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A decoupled numerical procedure for modelling soil interaction in the computation of the dynamic response of a rail track

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Abstract

The problem of vibration transmitted by train traffic to the soil in the case of railway lines in urban areas is gaining increasing attention in environmental impact analysis. An efficient method to consider both the train-track interaction in detail and the vibration transmitted to the soil nearby with an affordable computational cost is desirable. The paper proposes a numerical procedure based on a substructuring approach, in which the system is divided into three main subdomains: the train running on the track, the rail subjected to the loads coming from the train and the reactions from the sleepers and the "ground" sub-system, composed by the sleepers, the ballast with its subgrade and the actual ground. The overall procedure is divided into subsequent steps: first, the finite element modelling of the sleeper-ballast-subgrade combined system, characterized within the linear elastic field by means of frequency response functions at rail-sleeper interfaces. In a second step, moving loads transmitted to the track are computed by numerical time domain integration of the equations of motion of the train running on a model of the track only, in which the subgrade is modelled as a series of spring-damper elements, whose parameters are tuned according to the results of the FE model used in the first step and therefore consistent with it. Non-linear behavior of the rail-wheel interaction can be accounted for by the time-domain procedure. The track dynamics is finally computed via direct frequency domain analysis; the track is again modelled by Finite Elements, loaded by the forces transmitted by the train wheels and by the supporting sleepers. Finally, the vibrations propagated through the soil to a general receiver point are evaluated. The procedure can exploit favorable properties of frequency domain analysis in treating moving loads; in addition, frequency dependent properties of materials can be introduced.

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Keywords: Moving loads; track-train dynamics; track-soil interaction.

1. Introduction

The problem of vibration transmitted to the soil by train traffic has always been a topic of interest, especially when the presence of railway lines in densely populated areas is considered [1]. An efficient method for simulating in detail both the train-track interaction and the disturbance transmitted to the soil nearby with an affordable computational cost is desirable when dealing with vibrational impact analysis. An established literature exists on the topic of ground borne noise originated from train transit (see [2], [3]): aim of this paper is to add a contribution in this field proposing a numerical procedure based on a substructuring approach and a one-way decoupling scheme. The main purpose is to retain the essential features of the rail-vehicle dynamic characteristics, the

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track dynamic behavior and the soil propagation effect at a reasonable computational cost. A single/coupled Time Domain (TD) model considering all these aspects would be very heavy and inefficient from the computational point of view, also considering that short time steps would be necessary for train-track interaction. In addition, a substructuring approach can "confine" non-linearity in the train-track interaction analysis, while the rail and its supporting system can be analyzed in a linearized context.

In this light, a combination of TD and Frequency Domain (FD) sub-processes is used. The considered subsystems are: the train running on the track, the rail subjected to the loads coming from the train and the sleeper-ballast-subgrade-soil system supporting the rail through the fastenings located on the sleepers.

The procedure is divided into subsequent steps, as listed in the following (see Fig. 1).

- A finite element modelling of the sleeper-ballast-subgrade-soil combined system (subsequently referred to as "ground") is set up, characterized within the linear elastic field, by means of Frequency Response Functions (FRF) obtained by applying unit harmonic loads at the rail-sleeper interfaces. The FRFs are also used to tune a simplified model for the track to be used in the second step. The ground sub-system also comprises external "receivers", where the vibration level must be estimated.
- In the second step, the moving loads transmitted by train's wheels to the track are computed by numerical TD integration of the equations of motion of the train running on a simplified model of the track, in which the subgrade is modelled as a series of linear spring-damper elements, whose parameters are tuned according to the results of the Finite Element (FE) model used in the first step. Non-linear behavior of the track-wheel interaction can be accounted for by the TD procedure. The outputs are the moving loads corresponding the wheel-rail contact forces to be used in the third step.
- The rail dynamics is finally computed via direct FD analysis, exploiting its favorable properties in both modelling and response computation; the rail is modelled by FEs, loaded on one side by the forces transmitted by train's wheels and coupled on the other side, through the fastenings of the sleepers placed over the sleeper-ballast-subgrade-ground subsystem characterized by the above mentioned FRFs (step 1).
- Once the motion of the track is computed, it is possible to evaluate the vibrations propagated through the soil to a general receiver point.

It must be observed that a "one-way", i.e. with no interaction, decoupling scheme as the one here adopted can be justified within the context of a weak anticipated interaction between the train dynamics and the ground compliance. In different situations, typically for train-bridge interaction, a procedure of this type should be complemented by a suitable iteration scheme.

