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A new energy gradient-based model for LCF life prediction of turbine discs

Yunhan Liu[a], Shun-Peng Zhu[a, b, *], Zheng-Yong Yu[a], Qiang Liu[a]

[a]*Center for System Reliability & Safety, University of Electronic Science and Technology of China, Chengdu, 611731, P.R. China*
[b]*Department of Mechanical Engineering, Politecnico di Milano, Milan 20156, Italy*

Abstract

With continuous raising of thrust-weight ratio, low cycle fatigue (LCF) at high temperature is one of main failure modes for engine hot section components. Accurate life prediction of turbine discs has been critical for ensuring the engine integrity. According to this, a new LCF model through combining the energy gradient concept with critical distance theory is proposed for fatigue life prediction of turbine discs. In this paper, assuming that the processes of crack initiation and propagation in a LCF regime can be described by the cumulative strain energy. A relationship between the total strain energy in the fatigue process zone and the LCF life is explored. In particular, the energy parameters are weighted based on the energy gradient in the fatigue process zone. Using experimental data of GH4169 alloy at 650℃, a good agreement was achieved between model predictions and experimental results.

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Keywords: energy gradient; life prediction; low cycle fatigue; turbine disc; critical distance

1. Introduction

In the process of fatigue failure of engine components, fatigue crack usually initiates with the stress concentration in the structure especially notched region, like turbine disc grooves. The notch effect in engineering is one of the most common cause of high stress concentrations. It is worth noting that traditional fatigue life prediction methods based on the maximum stress or strain at the notch, is often inadequate to evaluate fatigue life of complicated mechanical structures with stress concentrations, which usually provides over-conservative life predictions.

To overcome this problem, several methods [1-4] have been developed to explore the effect of notch on fatigue life. Among them, the theory of critical distance (TCD), which evaluates the fatigue property based on effective stress/strain around the stress concentration [5, 6], has been employed for notch fatigue analysis because of its versatility. Recently, researches in [7-9] indicated that the TCD method can reasonably predict fatigue life of notched specimens made of titanium alloy under torsional loadings. In addition, an implicit gradient approach [10] is applied to V-notches and extended to other geometries like welded structures. Livieri et al. [11] combined the implicit gradient equation with the critical distance theory to predict high cycle fatigue life of U and V-notches specimens made of FeP04 steels.

However, most of these methods are mainly based on stress gradient in the stress field, which usually leads large scatters at the stress singularity in the vicinity of cracks or sharp notches [12]. In addition, considering only the stress is not enough to characterize the gradient effect on fatigue life. In this paper, in order to comprehensively consider the gradient change distribution of damage factors like the stress, strain or both. In particular, the notch of the component affects its fatigue process by influencing the energy gradient rather than the stress gradient, its fatigue life corresponds to the energy distribution. Until now, energy-based approaches

* Corresponding author. *E-mail address:* zspeng2007@uestc.edu.cn (S.P. Zhu)

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have been presented for low cycle fatigue (LCF) life prediction. These models can be mainly categorized as plastic strain energy density-based methods [13, 14] and total strain energy density-based methods [15, 16]. In this paper, a new model based on the total strain energy and critical distance theory is proposed for LCF life prediction of turbine discs. In particular, the fatigue life in a LCF regime is described by the cumulative strain energy which contributes to an effective damage accumulation zone.

2. Proposed energy gradient-based model

Most of notched fatigue researches are based on the stress gradient, which consider only the change of stress and cannot accurately characterize the complex relationship between stress and strain in the LCF process, especially in the local zone near the notch. According to this, this paper attempts to define and model the notch effect with a concept of strain energy gradient. How to define the concept of energy gradient which can reflect the effect of the stress/strain gradient effects encountered on the real structures is the key to determine the accuracy of a lifing model. Pluvinage et al. [17] defined the stress gradient χ of the elastic-plastic stress field as follows:

$$\chi = \frac{1}{\sigma_1(\theta=0,r)}\frac{d\sigma_1(\theta=0,r)}{dr} \tag{1}$$

