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## Thermo-Hydrodynamic Analysis of Plain and Tilting Pad Bearings

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### Abstract

The demand for higher efficiency and increased equipment compactness is pushing industrial compressors' designers towards the choice of higher rotor peripheral speed. As a consequence, modern bearing-rotor systems are subject to complex thermal phenomena inducing a renewed interest on their real working conditions. This work is about the validation of the in-house numerical code TILTPAD developed at the Department of Industrial Engineering of the University of Florence for the thermo-hydrodynamic analysis of both plain and tilting pad journal bearings performance. TILTPAD is a steady-state code based on a 2D thin-film approach able to find either the resulting hydrodynamic load using the shaft equilibrium position and the rotational speed (i.e., direct problem) or the shaft equilibrium position once the load and the rotational speed are prescribed (i.e., inverse problem). In order to calculate pads' pressure distribution a finite element approach is used to solve the Reynolds equation together with a mixed procedure to evaluate pads equilibrium positions. Two steady-state energy equations based on a Petroff-type simplification are implemented in the code. The first one is proposed in the work of Balbahadur and Kirk [1] while the second one is based on an improved mixing model and a temperature dependent viscosity. An iterative procedure is used between Reynolds and energy equations to account for the dependence of the dynamic viscosity on the temperature field. Super-laminar flow regimes are also modeled in the code by means of a simplified approach able to represent, with reasonable accuracy, the effects of Taylor-Couette vortex flows and of the transitional regimes up to the onset of a fully turbulent state. Under these hypotheses, the pressure field is slightly affected by the viscosity variation while dissipative effects are enhanced. The code has been validated by means of comparison with available experimental data. Particular attention is devoted to static working parameters (i.e., equilibrium position and frictional power loss), reproducing the global behavior of the bearing, although some local characteristic is also considered.

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## Nomenclature

B	Axial length [m]
C	Radial clearance [m]
D	Diameter [m]
h	Film thickness [m]
m	Preload ( $C_p/C_b - 1$ ) [-]
p	Pressure [Pa]
R	Radius [m]
Re	Reynolds number evaluated with the assembled clearance [-]
T	Temperature [°C]
x, y, z	Local coordinates [m]
X, Y	Global coordinates [m]

## Subscripts

b	Bearing (“assembled” if referred to the clearance)
i	Referred to the i-th element of the grid
j	Journal
0	Feeding oil
p	Pad (“pad machined” if referred to the clearance)
t	Turbulent regime (superlaminar regimes)

## Greek

$\alpha, \alpha', \alpha''$	Local angular coordinates (related to pad’s pivot position) [deg]
$\gamma$	Attitude angle [deg]
$\delta$	Angular coordinate [deg]
$\varepsilon$	Eccentricity ratio (with respect to the assembled clearance) [-]
$\vartheta$	Tilt angle [deg]
$\Omega$	Rotational speed [rad/s]

## 1. Introduction

The demand for higher efficiency and increased equipment compactness is pushing industrial compressors' designers towards the choice of higher rotor peripheral speed. Therefore, modern bearing-rotor systems are subject to complex thermal phenomena inducing a renewed interest on their real working conditions. Although Plain and Tilting Pad Journal Bearings (PJB and TPJB) are quite simple component, the physics behind their operating conditions could reach unexpected complexity. In fact, in a realistic operation the fluid is subject to viscosity variations, there could be strong effects of thermal and elastic deformations, and the film could be subject to oil cavitation. Moreover, modern machines could drive these components towards transitional or fully turbulent oil flow conditions.

Temperature calculation is of primary importance in order to account for dynamic viscosity variations. The problem has been soon considered and great effort has been dedicated over the years in order to overcome the isoviscous approach. One of the first proposed formulations is the adiabatic solution proposed in the work of Cope [2] in 1949. Since then, many works have followed and several approaches have been proposed: 1D formulations, as implemented in the present code, quasi-2D and quasi-3D formulations, as respectively described in the work of Lund and Hansen

[3] and in the work of Stefani and Rebori [4], up to fully 3D approaches, as in the work of Taniguchi et al. [5]. This latter also accounts for turbulent flow regimes.

The present work describes the main features of the in-house numerical code TILTPAD, developed at the Department of Industrial Engineering of the University of Florence for the thermo-hydrodynamic analysis of plain and tilting pad journal bearings. After some details of the code are given, results of the validation are reported.

