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Research Article

Planar Thinned Arrays: Optimization and Subarray Based Adaptive Processing

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A new approach is presented for the optimized design of a planar thinned array; the proposed strategy works with single antenna elements or with small sets of different subarray types, properly located on a planar surface. The optimization approach is based on the maximization of an objective function accounting for side lobe level and considering a fixed number of active elements/subarrays. The proposed technique is suitable for different shapes of the desired output array, allowing the achievement of the desired directivity properties on the corresponding antenna pattern. The use of subarrays with a limited number of different shapes is relevant for industrial production, which would benefit from reduced design and manufacturing costs. The resulting modularity allows scalable antenna designs for different applications. Moreover, subarrays can be arranged in a set of subapertures, each connected to an independent receiving channel. Therefore, adaptive processing techniques could be applied to cope with and mitigate clutter echoes and external electromagnetic interferences. The performance of adaptive techniques with subapertures taken from the optimized thinned array is evaluated against assigned clutter and jamming scenarios and compared to the performance achievable considering a subarray based filled array with the same number of active elements.

1. Introduction

As well known, the use of large array antennas allows obtaining patterns with good values of angular resolution. Amplitude tapering is typically used to lower the level of the side-lobes of the pattern. As a consequence usually a great number of Transmit/Receive (T/R) elements are needed, thus increasing the production cost.

Several techniques were studied in the past to reduce the number of active elements in the array with limited performance loss. Among them, thinning techniques allow to achieve low values of side-lobe level (SLL) without using amplitude tapering, thus with limited impact on the angular resolution. This is achieved when, among all the possible positions in the array, only a subset is actually occupied by T/R elements, inducing a density tapering.

Finding the best thinned array configuration is therefore the problem to be solved. The optimal result can be found only trying all the combinations of active elements among all the possible positions, but this brute approach is computationally consuming especially for large planar arrays. Therefore, several thinning techniques have been developed in the past, which can be divided in regular grid based and random location based techniques. In the first case, the possible positions form a regular grid; thinning can be performed (i) switching off several active elements depending on a statistic criterion or (ii) starting from an empty array and filling some positions according to a deterministic criterion. In the second case, T/R elements are displaced in random positions with proper statistical characteristics.

In [1] Sherman and Skolnik used an array with isotropic elements on concentric rings with different radial and angular spacing. This induces natural spatial tapering with effects on SLL depending on the uniformity of the radial spacing and on the number of allowed angular positions in each ring. In [2], Skolnik et al. proposed a technique where the spatial tapering is statistically determined, allowing a concentration of elements in the center of the array. Starting from [2], in [3]

Mailloux and Cohen studied a regular grid based thinning approach with the joint use of stepped amplitude tapering to further reduce the SLL. In [4, 5] Skolnik et al. proposed a trial-and-error technique called "Dynamic Programming," which has been devised to reduce the number of total trials with respect to the brute approach. As for the previous thinning approaches, "Dynamic Programming" is not supposed to reach the optimal array configuration. Moreover, its deterministic approach makes this technique suitable only for small arrays. More recently in [6], Keizer studied an iterative random technique, called Iterative Fourier Technique, to thin an array achieving a fixed number of active elements. The technique is based on the inverse discrete Fourier transform (IDFT) relation between the desired array pattern and the element excitations; moreover the iterative nature of this algorithm depends on the nonideal effects induced by the binary quantization of the actual element excitations.

The previous are regular grid based thinning approaches. Lo studied random location based thinning techniques in [7, 8], reaching some general and useful conclusions: for example, he stated that the number of active elements is linked with the desired SLL and that the angular resolution depends on the dimension of the resulting array and less on the elements distribution. Following the random location thinning approach, Steinberg succeeded in determining an estimate of the SLL knowing the number of elements, the dimension of the array, the wavelength, the steering direction of the pattern, and the frequency bandwidth of the signal, [9, 10]. Moreover, in [11] Steinberg and Attia used position and frequency diversity to reduce the SLL without impact on the main lobe of the pattern.

Also statistic global optimization algorithms have been applied to the problem of regular grid based thinning, where the objective function can be related to the SLL. Genetic algorithms [12–15] are well suited since the binary representation of the genes directly refers to the presence (1) or the absence (0) of an active element in the array. In [16], a genetic algorithm is used not to determine which element is active in the array but to determine the spacing of concentric rings according to Cantor sets where active elements should be placed, extending the original approach in [1]. In [17], a nested optimization algorithm integrating genetic algorithm and linear or quadratic programming is introduced to find the thinned array with the minimum number of active elements, whose excitations allow the achievement of a pattern with desired characteristics (such as minimum SLL or prescribed gain in a specific direction). Ant colony algorithm in [18] emulates the behavior of ants to determine the best thinned array configuration, based on the identification of the shortest path from nest to food. Here, the food is the desired SLL, the path is the set of active elements in the array, and the length is a figure of merit concerning the probability for an ant to pass from one node to another in the path that is the probability of an element in the array to be active. A Boolean version of the differential evolution algorithm is applied in [19], to cope with discrete-variable optimization problem. Also simulated annealing [20, 21] has been used to select simultaneously the active elements positions and weighting coefficient to reach the best SLL possible. Finally, particle swarm optimization has been proposed in [22] to cope with antenna design applications: in particular, the continuous and binary versions can be used to synthesize aperiodic or thinned arrays respectively, based on the minimum SLL criterion in the single-objective case or on a number of desired features of the antenna pattern in the multiobjective case.

In this paper, we consider two main issues: (i) the optimized design of planar thinned arrays at element or subarray level and (ii) the identification of suitable strategies to split the thinned array in multiple subapertures, adequate for the application of adaptive processing techniques for jammer cancellation.

The first issue addresses the practical problem of obtaining planar thinned arrays by disposing a certain number of active elements on a given planar surface. It is well known that good results can be obtained by randomly thinning the aperture and using irregular subarray configurations [23], but the absence of symmetry in the array structure strongly affect the design and production costs. Therefore, we aim at identifying thinned array structures with good performance in terms of SLL at the same time choosing the used subarrays from a limited number of different types. Subarray based thinning reduces the degrees of freedom in the array thus degrading the performance: therefore, suitable design approaches have to be identified to obtain good performance despite the constraints.

To this purpose we present two new statistical design techniques called sequential probabilistic element disposal (SPED) and sequential probabilistic subarray disposal (SPSD [24]), respectively. In our approach, the number of elements actually present in the array is fixed. Starting from the concepts in [1, 2], the proposed statistical approach does not decide whether or not to introduce a single element/subarray, but it selects the position inside the array that the new element/subarray should occupy. This is done according to a probability density function (PDF) derived from an amplitude tapering function favoring central positions with respect to the edges. This should correspond to a density tapering of the array and therefore results in an increased SLL. Moreover, starting from these two basic techniques, to improve efficiency and practical feasibility, several modified versions are also developed. The first modified version is based on the observation that there is a high probability of filling the central part of the array with active elements/subarrays. Therefore, a constrained version of SPED and SPSD is derived, where all the positions belonging to the center of the array (properly identified) are filled and the statistical procedure follows to reach the desired number of active elements. The second modified version is motivated by the observation that the use of larger subarrays is useful in terms of production costs but tends to degrade the thinned array performance unless high number of subarray types is considered. To avoid the use of SPSD with a large number of subarray types, we devise the wide subarray forcing (WFS) procedure; merging small adjacent subarrays allows obtaining an exponentially higher number of available types of larger subarrays, among which only a subset is considered valid. The performance analysis of the devised optimization techniques shows that it is possible to design planar thinned arrays with assigned desired performance in terms of SLL. Moreover this analysis proves that the obtained thinned arrays are effective in lowering the SLL with limited impacts on the main lobe aperture of the corresponding pattern, in contrast with what would be obtained by considering filled arrays with the same number of active elements.