According to the relationship between stress and energy, a similar function can be extended to define the energy gradient χ_W as:

$$\chi_W = \frac{1}{W_{O(x_0,y_0,z_0)}}\frac{\partial W_{xyz}}{\partial l} \tag{2}$$

where l is the distance between the point $P(x,y,z)$ in the effective damage process zone and the center $Q(x_0,y_0,z_0)$ of fatigue process zone. The zone's boundary will be discussed in Section 2.2, while the distance l is

$$l = \sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2} \tag{3}$$

In the cylindrical coordinate system, Equation (2) can be rewritten as

$$\chi_W = \frac{1}{W_{O(x_0,y_0,z_0)}}\frac{\partial W_{r\theta z}}{\partial l} \tag{4}$$

and

$$l = \sqrt{\left(\frac{r}{cos\theta}\right)^2+z^2} \tag{5}$$

$$\frac{\partial W_{r\theta z}}{\partial l} = \frac{\partial W}{\partial r}\frac{\partial r}{\partial l} = \frac{\partial W}{\partial r}\frac{l}{\sqrt{l^2-z^2}}cos\theta \tag{6}$$

In this study, a shape feature of a cylindrical bar with circumferential notch ($K_T = 3$) is used as example to show this energy gradient concept, where the influence of r in the bisector of notch is mainly investigated. To simplify, Equation (6) can be expressed as

$$\frac{\partial W_{r\theta z}}{\partial l} = \frac{\partial W}{\partial r} \tag{7}$$

Therefore, the energy distribution in the location shown in Figure 1 is used to build the energy gradient criterion in this paper. Based on the distribution of an energy gradient, the energy gradient effect can be quantified by introducing a weight function, as shown in Figure 1. Similarly, a relationship between the weighted cyclic strain energy density and LCF life can be established according to the Coffin-Manson equation as

$$\frac{1}{2L}\int \Delta W\varphi_{(\chi,r)}\,dR = 4\sigma_f^{'}\,\varepsilon_f^{'}\,\frac{(c-b)}{(c+b)}2N_f^{\,b+c} + \frac{\sigma_f^{'}\,(2N_f)^{2b}}{2E} \tag{8}$$

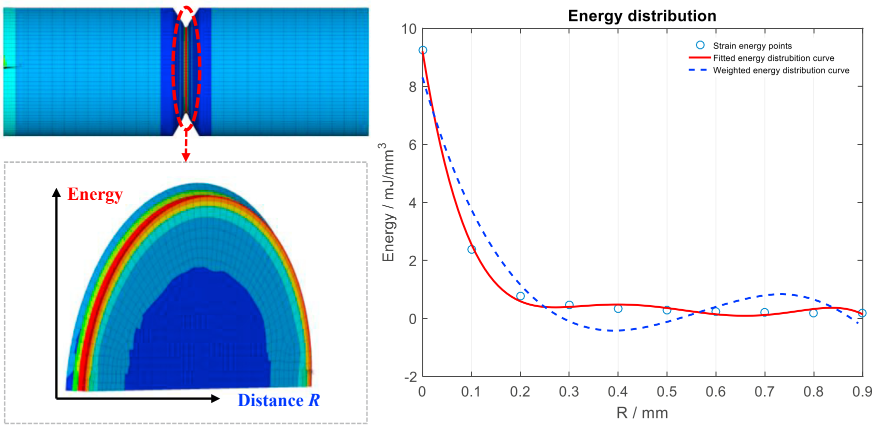

Figure 1. The location to calculate the energy

2.1 Calculation of the total strain energy

Various experimental results have shown that the hysteresis loop energy and the fatigue life can be expressed by a power law function [13, 18, 19]

$$\Delta W_p = \int \sigma_{ij}\,d\varepsilon_{ij} = a_0 N_f^{b_0} \tag{9}$$

Garud [20] extended the Morrow's uniaxial energy definition to multiaxial fatigue loading conditions. Assuming that the material satisfies the Masing's hypothesis, the Garud criterion is expressed as

$$\Delta W_c = \Delta\sigma\Delta\varepsilon^p\left(\frac{1-n'}{1+n'}\right) + \Delta\tau\Delta\gamma^p\left(\frac{1-n'}{1+n'}\right) \tag{10}$$

For non-proportional loadings, the cyclic plastic work can be calculated by

$$\Delta W_c = \Delta\sigma\Delta\varepsilon^p\left(\frac{1-n'}{1+n'}\right) + \xi\Delta\tau\Delta\gamma^p\left(\frac{1-n'}{1+n'}\right) \tag{11}$$

where ξ is a weighting factor. Garud [20] suggested a value of 0.5 for the shear strain energy.