## 2. Numerical code

TILTPAD is an in-house numerical code for the thermo-hydrodynamic analysis of both plain and tilting pad journal bearings' performance. The main hypothesis behind the code are summarized below and represent the classical assumptions to approach the hydrodynamic lubrication problem:  $(h/B) \ll 1$ ,  $(h/R_j) \ll 1$ , incompressible fluid, Newtonian fluid, laminar flow and negligible volume forces.

The obtained code is steady-state and is based on a 2D thin-film approach able to find either the resulting hydrodynamic load using the shaft equilibrium position and the rotational speed (i.e., direct problem) or the shaft equilibrium position once the load to be carried and the rotational speed are prescribed (i.e., inverse problem). More in detail, the code handles only direct problem data types hence a dedicated search procedure for the evaluation of the equilibrium's eccentricity and attitude angle is followed in case of inverse problems. The procedure uses the method of chords when dealing with eccentricity and a bisection method for the attitude angles. The choice of using two different procedures allowed obtaining faster solutions and a higher convergence rate.

### 2.1. Film thickness

Once values of eccentricity and attitude angle are provided, together with the geometrical features of the analyzed bearing, it is possible to calculate the film-thickness distribution along axial and tangential directions (in the present code neither misaligned bearings configurations nor thermal and elastic deformations are considered). Moreover, when dealing with tilting pad bearings, preload ( $m$ ) and pads' tilt angles ( $\vartheta$ ) have also to be provided. These latter, are evaluated by means of a dedicated procedure described below. The final formulation, shown in equation (1), is composed of three terms: the first one is the same in both plain and tilting pad bearings while the second and the third account for the variation of the film-thickness respectively due to pads' tilt angle and preload. In Figure 1a) a sketch of a tilting pad bearing and its local angular reference systems is reported.

$$h(\delta) = C_b (1 - \varepsilon \cos(\delta - \gamma)) - 2R_b \sin(\alpha') \vartheta + C_b m (1 - \sin(\alpha'')) \quad (1)$$

### 2.2. Pressure field calculation

The starting point to obtain the representative equation for the incompressible lubrication problem is the dimensional analysis of the Navier-Stokes equations. From such analysis, inertial terms can be disregarded within the usual geometrical and operating conditions. Moreover, pressure variation across the fluid film results to be negligible and the velocity component in the axial and tangential direction can be directly evaluated by means of integration of the momentum equations with suitable boundary conditions. Then, the continuity equation is integrated across the fluid film with the substitution of the obtained velocity components. The final result is the well-known Reynolds' equation that is widely reported in literature, as for example in the work of Szeri [6]. The present code uses a local reference system fixed with the bush and with the cross-film coordinate that increases moving from the bush towards the shaft. The code handles pressure and oil flow rates as boundary conditions. Moreover, Swift-Stieber condition is adopted in order to easily consider the effects of cavitation on the pressure field.

Usually, finite difference or Finite Element Methods (FEM) are considered for the solution of the incompressible lubrication problem that consists in finding the solution of the Reynolds' equation, with the prescribed boundary conditions. In the present work, the formulation proposed by Reddi [7] is implemented. In his work [7], a variational principle for the incompressible lubrication problem is presented and is proved to be a minimum principle. Hence, the solution of the Reynolds' equation is obtained finding the 2D pressure distribution that minimizes the functional. By

means of FEM discretization, using linear shape functions on the triangular elements of the grid, the problem is reduced to a linear system of equations with nodal pressure values as unknowns. For more details see also the work of Martelli and Manfreda [8], where isoviscous solution of plain bearings is presented. The linear system solution is obtained by means of the iterative Gauss-Seidel method with a relaxation factor adopted for a faster convergence. Once the pressure field is obtained an integration is performed over the domain in order to calculate the resulting load (module and angle).

### 2.3. Pads' equilibrium positions

The pads' equilibrium problem allows for the evaluation of the tilt angles representative of the equilibrium configuration (i.e. when the moment with respect to the pivot is zero and the resulting load is correct). Such a problem can be numerically posed as finding the first zero of the moment, as a function of the tilt angle. A mixed procedure is used for the purpose starting from a converging film condition. In the first part of the procedure (Fig. 1b) top) the code scans with prescribed tilt angle steps the whole available range in order to define an appropriate interval characterized by moment values at the extremes having opposite sign. Once the interval has been defined the code uses the method of chords to solve the root finding problem (Fig. 1b) bottom).