The second issue aims at verifying the feasibility of adaptive processing on a subarray based thinned aperture to mitigate either clutter or interferences. We identify suitable subaperture structures for the application of electronic counter-counter measures (ECCM) techniques. The performance analysis against simulated interference and clutter scenarios shows the quality of the obtained results.

The paper is organized as follows. Section 2 introduces the array structure model; Section 3 describes the optimization techniques to obtain element and subarray based thinned antennas with the desired characteristics. Section 4 shows some examples of thinned arrays obtained using the proposed design techniques, while Section 5 compares the performance of adaptive techniques applied to both thinned and filled arrays against jammer and clutter. Finally, we draw our conclusions in Section 6.

2. Array Structure Model

In order to develop the design techniques of a thinned planar array, we start from a filled array, containing $M \times N$ isotropic elements disposed on the intersections of a regular $M \times N$ grid.

Thinning consists in considering only a number N_A of active elements, over a total number of possible positions on the array $N_{\text{POS}} = MN$. Therefore, the thinning factor can be defined as

$$\eta = \frac{N_{\text{POS}}}{N_A}. \quad (1)$$

The procedure of thinning the array can be regarded as a transformation applied to the received signal vector at the element level, namely, to the $N_{\text{POS}} \times 1$ vector $\mathbf{x}_{\text{el}}$

$$\mathbf{x}_{\text{el}}(\theta, \varphi) = a \cdot \mathbf{s}_{\text{el}}(\theta, \varphi) + \mathbf{d}_{\text{el}}, \quad (2)$$

where $\mathbf{s}_{\text{el}}$ is target steering vector, a and (θ, φ) represent, respectively, the complex amplitude of the useful signal and its direction of arrival (DOA) angles in elevation and azimuth. We also define $\mathbf{d}_{\text{el}}$ as the disturbance signal (jammer plus noise) $N_{\text{POS}} \times 1$ vector and its $N_{\text{POS}} \times N_{\text{POS}}$ disturbance covariance matrix $\mathbf{M}_{\text{el}}$.

The steering and thinning procedure introduces a transformation applied on the received signal vector $\mathbf{x}_{\text{el}}(\theta, \varphi)$ that can be represented by the transformation matrix $\mathbf{T}_{\text{el}}$. Define the $N_{\text{POS}} \times N_{\text{POS}}$ matrix $\mathbf{S} = \text{diag}\{\mathbf{s}_{\text{el}}(\theta_0, \varphi_0)\}$ for steering purpose and the $N_{\text{POS}} \times N_A$ matrix $\mathbf{U}$, which selects the active elements in the array, for thinning. Thus, the global transformation is described by the $N_{\text{POS}} \times N_A$ matrix $\mathbf{T}_{\text{el}} = \mathbf{S} \cdot \mathbf{U}$. At element level, the $N_A \times 1$ received signal vector and the $N_A \times N_A$ disturbance covariance matrix will be given, respectively, by $\mathbf{x}(\theta, \varphi) = \mathbf{T}_{\text{el}}^H \mathbf{x}_{\text{el}}(\theta, \varphi)$ and $\mathbf{M} = \mathbf{T}_{\text{el}}^H \mathbf{M}_{\text{el}} \mathbf{T}_{\text{el}}$.

To generate subarray based thinned aperture, it is necessary to define also the shape, the size, and the orientation of the single subarray, as well as the grid of subarray centers. This grid has to be defined so that if we place a subarray on each element of the grid a full filling of the array area is obtained (i.e., absence of gaps).

In this case, the thinning procedure is described by a transformation of the received signal vector $\mathbf{x}_{\text{el}}(\theta, \varphi)$ in the $N_{\text{SUB}} \times 1$ thinned subarray vector $\mathbf{x}(\theta, \varphi)$. This is achieved considering the transformation matrix $\mathbf{T}_{\text{SUB}} = \mathbf{S} \cdot \mathbf{U} \cdot \mathbf{T}$, where $\mathbf{T}$ is a $N_A \times N_{\text{SUB}}$ transformation matrix which arranges N_A active elements in N_{SUB} subarrays. At subarray level, the $N_{\text{SUB}} \times 1$ received signal vector and the $N_{\text{SUB}} \times N_{\text{SUB}}$ disturbance covariance matrix will be given, respectively, by $\mathbf{x}(\theta, \varphi) = \mathbf{T}_{\text{SUB}}^H \mathbf{x}_{\text{el}}(\theta, \varphi)$ and $\mathbf{M} = \mathbf{T}_{\text{SUB}}^H \mathbf{M}_{\text{el}} \mathbf{T}_{\text{SUB}}$.

3. Thinned Array Design

The statistical technique for the thinned array generation described in [1, 2] starts from the conventional density tapering of the array elements. The single radiating element is introduced with a probability proportional to its weight depending on a given amplitude tapering function. This is effective in obtaining a set of thinned arrays with SLL close (in the average) to the level achievable by applying the selected amplitude tapering function to the corresponding filled array. Unfortunately this optimization algorithm does not allow setting a priori the number of active elements in the array and it does not cope with subarray based arrays.

To cope with these undesired features we introduce here two new techniques that we call Sequential Probabilistic Element Disposal (SPED) and sequential probabilistic subarray disposal (SPSD) for the design of a thinned antenna at element and subarray level, respectively. SPED and SPSD are still statistical approaches but properly modified to allow the design of the thinned array with a preassigned number of elements.

3.1. Sequential Probabilistic Element Disposal (SPED) Technique. The SPED technique is described in Figure 1. It uses the general scheme of Figure 1(a), with the single trial step described in Figure 1(b). Firstly, the set of possible positions is assigned and the counter and the minimum SLL are initialized (i.e., IND = 0 and $\text{SLL}_{\text{MIN}} = 0\,\text{dB}$). Then, the single trial of the technique follows. The first step of the single trial of the technique initializes the probability $p_1(j_k)$, $k = 1, 2, \ldots, N_{\text{POS}}$, to introduce an active element in the position j_k and the counter $m = 1$. Specifically, the probability $p_1(j_k)$ depends on a reference amplitude tapering $\mathbf{w} = [w_1, \ldots, w_{N_{\text{POS}}}]^T$ as follows:

$$p_1(j_k) = \frac{w_k}{\sum_{n=1}^{N_{\text{POS}}} w_n}, \qquad k = 1, \ldots, N_{\text{POS}}. \quad (3)$$

In each iteration of the SPED, a realization i_m is extracted from the set of possible active element positions j_k, $k = 1, \ldots, N_{\text{POS}}$, with probabilities $p_m(j_k)$. A new element is inserted in position (i_m) and the active elements position vector $\mathbf{i} = [\mathbf{i}, i_m]$ is updated.

