure 4 shows a comparison between numerical and experimental results in terms of applied axial load N vs. crack width w (at the concrete surface) and vs. crack spacing s_r, for all the three examined RC ties (Wu and Gilbert (2008), Gijsbers and Hehemann (1977)). As can be observed, the reduction of bond near transverse cracks exerts a valuable influence on the evolution of crack width w with applied loads (Fig. 4a, c, e), allowing a better fit of the experimental results, especially in terms of maximum crack width w_{max}. However, the same Figures 4a, c, e, highlight that experimental crack widths are characterized by a large scatter and go outside the lower bound of the crack width range (corresponding to minimum crack width and spacing).

This can be explained by remembering that the range model represents a simplified approach, which does not consider cohesive stresses across crack surfaces, nor the loading history. As a consequence, the presence of possible unloading, as well as crack closure and/or reopening or the simultaneous formation of more than one crack at the same load level, which can take place during an experimental test, are not taken into account.

Moreover, because of concrete plain strain hypothesis, the effect of shear lag strain in the cover is obviously not considered. Although its contribution in presence of slip at bar-concrete interface seems to be scarce (Pérez Caldentey et al. (2013), Bernardi et al. (2014)), its inclusion in the model could have more relevance when considering minimum crack widths. For these reasons, the lower bound of experimental crack widths could be better represented by a "perfect bond" curve, which does not account for any type of internal failure and maximizes the shear lag strain effect in concrete.

Consequently, in the graphs of Figures 4a, c, e, another curve labeled "*Perfect bond model*" (double dash-dot line) has been added. The latter has been obtained by performing a two-dimensional linear elastic finite element analysis and assuming no slip between concrete and reinforcement. The reinforcement has been schematized through 1D elements, while 4-node plane stress membrane elements have been applied for concrete. These analyses have been performed by considering different lengths of the tension block $2l_t$, and increasing the applied axial load N until the attainment of the cracking condition in the middle section, that is when the tensile stress in concrete reaches the material tensile strength f_{ct}. The obtained results highlight that experimental crack widths are included between a lower bound represented by the perfect bond model (as observed also in Forth and Beeby (2014)) and an upper bound represented by the proposed bond model, when referring to the maximum crack spacing configuration.

The evolution of crack spacing s_r with increasing load is reported in Figures 4b, d, f, for the same RC samples. As can be expected, the perfect bond model provides an incorrect evaluation of crack spacing, which is significantly underestimated; on the contrary, the stepwise trend of experimental data falls within both the ranges obtained with the proposed range model (by considering or not damage). As can be observed, the inclusion of bond deterioration in numerical analyses is less important in terms of crack spacing than in terms of crack width.

On the same graphs of Figure 4, design Code provisions have been also plotted. Both the relations suggested by MC2010 and ACI224 have been analyzed. According to MC2010, $w_{max}= 2l_{s,max}(\varepsilon_{sm} - \varepsilon_{cm})$ where $l_{s,max}$ is the length over which the slip between steel and concrete occurs, while ε_{sm} and ε_{cm} are the average strains in steel and concrete over the length $l_{s,max}$, respectively. The maximum crack spacing $s_{r,max}$ plotted in Fig. 4 has then been deduced as

$s_{r,max}$=2$l_{s,max}$. It can be observed that MC2010 provisions tend to overestimate the maximum experimental crack spacing and, in turn, the maximum crack width. This is probably due to the fact that the transmission length $l_{s,max}$ actually does not depend linearly from the ratio $\phi_s/\rho_{s,eff}$ (see e.g. Beeby et al. (2005)), whereas the MC2010 expressions does - $l_{s,max} = kc + (f_{ctm}\phi_s)/(4\tau_{bm}\rho_{s,eff})$, where c is the concrete cover and τ_{bm} the mean bond strength between steel and concrete.

As regards ACI224, the relation $w_{max}= s_{r,max}\varepsilon_s$ is provided, where the maximum crack spacing $s_{r,max}$ is assumed equal to 4 times the concrete cover d_c (intended as the distance from the center of the bar to the point of the surface where the crack width is considered, so being equal to $c + \phi_s/2$). ACI224 provision is almost superimposed with the upper bound curve provided by the proposed range model in presence of bond deterioration, even if the two approaches are based on completely different hypotheses (effect of concrete cover vs. bond).