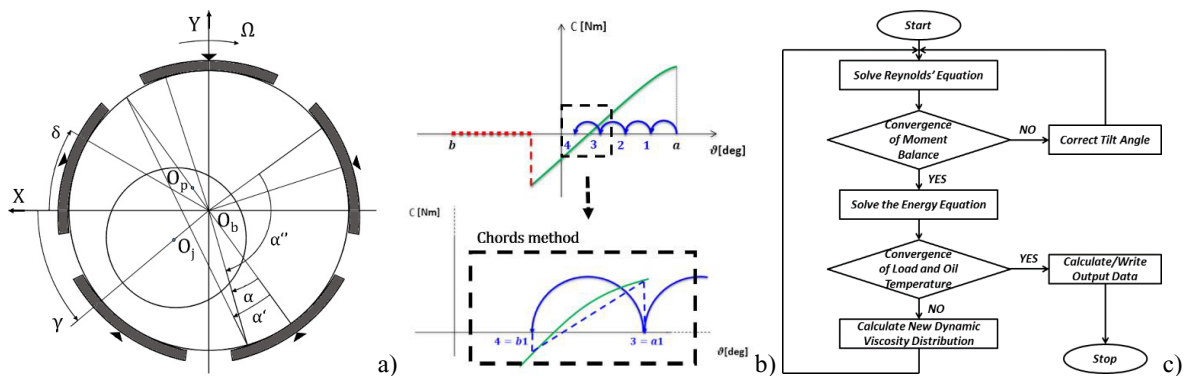


Fig. 1. (a) Sketch of a tilting pad bearing and its local angular reference systems; (b) Representation of the mixed procedure for the evaluation of pads' equilibrium position; (c) Flow chart of the iterative coupling procedure between Reynolds and energy equations.

### 2.4. Energy equations

Two simplified steady-state energy equations have been implemented in the code: the first one has been suggested by Balbahadur and Kirk [1] and is obtained by means of an energy balance executed on a control volume extended across the whole film-thickness, including a thin layer of bush/pad and shaft metal. Since axial temperature variations are neglected, a 1D mean temperature equation is provided. Besides, the equation is based upon a Petroff's type simplification. Under such hypothesis, Poiseuille terms of the velocity profile are neglected and hence, shear strain rates, involved in the calculation of the dissipation function, and oil flow rates are evaluated only considering Couette terms. Moreover, an effective viscosity is adopted based on the temperature variation obtained across a geometrically equivalent plain journal bearing with the same operating conditions. The obtained temperature raise is also adopted to evaluate pads' inlet temperatures. In particular, inlet temperature is considered to be the sum of lubricant supply temperature and the estimated temperature raise, this latter multiplied by a scaling ratio that is function of the minimum film thickness and of the inlet of the sector/pads film-thicknesses. Heat transfer effects are also considered by means of a coefficient that is derived from a regression analysis of experimental data provided by Ettles [9].

The second equation represents an evolution of the already described one in the way that it is able to overcome some of its drawbacks. In particular, authors decided to make the equation independent from the use of the effective temperature raise evaluated on an equivalent plain bearing. An improved mixing model, able to deal with realistic pad inlet temperatures and a temperature-dependent viscosity formulation have been adopted. The obtained equation is

numerically solved by means of a finite difference formulation. Iterations are necessary in order to account for convergence between energy and mixing model equations. Only results obtained with this latter approach will be shown.

The proposed mixing model is based on an energy and continuity balance for a control volume chosen between two consecutive pads. The Petroff's simplification is also adopted for the mixing. More details of the mixing model are shown in Panara et al. [10]. In case the code is used as a plain bearing solver (considering a constantly zero tilt angle for the pads), the mixing model is slightly modified. The loaded sector maintains the standard model using the sector's minimum film-thickness instead of the trailing edge one, while downstream of unloaded sectors a hot-oil carry over coefficient is used. This latter, has been selected by means of model assessment in comparison with experimental data and has shown good results when adopted both for different test-cases and operating conditions.

The coupling between the Reynolds' and energy equations is obtained by means of an iterative procedure (Figure 1c)). As it is possible to observe the coupling procedure includes the research of the pads' equilibrium positions. This has been chosen for handling realistic pads configurations while solving the energy equation in order to prevent the instability that could arise in unphysical configurations. A relaxation coefficient has been used for the temperature variation in order to limit numerical oscillations during the coupling procedure, thus increasing the convergence rate. Reynolds' formulation is adopted to model temperature-dependent viscosity.