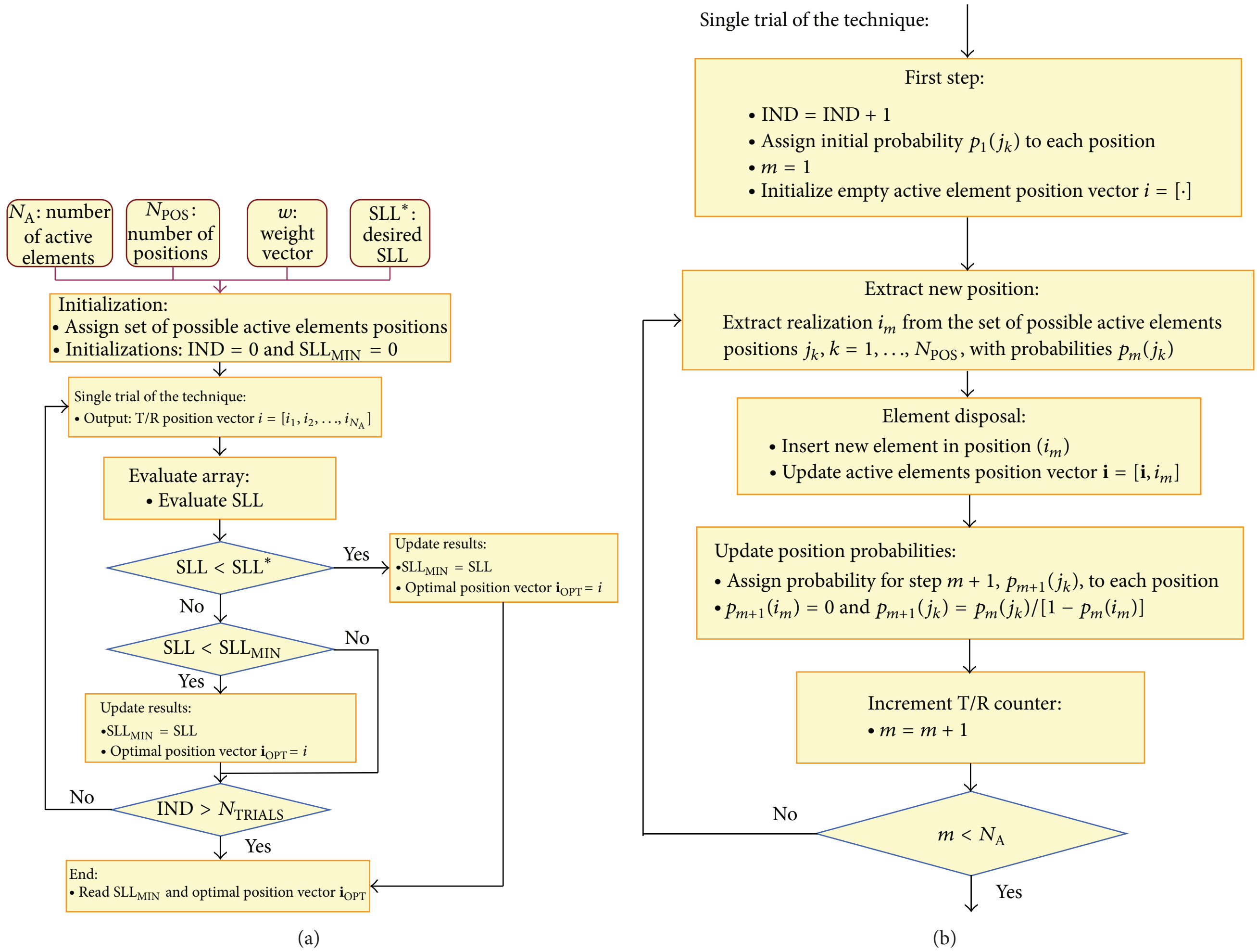


FIGURE 1: (a) SPED technique outer scheme (b) SPED single trial (inner block).

Then, the probabilities to assign each position are updated as follows:

$$p_{m+1}(i_m) = 0$$
$$p_{m+1}(j_k) = \frac{p_m(j_k)}{(1 - p_m(i_m))}. \qquad (4)$$

When the desired number of active elements is introduced in the array (i.e., $m > N_A$), the single trial is complete.

The output of the single trial of the SPED is a thinned array; therefore, the SLL can be evaluated. If it is lower than the desired value SLL^* the algorithm stops; otherwise, the SPED continues with the next trials. Anyway, a maximum number of allowed trials N_{TRIALS} is defined: in case the desired SLL^* is not achieved, the algorithm stops when N_{TRIALS} have been executed. In such a case the output is given by the best achieved result represented by the vector of optimum positions $\mathbf{i}_{OPT}$ providing the lowest SLL_{MIN}.

The novelty introduced by the SPED is that the statistical approach is not used to decide whether or not to introduce a single element but to select the position inside the array that the new element should occupy. In this manner, the number of elements in the array is fixed and it is deterministically set by the thinning factor. This is different from thinning using dynamic programming [4, 5], where a deterministic approach is used to test all the possible insertions of an active element, including a pruning strategy to reduce the number of trials.

3.2. Sequential Probabilistic Subarray Disposal (SPSD) Technique. The "random" displacement of the active elements in the array, while guaranteeing an improvement of SLL, is a critical point in terms of ease and cost of design and manufacturing. A regular structure would be more convenient and would allow the scalability of the antenna design in relation to specific applications. As a trade-off between these two aspects, the thinned array can be designed based on subarrays; indeed keeping small the number of different subarray shapes and sizes is relevant for industrial production, to reduce design and manufacturing costs, as well as to allow scalable antenna designs.

A subarray is formed by several antenna elements and has to be considered as the basic tile of the array. When thinning is involved in the design step, the position of the entire subarray has to be set in lieu of the single active element. Moreover,

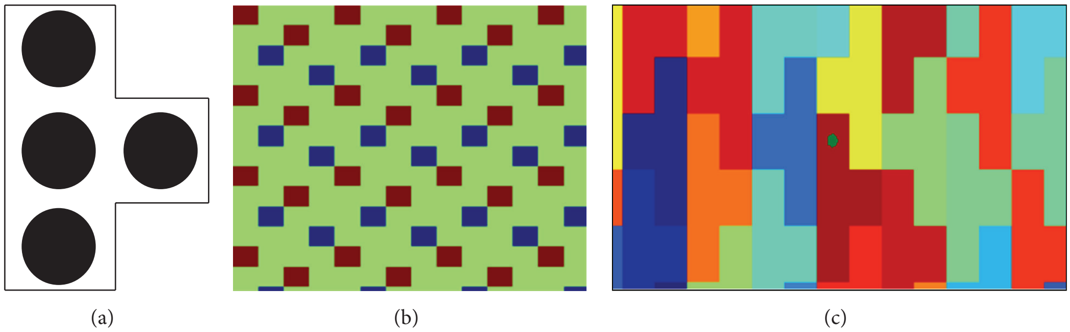


FIGURE 2: Example of (a) subarray, (b) grid of subarray centers, and (c) possible subarray displacement.

if subarrays with different shape and size are available, also the type of subarray to be filled in the array has to be decided.

To design a thinned array based on subarrays, a modified version of the SPED is here introduced called SPSD. The SPSD technique operates as follows. The first step involves the selection of the subarray shape and size D (as an example Figure 2(a) shows a triangular subarray with $D = 4$), as well as the subarray center and its original orientation. Moreover, it is necessary to define a grid containing all the possible positions $N_{\text{POS-SUB}}$ for the subarray centers (e.g., Figure 2(b)). Each subarray can be inserted with different orientations (Figure 2(b), red: original orientation and blue: rotation of 180°) starting from the center of the grid, to completely cover the array without overlapping (Figure 2(c)).

Thereafter, the N_{SUB} subarrays are sequentially introduced in a position that is randomly selected, in accordance with the weight corresponding to an assigned reference taper function. Specifically, we generalize the SPED procedure to cope with the subarray case. The technique is described in Figure 3(a), with the single trial described in Figure 3(b).

The first step of the single trial initializes the probability $p_1(j_k)$, $k = 1, 2, \ldots, N_{\text{POS-SUB}}$, to introduce a subarray in the position j_k and the counter $m = 1$.

Specifically, the probability $p_1(j_k)$ depends on reference amplitude tapering $\mathbf{w} = [w_1, \ldots, w_{N_{\text{POS-SUB}}}]^T$ as follows:

$$p_1(j_k) = \frac{w_k}{\sum_{n=1}^{N_{\text{POS-SUB}}} w_n} \quad k = 1, \ldots, N_{\text{POS-SUB}}. \tag{5}$$

At each iteration of the SPSD a realization i_m is extracted from the set of possible subarray positions j_k, $k = 1, \ldots, N_{\text{POS-SUB}}$, with probabilities $p_m(j_k)$. A new subarray is inserted in position (i_m) and the subarray position vector $\mathbf{i} = [\mathbf{i}, i_m]$ is updated.