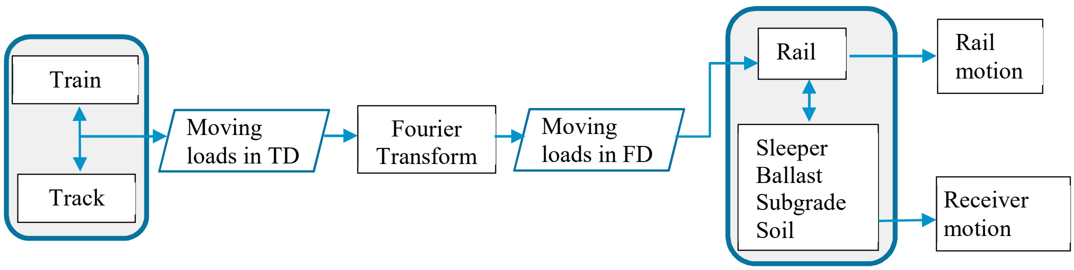

Fig. 1. Overall flow diagram of the procedure.

2. Problem formulation

In light of the above sketched one-way decoupling procedure, the dynamic behaviour of the railway track can be studied by solving the equations of motion of a beam subject to the forces transmitted by the wheels and supported by an elastic subsystem (sleepers-ballast-subgrade-soil). If the problem is formulated in the frequency domain, the support behaviour can be described by mechanical impedance functions. The latter can be obtained by different numerical procedures, such as finite element or boundary element analysis.

Assuming that both wheel-track forces and support impedance functions are available, the equations of dynamic equilibrium of the track can be written, in the frequency domain, in the usual matrix form:

$$\left[\mathbf{E}^s(f)+\mathbf{E}^g(f)\right]\tilde{\mathbf{q}}(f)=\tilde{\mathbf{Q}}(f) \tag{1}$$

where $\tilde{\mathbf{q}}(f)$ is the Fourier transform of the configuration vector; $\mathbf{E}^s(f)$ and $\mathbf{E}^g(f)$ are respectively the mechanical impedance matrices of the rail and of the supporting system. The vector $\tilde{\mathbf{Q}}(f)$ lists the Fourier Transforms of the generalized moving loads transmitted to the track due to the passage of the train.

The vector $\mathbf{q}$ can be first partitioned by expliciting the Lagrangian coordinates related to the motion of rail structure (subscript "s") and supporting "ground" (subscript "g") nodal displacements. The latter comprises "contact" nodes, where ground impedances are ideally connected to the rail, and nodes acting as external "receivers", located within the model of the surroundings of the track. Consistently, the vector, the impedance matrices and the load vector are partitioned as

$$\mathbf{q}=\begin{Bmatrix}\mathbf{q}_s\\ \mathbf{q}_g\end{Bmatrix}\quad;\quad \mathbf{E}^s(f)=\begin{bmatrix}\mathbf{E}^s_{ss} & \mathbf{E}^s_{sg}\\ \mathbf{E}^s_{gs} & \mathbf{E}^s_{gg}\end{bmatrix}\quad;\quad \mathbf{E}^g(f)=\begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{E}^g_{gg}\end{bmatrix}\quad;\quad \mathbf{Q}(f)=\begin{Bmatrix}\mathbf{Q}_s\\ \mathbf{Q}_g\end{Bmatrix} \tag{2}$$

Replacement of (2) into the system of equations (1) leads to the partitioned form:

$$\begin{aligned}&\mathbf{E}^s_{ss}\tilde{\mathbf{q}}_s+\mathbf{E}^s_{sg}\tilde{\mathbf{q}}_g=\tilde{\mathbf{Q}}_s\\ &\mathbf{E}^s_{gs}\tilde{\mathbf{q}}_s+\left[\mathbf{E}^s_{gg}+\mathbf{E}^g_{gg}\right]\tilde{\mathbf{q}}_g=\tilde{\mathbf{Q}}_g\end{aligned} \tag{3a,b}$$

To simplify the treatment of the support system, equations (3b) can be modified by pre-multiplying all terms by its frequency response function (FRF) matrix, this being the inverse of the impedance, delivering:

$$\mathbf{H}^g_{gg}\mathbf{E}^s_{gs}\tilde{\mathbf{q}}_s+\left[\mathbf{H}^g_{gg}\mathbf{E}^s_{gg}+\mathbf{I}\right]\tilde{\mathbf{q}}_g=\mathbf{H}^g_{gg}\tilde{\mathbf{Q}}_g \tag{4}$$