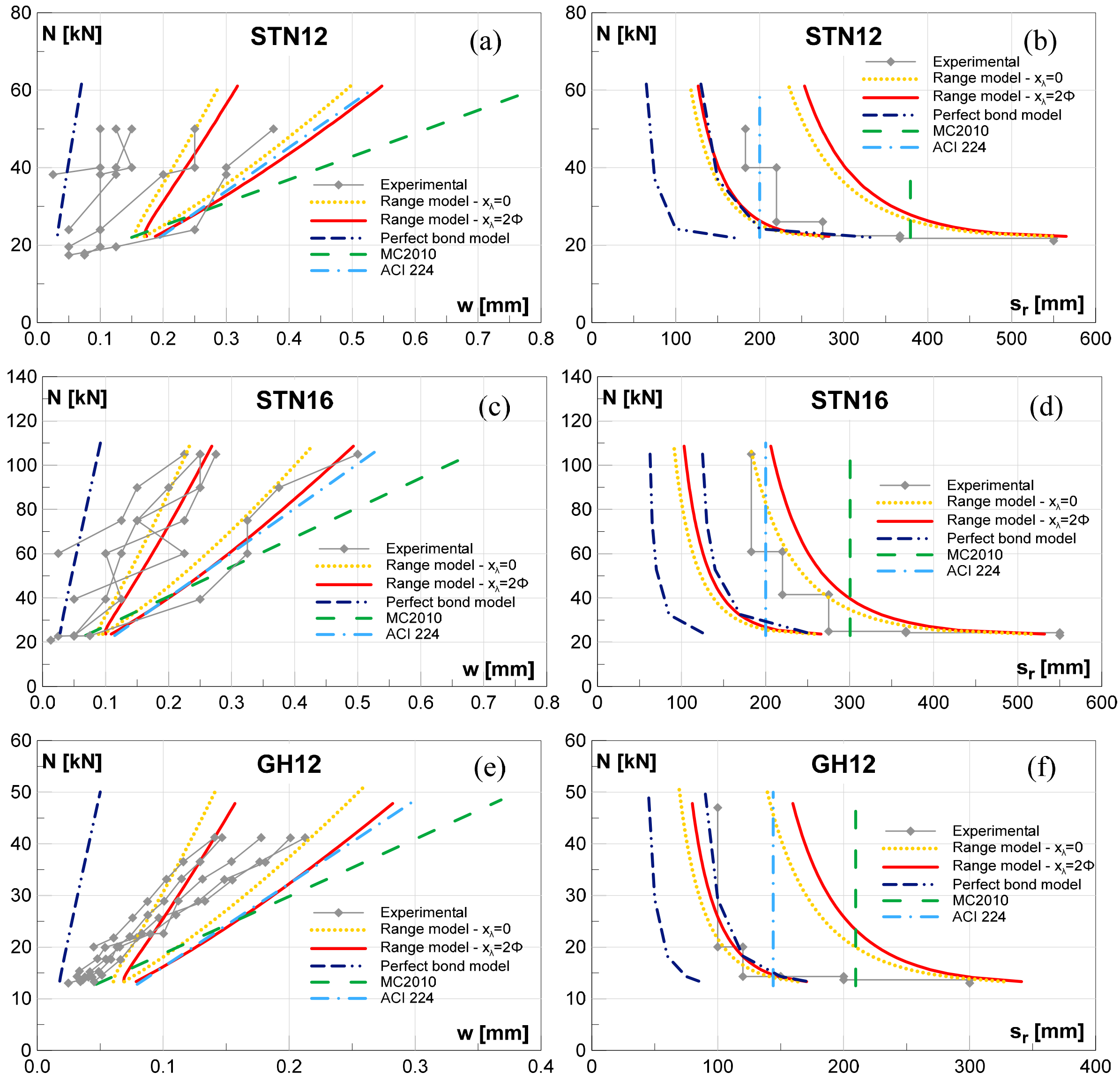


Fig. 4. Comparison between numerical and experimental (Wu and Gilbert (2008), Gijsbers and Hehemann (1977)) results for RC ties (a), (b) STN12; (c), (d) STN16; (e), (f) GH12, in terms of applied load N vs. crack width w and vs. crack spacing s_r. On the same graphs, the ACI224 and MC2010 provisions are also reported.

A good correlation with experimental results can be also observed, probably due to the empirical nature of ACI224 crack width expression. As a matter of fact, even if this formula is substantially based on the assumption that there is no significant slip and that the crack spacing is strictly related to concrete cover dimension, its expression has been properly adjusted on the basis of experimental results on tension members and at the end it is able to consider the most important factors, Broms (1965).

4. Conclusions

In this paper, a numerical model based on bond has been applied to the analysis of RC tension members. The approach has been validated against significant experimental results available in technical literature and compared with well-known Code provisions (MC2010 and ACI 224).