Then, the probabilities to assign each position are updated as follows:

$$\begin{aligned} p_{m+1}(i_m) &= 0, \\ p_{m+1}(j_k) &= \frac{p_m(j_k)}{(1 - p_m(i_m))}. \end{aligned} \tag{6}$$

When the desired number of fully active subarrays N_{SUB} (and thus active elements N_A) is introduced in the array, the single trial is complete. Note that if both fully active and thinned subarrays are available, when a realization i_m is extracted the algorithm needs to decide what kind of subarray has to be inserted in the array. Again, this can be done using a statistical approach that promotes the selection of a fully active subarray near the center of the array or of a thinned subarray near the edge.

The same stop condition of the SPED technique applies also to SPSD.

3.3. Strategies for Elements/Subarrays Sorting. Equations (3) and (5) associate a certain amplitude weight and therefore a corresponding probability to a specific position in the array. The basic rules of this association, based on the sorting of all the possible positions in the array, are important in adapting the SPED and SPSD algorithms to different kinds of planar arrays and therefore to different directivity characteristics of the corresponding antenna pattern.

First of all consider a circular array; in this case, according to [1, 2], a sorting of all the possible positions based on the radial distance from the center of the array is adequate, due to the particular symmetry (Figure 4(a): dark red subarrays are the farthest from the center). In this way, positions near to the center should benefit of a higher probability than the side positions, according to the characteristics of the amplitude tapering functions. Tie situations can be solved using the angular displacement of each position with respect to a reference direction.

In a rectangular array, this ordering strategy greatly penalizes the farthest elements outside a circular region around the array center. To this purpose, instead of sorting the positions according to concentric circles, it is more appropriate to sort them in concentric frames, where each frame is characterized by the same ratio between the greatest and the smallest dimensions of the array (Figure 4(b): dark red subarrays belong to the outer frame). Positions are therefore ordered first of all according to the frame (from the inner to the outer). Positions belonging to the same frame are then sorted according to the distance from the center of the array: positions in the same frame and at the same distance are finally ordered using the angle between the position and a reference direction.

Obviously SPED and SPSD techniques can also be used jointly with other sorting strategies; moreover, array shapes

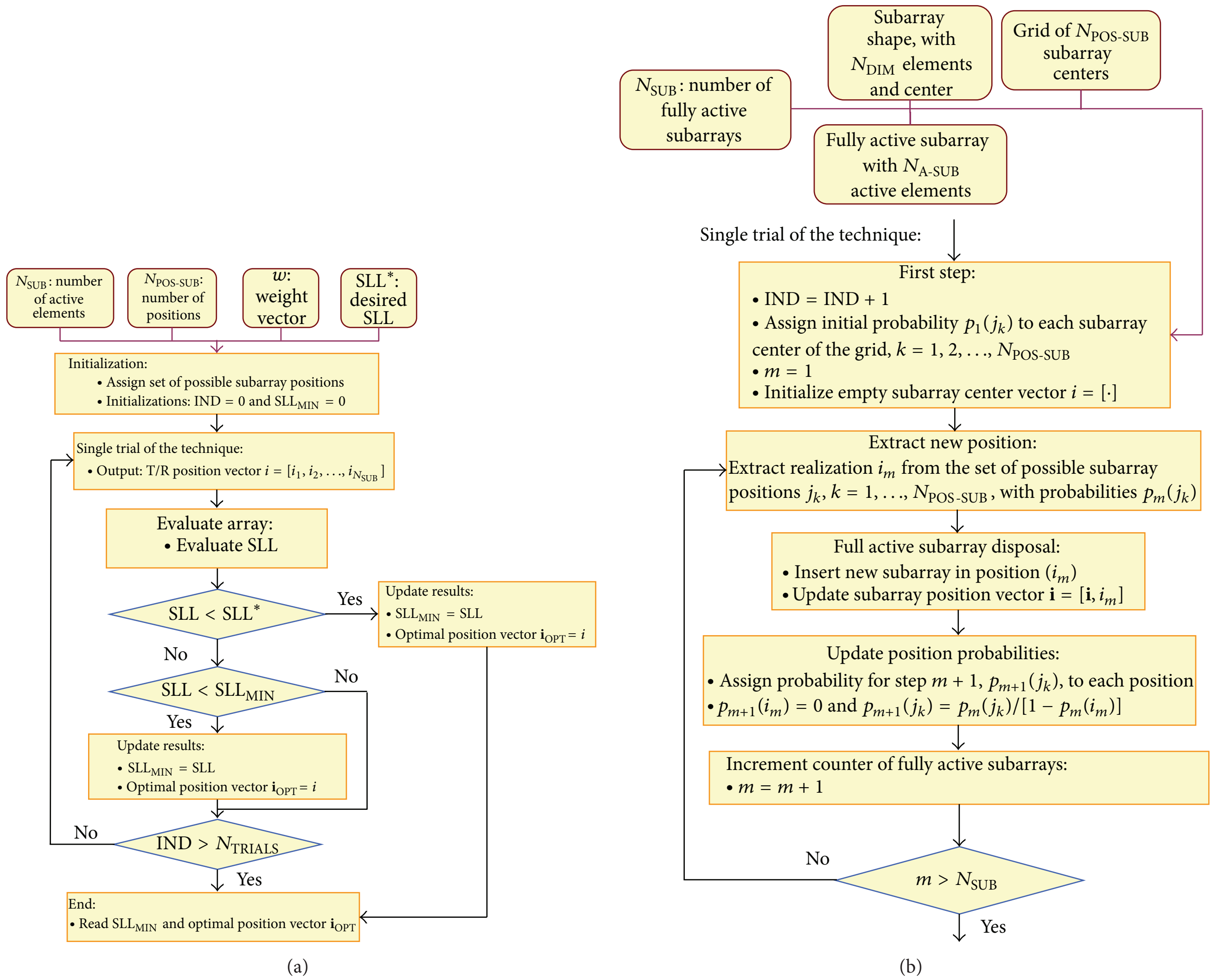


Figure 3: (a) General statistical technique outer scheme (b) SPSD single trial (inner block).

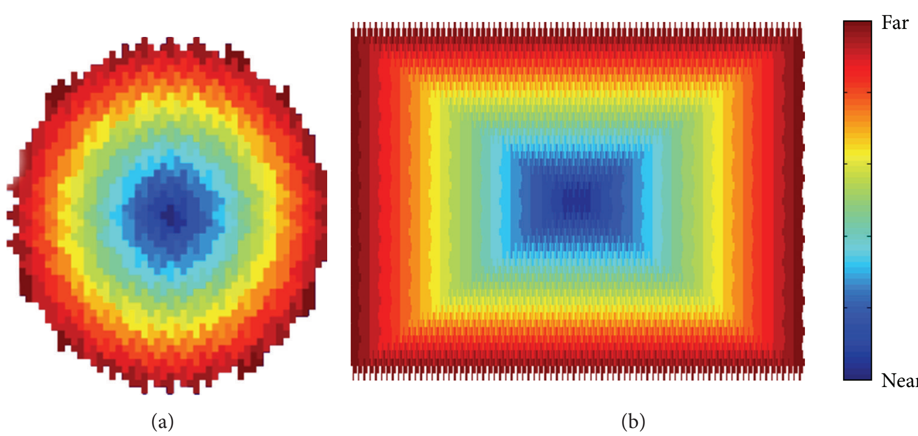
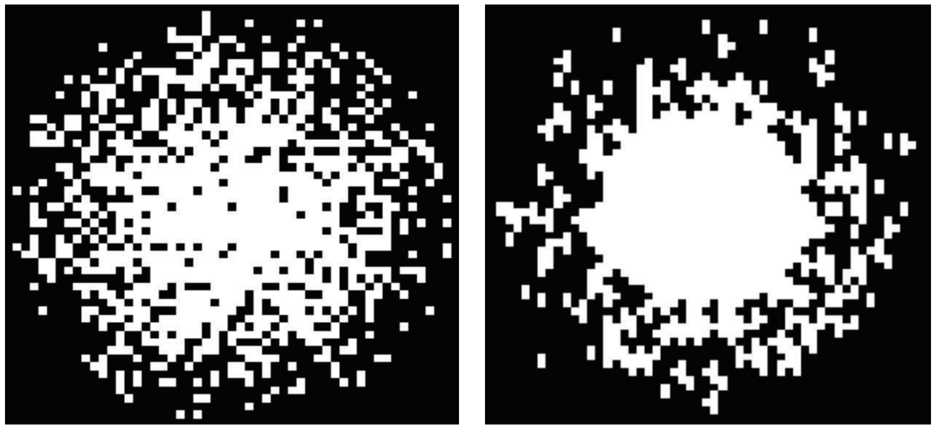


Figure 4: Subarray sorting approach for (a) circular and (b) rectangular arrays.