The ground motion vector can be further partitioned between "contact" (subscript "c") and "external receiver" (subscript "r") nodes, leading to:

$$\mathbf{q}_g=\begin{Bmatrix}\mathbf{q}_c\\ \mathbf{q}_r\end{Bmatrix}\quad;\quad \mathbf{H}^g_{gg}(f)=\begin{bmatrix}\mathbf{H}^g_{cc} & \mathbf{H}^g_{cr}\\ \mathbf{H}^g_{rc} & \mathbf{H}^g_{rr}\end{bmatrix}\quad;\quad \mathbf{E}^s_{gg}(f)=\begin{bmatrix}\mathbf{E}^s_{cc} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\quad;\quad \mathbf{Q}_g(f)=\begin{Bmatrix}\mathbf{Q}_c\\ \mathbf{0}\end{Bmatrix} \tag{5}$$

Replacing equations (5) into (4) leads, after some manipulation, to the following form of the (3b) sub-system, involving only contact ground nodes:

$$\mathbf{H}^g_{cc}\mathbf{E}^s_{cs}\tilde{\mathbf{q}}_s+\left[\mathbf{H}^g_{cc}\mathbf{E}^s_{cc}+\mathbf{I}_{cc}\right]\tilde{\mathbf{q}}_c=\mathbf{H}^g_{cc}\tilde{\mathbf{Q}}_c \tag{6}$$

The simultaneous solution of (3a) and (6) delivers the rail motion: subsequently, the motion transmitted to the external receivers can be obtained as

$$\tilde{\mathbf{q}}_r=-\mathbf{H}^g_{rc}\mathbf{E}^s_{cs}\tilde{\mathbf{q}}_s-\mathbf{H}^g_{rc}\mathbf{E}^s_{cc}\tilde{\mathbf{q}}_c+\mathbf{H}^g_{rc}\tilde{\mathbf{Q}}_c \tag{7}$$

Time domain response can be finally obtained by Inverse Fourier Transform of vectors computed from equations (3a) and (6) for the rail and from equations (7) for the external receivers.

Looking at the load vector, this is related to moving loads showing vertical, horizontal and shearing components at the interface between the track and each wheel. The generalized load transmitted to the track can be obtained upon superposition of the effects of the forces transmitted by each wheel. By making reference to the k-th degree of freedom of the rail system, the corresponding load can thus be expressed as:

$$Q_k(t)=\sum_{j=1}^{n}Q_k^j(t)=\sum_{j=1}^{n}P^j(t)\psi_k(t) \tag{8}$$

where n denotes the number of axles of the train, $P^j(t)$ is the time history of the force exerted by the j-th wheel of the train, while the function $\psi_k(t)$ transforms each moving load in its generalized components with respect to the k-th degree of freedom of the system. The latter function is dependent on the train velocity and on the position of the loaded node along the track. Switching to the frequency domain, a product between time functions delivers a convolution between their Fourier Transform, i.e.:

$$\tilde{Q}_k(f)=\sum_{j=1}^{n}\tilde{Q}_k^j(f)=\sum_{j=1}^{n}\int_{-\infty}^{+\infty}\tilde{P}^j(\alpha)\tilde{\psi}_k(f-\alpha)\,\mathrm{d}\alpha \tag{9}$$

Regarding the functions $\psi_k(t)$, a formulation of the aforementioned function is proposed on the basis of the fixed-end reactions of a beam finite element subjected to a concentrated force.

2.1. Dynamic characterization of the superstructure+subgrade+soil subsystem

A 3D model of a portion of the sleeper-ballast-subgrade combined system was developed by means of the Abaqus software using eight-node quadrilateral solid elements with a reduced integration scheme. Direct steady-state solution is performed in order to calculate the linearized response of the system to harmonic excitation. The geometry of the model is depicted in Fig. 2.

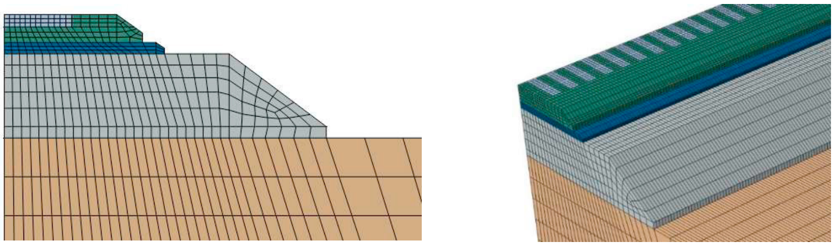

Fig. 2. Geometry of the FE model of the sleeper-ballast-subgrade-soil model.