### 2.5. Super-laminar flow regimes

Super-laminar effects are also modeled in TILTPAD. A simplified approach proposed by Frene et al. [11] has been used for the purpose. It gives a reasonable accuracy for Taylor-Couette vortex flows and transitional regimes, where many modern oil-lubricated bearings operate, until the fully turbulent condition is reached. By means of the proposed approach, pressure field seems to be slightly affected by turbulence level while, on the contrary, dissipative effects are enhanced. Therefore, the code has been modified to take into consideration super-laminar effects during the evaluation of the dissipation function. For this purpose, local turbulent viscosity, shown in equation (2), is evaluated as a function of the local Reynolds number and of two constants  $C_1$  and  $C_2$ . Among the several constants tested, coming from works proposed in literature, results shown in the present paper (referred to as MT3 in Fig. 4) are obtained considering the ones proposed in [12] and derived from the work of Hirs [13] ( $C_1 = 0.00818$  and  $C_2 = 0.75$ ).

$$\mu_{it} = \mu_i \left( 1 + C_1 \text{Re}_i^{C_2} \right) \text{ where } \text{Re}_i = \rho \Omega R_j h_i / \mu_i \quad (2)$$

### 3. Test cases

Three different test cases have been used for the validation of the present code. Results will be analyzed in terms of equilibrium position of the journal, frictional power loss (or friction coefficient) and local temperature distributions. The experimental works will be here referenced and, for a sake of brevity, only a few information will be given about their characteristics.

The first test case, proposed in the work of Lund and Tonnesen [14], shows results of a PJB with two feeding grooves. The others concern TPJBs: a five pads bearing from the work of Brockwell and Kleinbub [15], and a large bearing operating in the transitional and fully turbulent regime from the work of Taniguchi et al. [5]. The main characteristics of the three test cases are reported in Table 1.

Table 1. Experimental test case characteristics.

Reference	$R_j$ [mm]	B/D	$C_b$ [ $\mu\text{m}$ ]	$m$ [-]	pads [-]	Oil type	$T_0$ [ $^{\circ}\text{C}$ ]
Lund and Tonnesen [14]	50	0.55	75	-	-	ISOVG 32	50
Brockwell and Kleinbub [15]	38	0.75	60	0	5	ISOVG 32	50
Taniguchi et al. [5]	239.5	0.626	612	0	4	ISOVG 32	40

## 4. Results

### 4.1. Plain journal bearing (PJB): Lund and Tonnesen [14].

A very good agreement with experiments is obtained in terms of eccentricity ratio (Fig. 2a)) and frictional power loss (Fig. 2b)) both for 3500 rpm and for 5000 rpm along the whole range of tested loads with maximum differences always below 2.5% with respect to the experimental values. It has to be underlined that the assessment of the mixing model has been performed on the present test case for a rotational speed of 3500 rpm and with a load of 5600N. In terms of local temperature distribution, shown in Fig. 2c), orders of magnitude and trends are quite well predicted, particularly considering that local experimental temperatures are measured at the bearing bush while TILTPAD data are mean film values. More in detail, in the unloaded section of the bearing, where the ruptured film zone is located, the trend is not well predicted. This has to be ascribed to a backflow recirculation from the groove into the unfilled space, where the film is striated, as described in Lund's work [14]. Comparing the obtained results with the mean temperature presented in the referenced paper, both trends and values are in better agreement. Considering the loaded sector, the maximum temperature differences are obtained at its inlet, with values of respectively +9.2 °C and +4.9 °C with respect to the experiments and to the Lund's calculated mean values respectively.

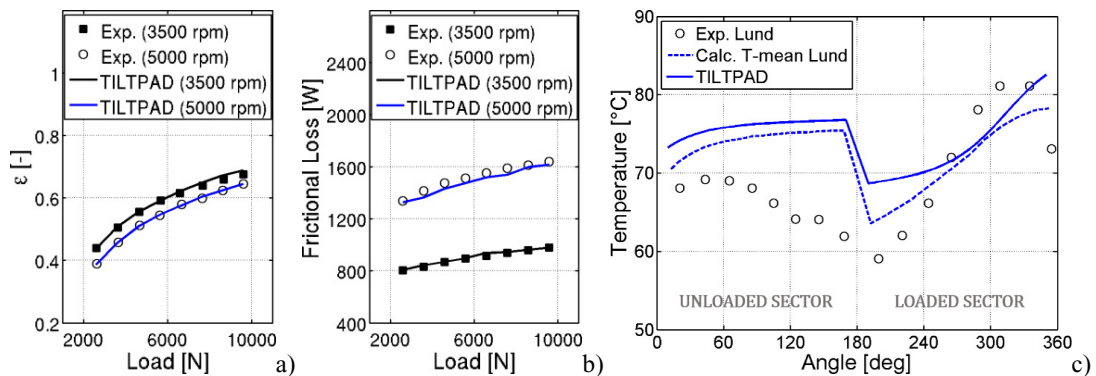


Fig. 2. Comparison between numerical and experimental results from Lund and Tonnesen [14]: a) eccentricity ratio for 3500-5000 rpm varying the applied load; b) frictional power loss for 3500-5000 rpm varying the applied load; c) temperature distribution (5000 rpm, 5600 N)