(a) (b)

Figure 5: Circular thinned array with (a) SPED and (b) Constrained SPSD.

different from circular and rectangular could also be considered. Such flexibility of the proposed techniques is well suited for the achievement of a thinned array able to fulfill the requirements not only relative to the SLL (that depends on the displacement of the active elements) but also on the directivity of the achievable pattern (which is more sensitive to the maximum distance between active elements in the array).

3.4. Improving SPED/SPSD Efficiency and Practical Feasibility. Since SPED and SPSD are statistical approaches that use a PDF achieved from an amplitude tapering function, it is straightforward to expect best results in terms of SLL when active elements fill the central area of the array. Following this observation, we propose a modified version of the SPED/SPSD techniques, named Constrained SPED/SPSD, which starts by filling the central part of the array (with proper dimension). In this way the statistical procedure is applied only to the remaining elements to reach the desired number of active elements. This approach guarantees that only configurations with "reasonably good" SLL are generated and makes the algorithm faster than the initial SPED/SPSD.

As it will be clear in the following section, good SLL results can be reasonably obtained by the SPSD and by the Constrained SPSD, when using subarrays with a small size. Large subarrays are preferable in terms of design and production costs but tend to degrade the thinned array performance, in terms of achievable SLL. This degradation can be partially mitigated by using multiple types of subarray, which also allows a reasonable convergence. However, a considerable increase of the number of subarray types is not a desirable condition. Therefore, a trade-off is needed between the achievement of the desired performance and the cost avoidance. To this aim, we propose a further approach that allows us to obtain a thinned array based on a set of wider subarrays, starting from the thinned array provided in output by SPSD based on smaller subarrays.

Wider subarrays can be obtained by merging adjacent smaller subarrays, thus considerably increasing the number of available larger subarrays types. The wide subarray forcing (WSF) technique allows reducing this number, by operating as follows:

(i) adjacent small subarrays in the original thinned array are merged;

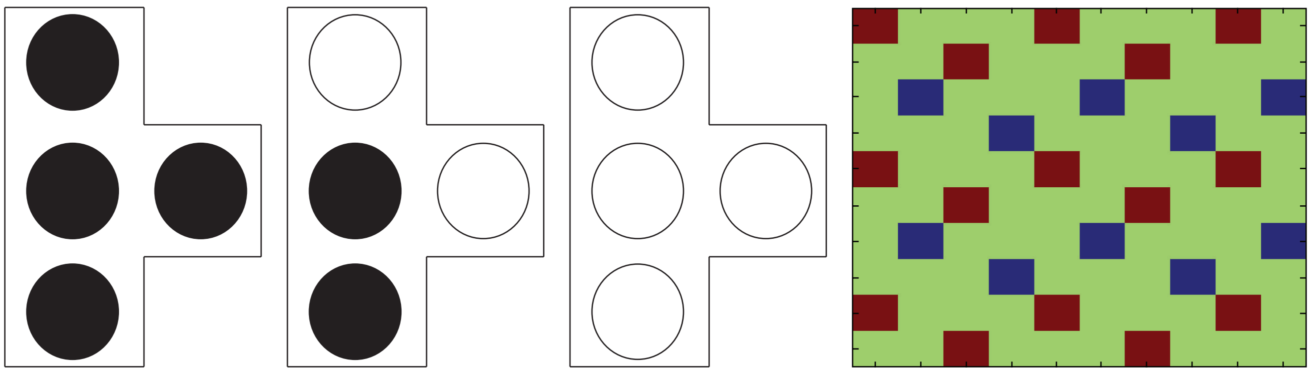
(ii) among all the possible large subarrays, achievable when merging all used small subarrays, a valid sub-set is defined, that is, containing the more frequent large subarrays in the merged thinned array;

(iii) for each larger subarray in the merged thinned array, we measure the Hamming distance for all selected subarray types. (The Hamming distance between two subarrays is the number of positions where they differ in terms of active elements). Now, we have two possibilities: (a) to replace the merged subarray with the one with closer measure, or (b) to randomly replace it with one of the selected subarray types, with a probability inversely proportional to their distance measure. Next section provides an example of this procedure.

4. Performance Analysis

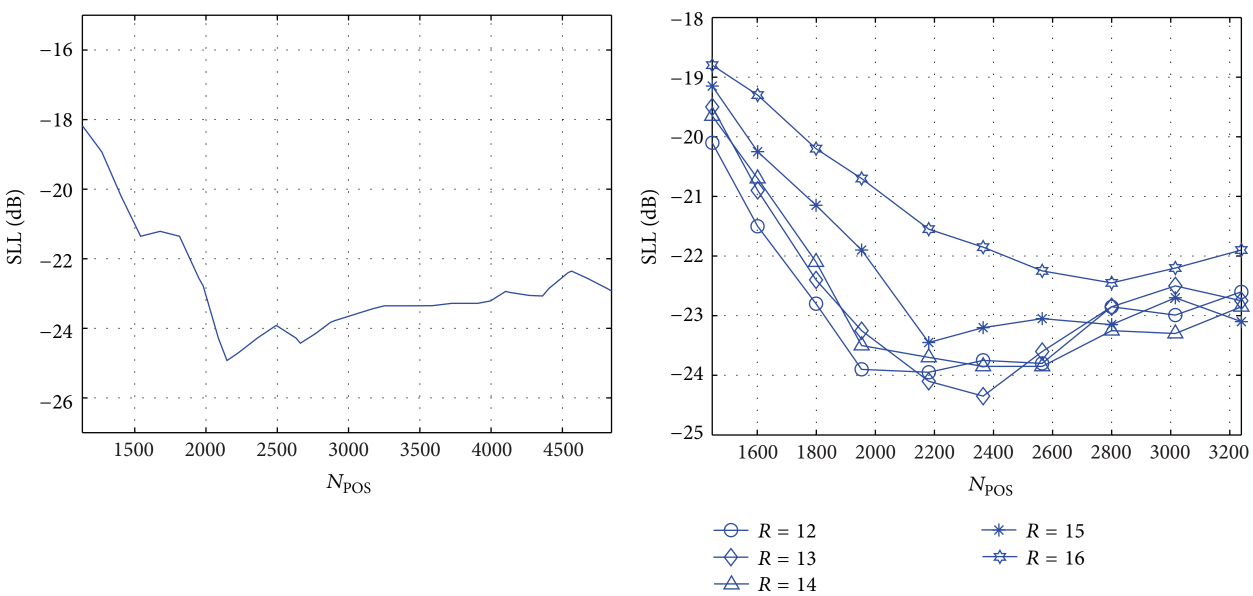
In this section, we discuss the design of thinned arrays with a SLL better than −23 dB. We consider both the cases of a circular array (with $N_A = 1024$) and a rectangular array (with $N_A = 2500$). Section 4.1 presents the results of the design techniques while Section 4.2 compares the performance of the achieved thinned arrays to those of the corresponding filled arrays.

4.1. Design Examples. For the circular case, the output thinned array resulting from SPED is shown in Figure 5(a); in particular, this result has been obtained staring from an array with $N_{POS} = 2121$ positions: the designed one has $N_A = 1024$ active elements and assures an SLL = −24.93 dB. As it is apparent in Figure 5(a), the central disk of the array is quite full, justifying the eventual use of the constrained version



(a) (b) (c) (d)

Figure 6: Subarray type of size 4 (a) fully active, (b) thinned for SPSD, and (c) grid of centers scheme.