The combined system is characterized by a series of overlaid layers with different linear elastic materials. The mechanical features of each component of the combined system are summarized in Table 1, where γ identifies the density, E is the Young Modulus, ν and μ are the Poisson ratio and the structural hysteretic damping, respectively.

Table 1 .Mechanical properties of stratigraphic components of the system.

	γ (*kN/m³*)	E (*MPa*)	ν (-)	μ (-)
Sleeper	25	40000	0.2	0.06
Ballast	16	310	0.2	0.2
Subballast	16	130	0.2	0.1
Subgrade	16	100	0.3	0.1
Soil	20	80	0.3	0.1

The FRFs of the system are directly evaluated through the application of a harmonic load of unit amplitude over the area of contact between the railway track and the sleeper located in a central position with respect to the size of the geometrical model. The standard viscous boundary [4] has been implemented to avoid reflections at the ground mesh boundaries.

2.2. Train modelling and response

The mathematical model of the train-track interaction used in the simulations was developed at Department of Mechanical Engineering of Politecnico di Milano [5] and is based on a multi-body schematization of the train vehicles and a FE schematization of the track, with the two subsystems coupled by the contact forces at the wheel-rail interface.

The multi-body model of the train assumes car-bodies and bogie frames as rigid elements whereas the flexibility of the wheelsets is taken into account by means of a modal superposition approach. The bodies are connected by primary and secondary suspensions represented by means of both linear visco-elastic elements and non-linear elements (e.g. the ones representing the bumpstops). The motion of each rigid body is defined by 5 degrees of freedom, being the forward speed of the train assigned to a constant value V. The equation of motion of the trainset assumes the form:

$$\mathbf{M}_v\ddot{\mathbf{x}}_v + \mathbf{C}_v\dot{\mathbf{x}}_v + \mathbf{K}_v\mathbf{x}_v = \mathbf{F}_{nl}\left(\mathbf{x}_v, \dot{\mathbf{x}}_v\right) + \mathbf{F}_{cv}\left(\mathbf{x}_v, \dot{\mathbf{x}}_v, \mathbf{x}_t, \dot{\mathbf{x}}_t, V, t\right) \quad (10)$$

where $\mathbf{M}_v$, $\mathbf{C}_v$ and $\mathbf{K}_v$ are respectively the mass, damping and stiffness matrices of the trainset, $\mathbf{F}_{nl}$ is the vector of the forces due to the non-linear elements in the suspension and $\mathbf{F}_{cv}$ is the vector of the generalized forces due to wheel-rail contact. The latter depend on time on the motion (displacement and velocity) of both the trainset and the track. The model of the track is based on a FE schematization in which rail elements are represented by Euler-Bernoulli beams, rail-pads are introduced as lumped visco-elastic elements while sleepers and ballast are represented by lumped mass elements connected to the ground by means of a distributed visco-elastic layer. The equation of motion of the track subsystem is given by

$$\mathbf{M}_t\ddot{\mathbf{x}}_t + \mathbf{C}_t\dot{\mathbf{x}}_t + \mathbf{K}_t\mathbf{x}_t = \mathbf{F}_{ct}\left(\mathbf{x}_v, \dot{\mathbf{x}}_v, \mathbf{x}_t, \dot{\mathbf{x}}_t, V, t\right) \quad (11)$$

where $\mathbf{M}_t$, $\mathbf{C}_t$ and $\mathbf{K}_t$ are respectively the mass, damping and stiffness matrices of the track, and $\mathbf{F}_{ct}$ is the vector of the generalized components of the forces acting on the track due to wheel rail contact forces.

The model of wheel-rail contact used to reproduce the dynamic coupling between the vehicle and the track is multi-Hertzian, enabling either to include the occurrence of multiple contacts or to approximate complex non-Hertzian contact patches by one or more elliptic patches, as it happens in case of highly worn wheel and rail profile. Based on wheel and rail displacements and velocities at each contact point obtained from the train and track coordinates, the value of the contact forces is derived. The contact forces are then transformed to the reference in which train and track coordinates are defined and vectors of generalized forces $\mathbf{F}_{cv}$ and $\mathbf{F}_{ct}$ are derived. On account of the non-linearities involved in the problem, the two set of equations are simultaneously integrated using a modified Newmark time-step procedure. At each step, convergence on the wheel-rail contact forces is reached iteratively.