### 4.2. Tilting pad journal bearing (TPJB): Brockwell and Kleinbub [15].

The second test case is a TPJB equipped with five pads, in a load between pads configuration. Data are presented in terms of differential eccentricity ratio, friction coefficient and temperature field. Differential eccentricity is here evaluated as the vertical distance (nondimensional with respect to  $C_b$ ) between the obtained results and results from a reference case (83 Hz and 450 N). The friction coefficient is evaluated dividing the obtained friction force by the applied load. A good prediction of the equilibrium position (Figure 3a)) is obtained for both the tested configurations (i.e. 2225 and 4450 N at various shaft rotations) although with a general underestimation. The maximum difference between the calculated data and the experiments is a -14% obtained for the 3600 rpm and 4450 N case. It is necessary to observe also a comparable scatter of the reported experimental data. Same considerations can be done for the friction coefficient, shown in Fig. 3b), where a good agreement is globally obtained, yet again, numerical variations are comparable with the scatter of the experimental data.

Two configurations have been tested in terms of temperature distribution (i.e., 1800 and 5000 rpm, with 8900 N). Trends and orders of magnitude are quite well predicted considering what reported in the previous subsection about mean values calculated from the code and locally measured temperatures. A misprediction of the trend is highlighted for the three unloaded pads of the 1800 rpm configuration. Each time a mixing is performed the code obtains a lower mean temperature value of the "mixed" oil at the leading edge of the pad while, particularly for pad number three, experiments show an increase of temperature after mixing. The maximum differences on the loaded pads is obtained



at the inlet of the fourth pad and at the outlet of the fifth one with respectively a  $-6.2\text{ }^{\circ}\text{C}$  and  $+9.6\text{ }^{\circ}\text{C}$  with respect to the experimental data for the 1800 rpm case.

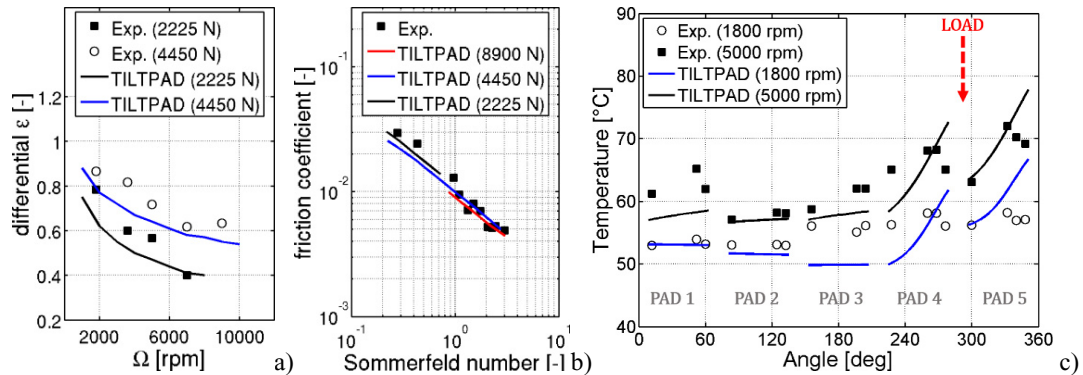


Fig. 3. Comparison between numerical and experimental results from the work of Brockwell and Kleinbub [15]: (a) differential eccentricity ratio for 2225-4450 N varying rotational speed; (b) friction coefficient; (c) temperature distributions (1800-5000 rpm, 8900 N)

#### 4.3. Large tilting pad journal bearing (TPJB): Taniguchi et al. [5].

The last test case is a large four pads TPJB in a load between pads configuration [5]. Results are compared also with the FEM code presented by the authors, which is quite complete (i.e. both pressure and energy equations consider the effect of turbulence) and very well predicts all of the presented data.