(a) (b)

Figure 7: SLL for different N_{POS} for (a) SPED and (b) Constrained SPSD varying with radius R.

of the algorithm. Figure 7(a) shows the best SLL provided by SPED technique when varying the number of possible positions N_{POS} (namely varying the size of the array) and keeping fixed the number of active elements. This figure highlights the dependence of the achievable performance on the thinning factor: the minimum value is reached in correspondence of the thinned array shown in Figure 5(a), therefore with $\eta = 2021/1024 = 2.07$.

Figure 5(b) shows the output of the Constrained SPSD using three kinds of subarrays of size 4 (Figures 6(a), 6(b), and 6(c)): one fully active, the second one thinned with two active elements, and the third one empty, the latter being inserted when a position is not selected. Moreover, a suitable grid of centers is defined (a zoom of the central part in Figure 6(d)) and different values of the radius R (normalized to the inter-element distance) of the constrained inner circle are considered. The SLLs reported in Figure 7 are obtained again while varying N_{POS} and keeping fixed $N_A = 1024$. As it is apparent also in this case the best results are achieved with a thinning factor near 2. Figure 5(b) reports the achieved array configuration that yields a SLL of −24.35 dB, occurring for $R = 13$. It appears that the combined use of two different types of subarrays and the constrained full center allows almost the same SLL of the SPED.

To show the results achievable by the WSF technique, introduced in the previous section, we start from subarrays of small size. To optimize the result, we use appropriate suitable grid of centers (Figure 8) and set a filled inner circle radius R of 14 elements. Starting with the subarrays with size $D = 4$ shown in Figure 6, we first obtain the thinned array of Figure 9(a). All the possible subarrays of size 8 achievable by merging adjacent smaller subarrays are shown in Figure 10. By applying the WSF technique, only subarrays from 1 to 4 are considered acceptable and we obtain the thinned array

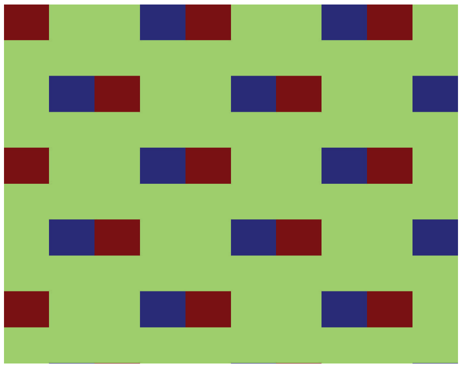


Figure 8: Used grid of centers.

Table 1: Comparison between thinned and filled arrays.

Scheme	Taper	
Circular thinned array (N_A = 1024)	No tapering	SLL = −24 dB
Circular filled array (N_A = 1024)	No tapering	SLL = −17 dB
Circular filled array (N_A = 1024)	Tapering	SLL = −24 dB
Rectangular thinned array (N_A = 2500)	No tapering	SLL = −23 dB
Rectangular filled array (N_A = 2500)	No tapering	SLL = −13 dB
Rectangular filled array (N_A = 2500)	Tapering	SLL = −23 dB

of Figure 9(b). Both the thinned arrays have a SLL of nearly −24.3 dB.

Similar results could be shown also for the rectangular case. As an example, the output of the constrained SPSD technique is shown in Figure 11. By using a thinning ratio $\eta = 2.7$ and constraining a full center of 343 fully active subarrays a SLL of −23.17 dB is achieved.

4.2. Comparison between Thinned and Filled Arrays. The cir- cular and rectangular thinned arrays in Figure 5(b) and Figure 11 are compared with the corresponding filled arrays, with the same number of active elements. In Table 1, we report the array characteristics.

Circular and rectangular filled arrays provide a pattern with a SLL of about −17 dB and −13 dB, respectively. To achieve the same SLL of the thinned array, a taper function must be applied so that the main lobe widens. The use of the thinned array is therefore effective in achieving the improved SLL with reduced effects on the main lobe width. This is apparent from Figures 12 and 13 where the main lobe width is reported for both thinned and filled arrays, for the circular and the rectangular arrays, respectively, being the u and v axes defined as follows:

$$u = \cos(\theta)\cos(\varphi) \qquad v = \cos(\theta)\sin(\varphi). \tag{7}$$

5. Adaptive Techniques for SPSD Based Thinned Arrays

If multiple receiving channels are available, connected to different antenna apertures, multi-channel adaptive techniques can be exploited to mitigate the effect of electromagnetic interferences and clutter returns for the thinned array. Consider the design of the subapertures to be connected to the independent receiving channels. With Constrained SPSD, we observe that the inner part of the array is always filled: this part presumably yields a nice main beam, with a slight increase of the beam width with respect to the one achieved if using the entire antenna. This central part can be connected to the main channel, while the external subarrays or groups of them can be eventually connected to a few auxiliary channels for a side-lobe canceller scheme, as shown in Figure 14.

Two different adaptive cancellation filters are applied. They are forced to achieve the quiescent design pattern in the absence of interference. The filters are mismatched optimum detector (MOD) filter [25, 26] and a generalized side-lobe canceller (GSLC) filter [27, 28].

The MOD filter is based on the definition of a mismatched target vector $\mathbf{s}_T$, selected as $\mathbf{s}_T = \mathbf{M}^{(0)}\mathbf{q}$, where $\mathbf{M}^{(0)}$ is the thermal noise covariance matrix and $\mathbf{q}$ is the desired subarray weight vector when only the thermal noise is present. The optimum filter to detect $\mathbf{s}_T$ is used:

$$\mathbf{w}_{\text{MOD}} = \mathbf{M}^{-1}\mathbf{s}_T, \tag{8}$$

where $\mathbf{M}$ is the covariance matrix of disturbance. In absence of jammer $\mathbf{M} = \mathbf{M}^{(0)}$ and $\mathbf{w}_{\text{MOD}} = \mathbf{q}$.

The GSLC filter is based on the generation of an orthogonal space formed by the matrix $\mathbf{B}$, which selects the subarrays of the auxiliary channels. Besides, we define the vector $\mathbf{t}$ which selects the main channel. The weight vector of the GSLC filter can be expressed as follows:

$$\mathbf{w}_A = -\left(\mathbf{t}^H\mathbf{M}\mathbf{B}\right)\cdot\left(\mathbf{B}^H\mathbf{M}\mathbf{B}\right)^{-1}. \tag{9}$$

The performance is evaluated by considering the signal to clutter ratio (SCR), defined as the ratio between the received useful signal power and the clutter power, and the signal to disturbance ratio (SDR), defined as the ratio between the received useful signal power and the disturbance power where disturbance is the sum of the thermal noise and the jammer.

To show the performance of the proposed algorithms the circular thinned array in Figure 9(b) is considered and it is compared to several circular filled arrays, with the same number of active elements. In Table 2, we show the tested array configurations. "A" schemes refer to the thinned array obtained with Constrained SPSD and WSF: therefore amplitude tapering is not used to achieve the desired level of SLL = −24 dB. The "C" schemes refer to the filled arrays: we considered also the possibility of applying amplitude tapering (a_i coefficients in Figure 14) both in transmission (TX) and in reception (RX). In Table 2, the SLLs of the TX and RX patterns are highlighted.