3. Example of application

The procedure was applied to a standard configuration of ballasted track, with a fast commuter train, for a total of 12 axes, with mean axle load equal to 145 kN. Speeds from 130 km/h to 170 km/h were considered. Track vertical level irregularity was introduced in the time domain step, considering a minimum wavelength equal to 0.25 m.

An example of the time variation of the wheel- rail contact force is given in Figure 3 along with its Fourier Amplitude Spectrum (FAS). Typical results of the rail dynamic analysis are reported in Figure 4 in terms of vertical rail deflection at wheels' transits.

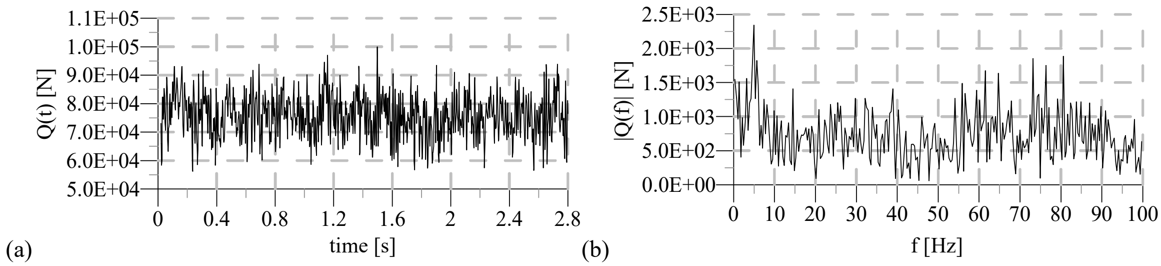

Fig. 3. wheel-rail contact force (vertical component) for v=130 km/h [Q_0=77.2 kN]: (a) time-history, (b) FAS.

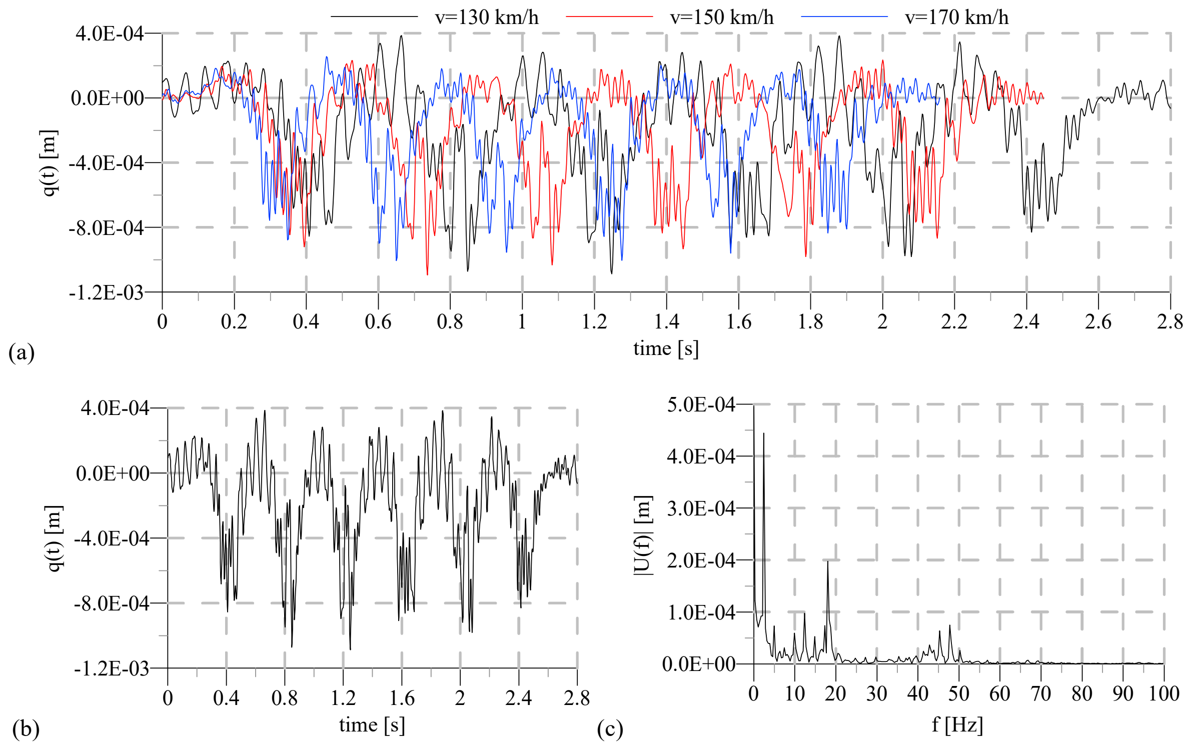

Fig. 4. (a) displacement time histories for the different velocities. (b) (c) time history and Fourier Amplitude Spectrum for v=130 km/h