In terms of eccentricity ratio, shown in Fig. 4a) for a global Re of about 1500, an improvement of TILTPAD's prediction is obtained when modeling the superlaminar flow effects (referred to as MT3) with respect to the laminar calculation. A slight overestimation is obtained along the whole tested range of loads and could be mainly ascribed to the fact that TILTPAD neglects the effects of turbulence on the pressure calculation, hence reducing the load carrying capacity of the bearing for high Reynolds numbers. It has to be noticed that above 210 kN the code is not able to follow the trend because it has an imposed limit for the maximum allowable eccentricity ratio.

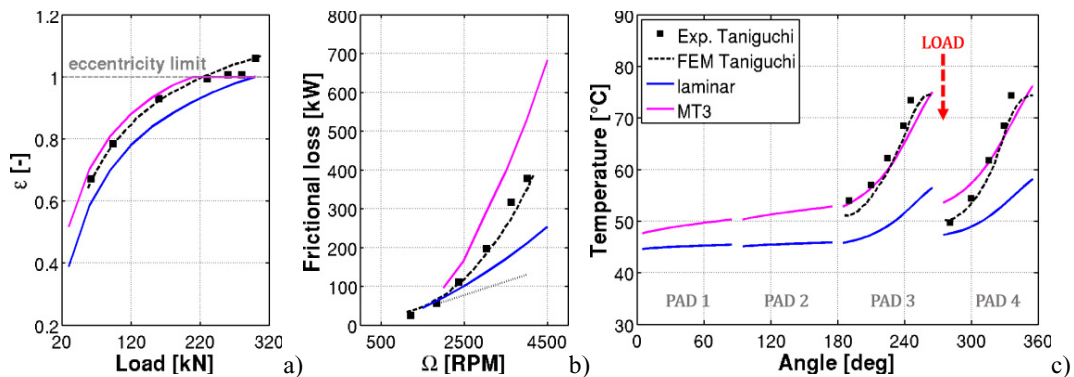


Fig. 4. Comparison between numerical and experimental results from the work of Taniguchi et al. [5]: (a) eccentricity ratio for 3000 rpm varying the applied load; (b) friction coefficient for 180000N varying the rotational speed; (c) temperature distributions (3000 rpm, 180000 N).

In terms of friction power loss, better trends are again obtained for the MT3 calculation instead of the laminar one, although the overestimation is here high and increases with increasing rotational speed, so with increasing Reynolds number. The maximum difference is about +40% at 4000 rpm when the Re is about 2000. It is not easy to identify the reason for such a result due to the number of simplification considered in the code, e.g., Poiseuille terms are neglected,

turbulent viscosity effects on the pressure field are neglected, unloaded pads are set in a prescribed position. Moreover, the selection of different constants and critical values from different turbulence models proposed in literature drives to different results. Anyway some considerations can be done: the more the Reynolds number increases the less neglecting turbulence effects on the pressure field is acceptable, driving the code towards overestimations of the equilibrium eccentricity, and hence, towards the overestimation of the shear stress, that is here evaluated only considering the Couette term. Such an increase in shear stress could directly drive an overestimation of the frictional power loss. Moreover, the overprediction of the equilibrium position increases the local Reynolds number on the unloaded pads and hence their dissipation due to the increase in turbulent viscosity. Unloaded pads contribution to the total frictional power loss raises up to about 50% for high rotational speed. In terms of temperature, shown in Fig. 4c), a good agreement is obtained with the experimental data, where instead, laminar calculation showed remarkable discrepancies in particular at the outlet of pad number three with an underestimation of -18.4 °C.

## 5. Conclusions

In the present work the in-house code TILTPAD for the thermo-hydrodynamic analysis of PJBs and TPJBs is presented. After the main features of the code are described, with particular attention to the 1D energy equation able to model superlaminar flow regimes with a simplified approach, results of the validation over three very different test cases is reported. Good agreement is obtained for all of the test cases in terms of eccentricity ratio, with a slight overprediction obtained in comparison with experiments from Taniguchi et al. [5]. Such an overestimation has been ascribed to the simplifications affecting the code when superlaminar regimes are considered. Same considerations can be done when dealing with frictional power losses, where an overestimation, up to +40%, is predicted by the code for high rotational speed. In terms of temperature, results showed good trend prediction and slight variations with respect to the experiments, with a maximum peak at the outlet of the fifth pad of +9.6 °C with respect to the experimental data of Brockwell and Kleinbub [15] for the 1800 rpm case. In conclusion, although the code is based on strong simplifications, it allows to obtain accurate results, particularly in terms of global system behavior, resulting in a good trade-off between accuracy and computational costs.

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