The performance of the adaptive schemes against electromagnetic interference is analyzed by simulating two different

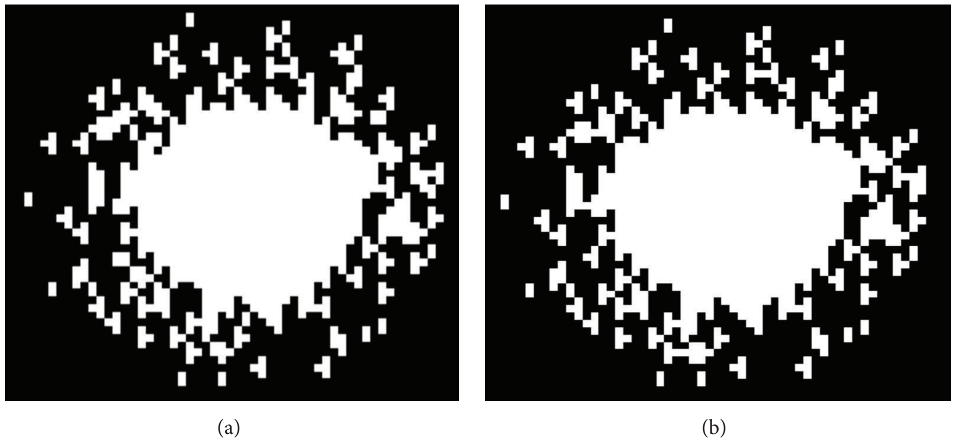


Figure 9: Achieved array configuration (a) with subarray of dimension 4 and (b) with subarray of dimension 8 after WSF.

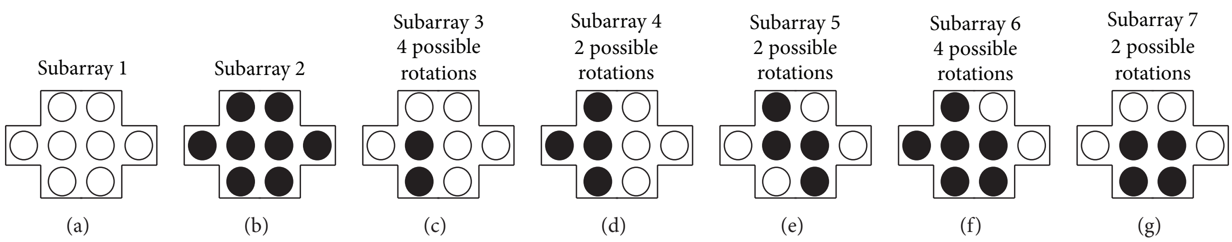


Figure 10: Possible Subarray type of size 8.

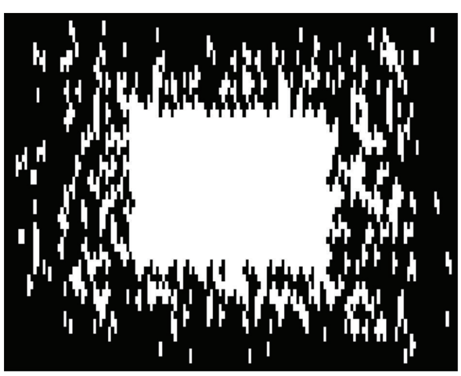


Figure 11: Rectangular thinned array using subarrays and grid of centers of Figure 6.

Table 2: TX/RX schemes.

Scheme	TX	RX
A1	No tapering (SLL = −24 dB)	No tapering (SLL = −24 dB)/MOD
A2	No tapering (SLL = −24 dB)	No tapering (SLL = −24 dB)/GSLC
C1	No tapering (SLL = −17 dB)	Tapering (SLL = −24 dB)/MOD
C2	Tapering (SLL = −24 dB)	Tapering (SLL = −24 dB)/MOD
C3	No tapering (SLL = −17 dB)	Tapering (SLL = −30 dB)/MOD

scenarios, with characteristics reported in Table 3. In the first a single jammer is considered with a variable DoA, while in the second scenario a second jammer impinges on the antenna from a fixed direction. For both jammers a Jammer to Noise Ratio (JNR) of 40 dB is considered.

Figure 15 shows the SDR as a function of the variable jammer azimuth angle obtained using the 16 outer subarrays as subapertures connected to auxiliary receiving channels and the remaining connected to the main receiving channel. The performance of the thinned arrays are comparable with the filled ones, except for the filled array with an amplitude tapering applied in transmission that lowers the transmitted power while reaching the thinned array SLL.

To evaluate the performance of the adaptive scheme against clutter returns, we simulated a scenario with the Constrained SPSD and WSF thinned antenna 20 m above earth surface, being the elevation steering angle 5°. In this case, clutter echoes are received by the first side-lobes of the antenna and the acquisition configuration could suffer from clutter since returns from the ground could share the same resolution cell of a potential target with comparable powers.

Figure 16 shows the SCR versus the target range. It is apparent that the thinned array yields better performance than the filled ones. The superiority of the thinned array against clutter is demonstrated when no attempt is made to

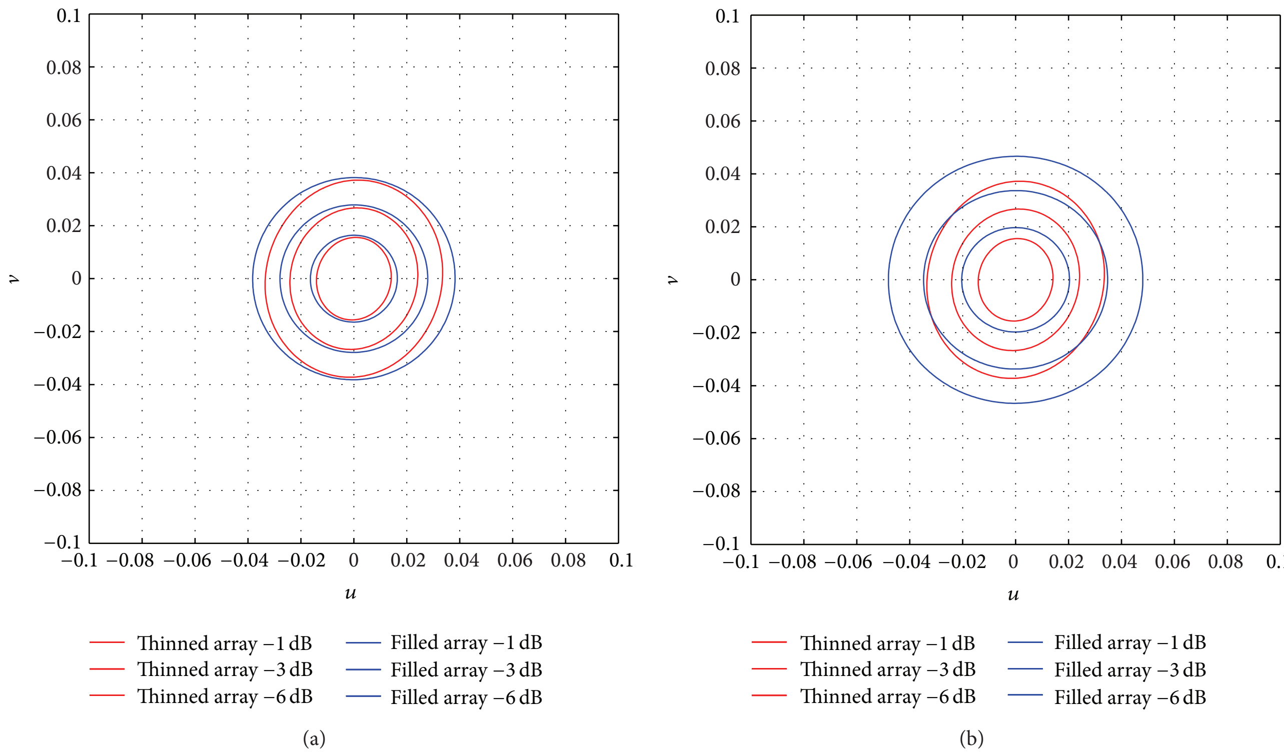


Figure 12: Comparison between the main lobe aperture of a thinned circular array and a filled array with the same number of active elements (a) without and (b) with amplitude tapering.