In Figure 5 the time histories of vertical acceleration and the corresponding Fourier Amplitude Spectra are reported, both on the rail at a typical sleeper and on the ground, at a distance of 5.5 m from the track. The latter response can be regarded as the "free-field" input for a hypothetical structure having a surface foundation at that location.

In the following Table 2 the RMS values of vertical acceleration are given for the different values of the train speed.

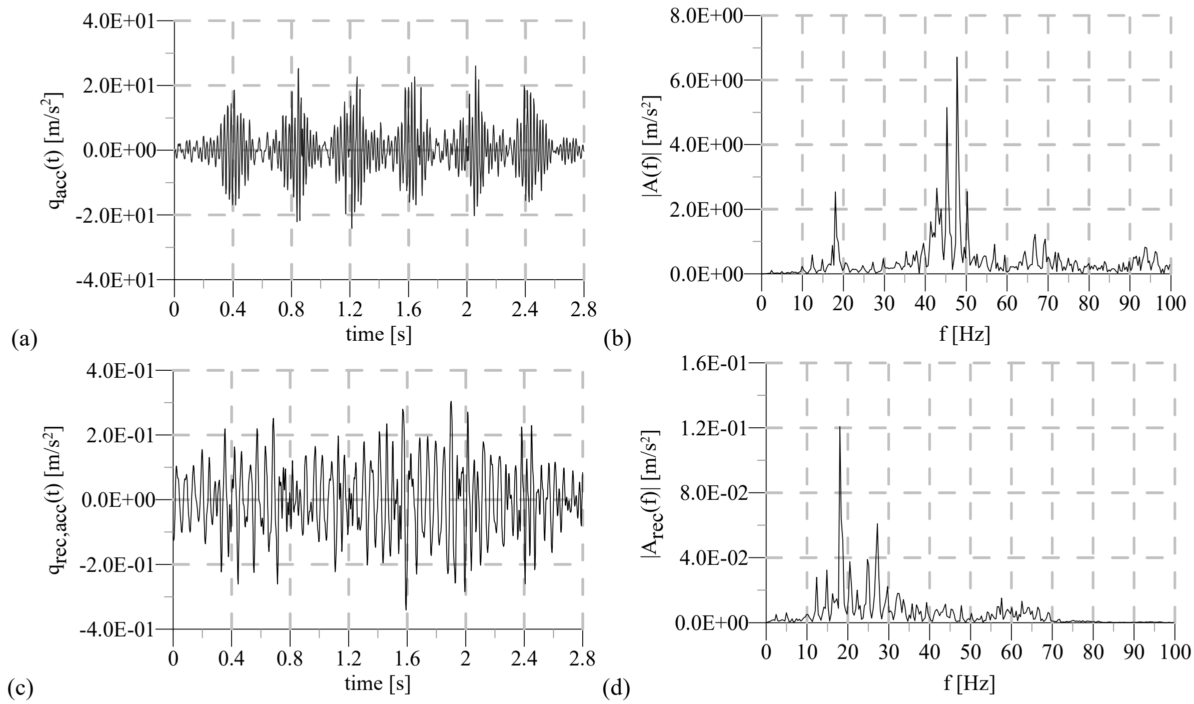

Fig. 5. Acceleration at the rail (a) (b) and in the soil (5.5 m from the track) (c) (d). Time history on the left and FAS on the right.

Table 2. RMS values of acceleration [m/s²] on rail and soil for different train speeds.

	130 km/h	150 km/h	170 km/h
Rail	6.63	8.25	10.06
Soil	0.1040	0.1123	0.1039

4. Conclusions

A numerical procedure able to combine both time domain approach for train-track interaction and frequency domain approach for superstructure and soil analysis was presented. The aim was to develop a detailed model of the train and the track and also to retain the propagating effect of the soil, with the possibility to include inhomogeneity in the soil characteristic.

An example of the influence of train speed considering a fast commuter train was presented as first results. Further use of the procedure can cover topics as types of track system and its characteristics, insertion of additional resilient elements, stats of the maintenance of the track and of the train and soil parameters.

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