The study confirms that cracking behavior of RC ties can be correctly predicted through the classical bond theory, which can be still successfully applied for serviceability verifications. Moreover, numerical predictions of maximum crack width can be further refined by considering the presence of a bond deterioration zone near transverse cracks, due to splitting and crushing of concrete around the bar beside the crack surface. The empirical relation suggested in ACI 224 is also shown to provide accurate predictions of maximum crack width, whereas MC2010 formula seems to overestimate it, probably since bond contribution is included in a too simplified way, through the ratio $\phi_s / \rho_{s,eff}$.

References

ACI224.2R-92, 1992. Cracking of concrete members in direct tension. ACI Committee 224. Detroit (USA), American Concrete Institute.

Avalle, M., Ferretti, D., Iori, I., Vallini, P., 1994. On the deformability of reinforced concrete members in tension and bending. In: Mang H. et al (Eds) Proceedings of computational modelling of concrete structures (EURO-C 1994), Pineridge Press, Innsbruck, Austria, 723-734.

Beeby, A.W., 2004. The influence of the parameter ϕ / ρ_{eff} on crack widths. Structural Concrete 5, 71-83.

Beeby, A.W., et al., 2005. Discussion: The influence of the parameter ϕ/ρ_{eff} on crack widths. Structural Concrete 6, 155-165.

Bernardi, P., Cerioni, R., Ferretti, D., Michelini, E., 2014. Role of multiaxial state of stress on cracking of RC ties. Engineering Fracture Mechanics 123, 21-33.

Borosnyói, A., Balázs, G.L., 2005. Models for flexural cracking in concrete: the state of the art. Structural Concrete 6, 53-62.

Broms, B.B, Lutz, L.A., 1965. Effects of arrangement of reinforcement on crack width and spacing of reinforced concrete members. Journal of American Concrete Institute 62(11), 1395-1410.

Broms, B.B., 1965. Crack width and spacing in reinforced concrete members. Journal of American Concrete Institute 62, 1237-1256.

Bruggeling, A.S.G., 1991. Structural concrete: theory and its applications. Balkema, Rotterdam, pp.504.

CEB-FIP Bulletin No.10, 2000. Bond in reinforced concrete. Lausanne: International Federation for Structural Concrete.

CEB-FIP Bulletin No.65, 2012. Model Code 2010. Final draft – vol. 1. Lausanne: International Federation for Structural Concrete.

Chiaia, B., Fantilli, A.P., Vallini, P,. 2009. Evaluation of crack width in FRC structures and application to tunnel linings. Materials and Structures 42, 339-351.

Fantilli, A.P., Ferretti, D., Iori I., Vallini P., 1998. Flexural deformability of reinforced concrete beams. Journal of Structural Engineering 124(9), 1041-1049.

Ferry-Borges J., 1966. Cracking and deformability of reinforced concrete beams. IABSE Publication, Zürich, 26, 75-95.

Forth, J.P., Beeby, A.W., 2014. Study of composite behavior of reinforcement and concrete in tension. ACI Structural Journal 111(2), 397-406.

Gijsbers, F.B.J., Hehemann, A.A., 1977. Enige trekproven op gewapend beton (Some tensile tests on reinforced concrete). Report BI-77-61, IBBC Institute TNO for Building Materials and Building Structures, Delft, The Netherlands.

Harajli, M.H., Mabsout, M.E., 2002. Evaluation of bond strength of steel reinforcing bars in plain and fiber-reinforced concrete. ACI Structural Journal 99, 509-517.

Pérez Caldentey, A., Corres Peiretti, H., Peset Iribarren, J., Giraldo Soto, A, 2013. Cracking of RC members revisited: influence of cover, $\phi/\rho_{s,ef}$ and stirrup spacing – an experimental and theoretical study. Structural Concrete 14(1), 69-78.

Rizkalla, S.H., Hwang, L.S., 1984. Crack prediction for members in uniaxial tension. Journal of American Concrete Institute 81, 572-579.

Shampine, L.F., Gladwell, I., Thompson, S., 2003. Solving ODEs with MATLAB, Cambridge University Press.

Somayaji, S., Shah, S.P., 1981. Bond stress versus slip relationship and cracking response of tension members. Journal of American Concrete Institute 78(3), 217-225.

Watstein, D, Sees Jr., N.A., 1945. Effect of type of bar on width of cracks in reinforced concrete subjected to tension. Journal of American Concrete Institute 41, 293-304.

Wu, H.Q., Gilbert, R.I., 2008, An experimental study of tension stiffening in reinforced concrete members under short-term and long-term loads. UNICIV Report No. R-449, The University of New South Wales, Sidney, Australia.

Ziari, A., Kianoush, M.R., 2009. Investigation of direct tension cracking and leakage in RC elements. Engineering Structures 31(2), 466-474.