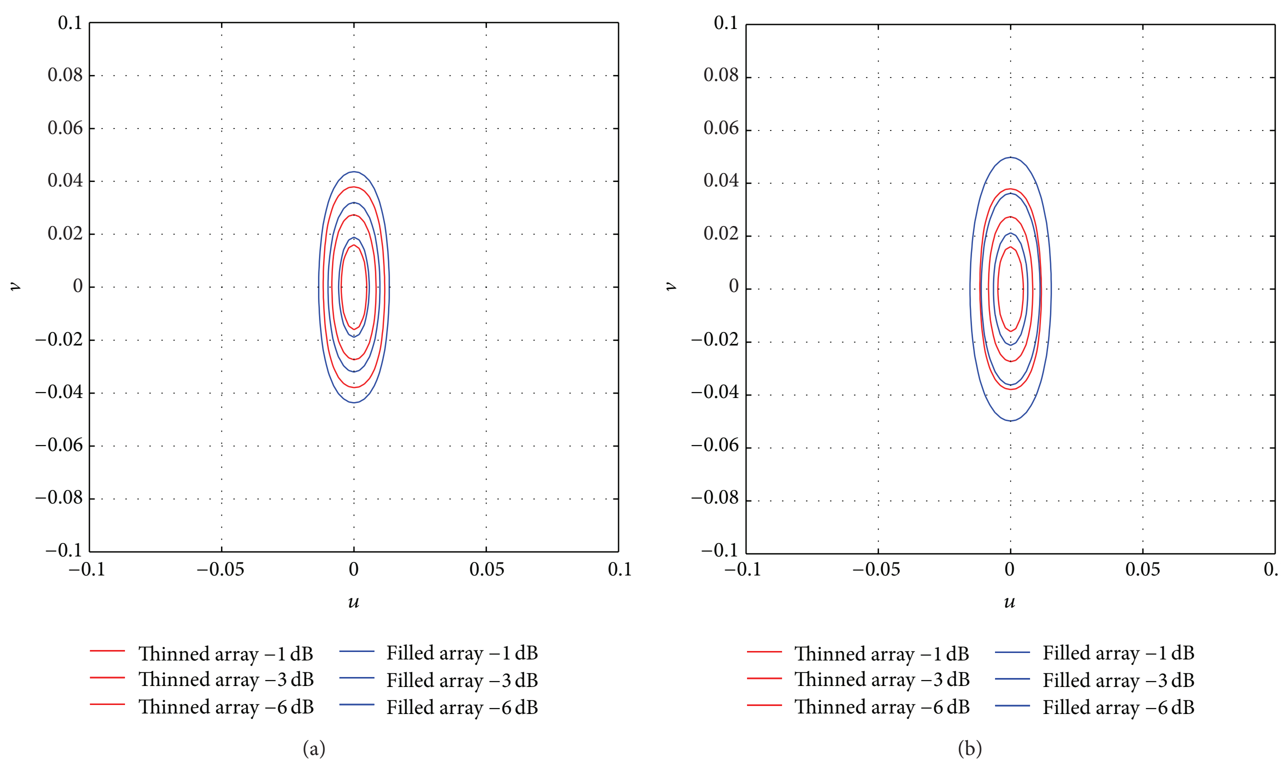
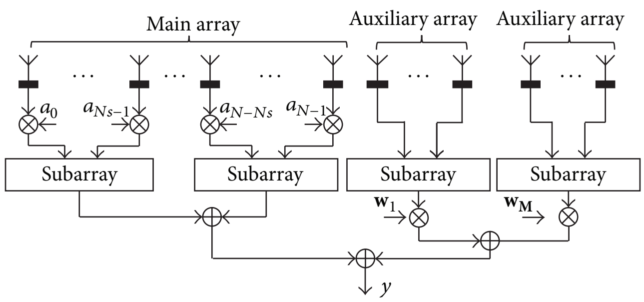


Figure 13: Comparison between the main lobe aperture of a thinned rectangular array and a filled array with the same number of active elements (a) without and (b) with amplitude tapering.

Table 3: Scenarios for the performance study.

Scenario	Target DOA (θ_T, φ_T)		jammer DOA (θ_j, φ_j)		JNR (dB)
1 jammer	$\theta_T = 5^\circ$	$\varphi_T = 0^\circ$	$\theta_j = 5^\circ$	$-60^\circ \leq \varphi_j \geq 60^\circ$	40
2 jammers	$\theta_T = 5^\circ$	$\varphi_T = 0^\circ$	$\theta_j = 5^\circ$	$\varphi_j = 10^\circ$	40
			$\theta_j = 5^\circ$	$-60^\circ \leq \varphi_j \geq 60^\circ$	40



Ns: subarray size

N: centre elements

M: number of auxiliary channels

Figure 14: Adaptive cancellation scheme.

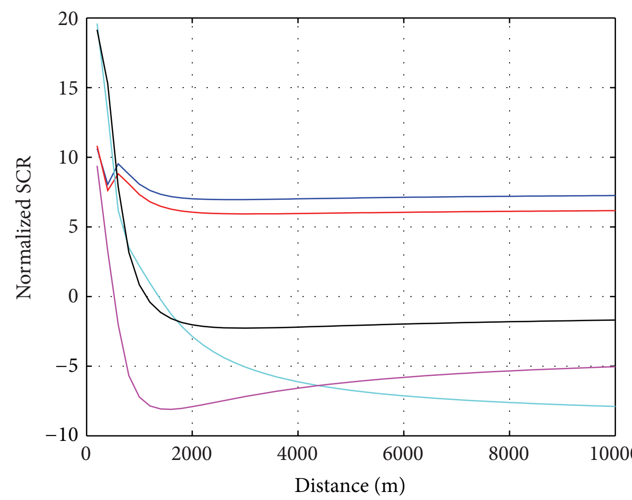


Figure 16: SCR versus target distance.

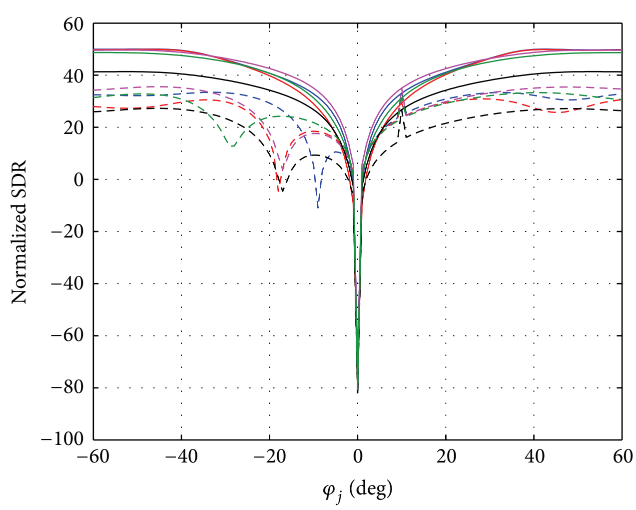


Figure 15: SDR versus jammer azimuth angle.

impose amplitude taper for further side-lobe suppression. This mainly depends on the fact that thinned arrays reach low SLL at clutter DoA without a significant increase of the main-lobe aperture, differently from the filled array cases where the reduction of the side-lobes using a tapering function is paid in terms of pattern main-lobe widening and gain reduction.

6. Conclusions

The SPED and SPSD optimization techniques and their Constrained versions have been proposed for the design of optimized element and subarray based planar thinned arrays using an assigned number of active elements and assuring a desired level of SLL. Since the SPSD technique operates effectively for a limited number of small-sized subarrays, a WSF approach has been introduced to increase the subarray size; this approach does not need a significant increase of the number of different subarray types and achieves comparable performance in terms of SLL. All the proposed techniques have been shown to be effective for practical planar thinned array design and adaptable to different planar array shapes. Moreover, possible adaptive solutions have been discussed suitable for SLC schemes based on the MOD and GSLC filters using the circular thinned arrays produced by the use of Constrained SPSD and WSF. An analysis has been conducted to compare the performance in terms of jammer and clutter cancellation of the thinned array with respect to several configurations of filled arrays with the same number of active elements using amplitude tapering to lower the SLL. The proposed comparative analysis showed that for the considered study cases the adaptive thinned array is able to yield better performance than the adaptive filled arrays when clutter cancelation is considered and has a comparable behavior in terms of jammer reduction.

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