In this analysis, the above-mentioned total strain energy can be expressed as:

$$\Delta W_c = \Delta W_p + \Delta W_e \tag{12}$$

Similar to the Garud's method, the weighting factor to the shear strain energy is given as 0.5 for non-proportional loading conditions. For simplify, a relationship between the weighted cyclic strain energy density and LCF life can be derived as:

$$\frac{1}{R}\int_{cycle}\Delta W\varphi_{(\chi,r)}\,dR = f(N_f) \tag{13}$$

where R is the effective zone radius; $\varphi_{1(\chi,r)}$ is the weighted function, two forms can be generally used to describe the energy gradient effect

$$\varphi_1(\chi,r) = 1 - |\chi|r \tag{14}$$

$$\varphi_2(\chi,r) = 1 - |\chi|(r/R) \tag{15}$$

2.2 Boundary condition to determine the radius of effective damage zone

It should be pointed out that the effective damage zone is an important part for the proposed model in Equation (13). In order to determine the radius of the effective fatigue damage zone, the critical distance theory is introduced into Equation (13) to consider the notch effect. The boundary applied to fatigue problems which contain stress concentration caused by the notch effect can be mainly divided into four kinds of characteristic length parameters [21]. In this analysis, according to the critical distance theory proposed by Haddad [22] based on the linear elastic fracture mechanics, the material characteristic length L can be defined as

$$L = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{\sigma_0}\right)^2 \tag{16}$$

where $\Delta K_{th}$is the threshold range of the stress intensity factor,σ_0 is the plain fatigue limit. Our case is very closely to the LM [23], the effective damage zone can reflect the damage of the notch in order to obtain the effective damage zone radius, the definition of this radius is given as:

$$\frac{1}{R}\int_{cycle}\Delta W\varphi_{(\chi,r)}\,dR = 4\sigma_f'\varepsilon_f'\frac{(c-b)}{(c+b)}2N_f^{b+c} + \frac{\sigma_f'(2N_f)^{2b}}{2E} \tag{17}$$

The relationship in Equation (17) can be fitted from fatigue tests of notch specimens. In this paper, the effective damage zone radius is 0.14mm.

3 Model application to turbine disc alloys

In this section, the proposed model is verified by using experimental data of turbine disc alloy GH4169 under multiaxial fatigue testing on notch specimens [24]. Material properties and test results are listed in Tables 1 and 2, respectively. When utilizing the proposed model in Equation (13) for life prediction, an elasto-plastic analysis of the notched specimen is conducted by using FE simulations with ANSYS 14.5, where the Ramberg-Osgood relation is introduced for stress-strain analysis together with the Chaboche plasticity model:

$$\varepsilon_a = \sigma_a/E + (\sigma_a/K)^{1/n'} \tag{18}$$

where K' is the cyclic hardening coefficient, n' is the cyclic strain hardening exponent.

Table 1 Mechanical properties of GH4169

Temperature	Tensile strength	Yield strength	Elongation at failure	E	K'	n'
650°C	1005*MPa*	965*MPa*	12%	153000 *MPa*	1950*MPa*	0.15

Table 2 Multiaxial fatigue results of GH4169

Specimens No.	Phase (°C)	Tensile strain /%	Torsional strain /%	Tested life /N
R1	90	0.297	0.410	2086
R2	90	0.395	0.553	425
R3	90	0.397	0.550	469
R4	0	0.281	0.392	871
R5	0	0.282	0.370	1076
R6	0	0.419	0.675	139
R7	45	0.354	0.479	642
R8	45	0.357	0.487	509

Moreover, the proposed model predictions are compared with that of Graud [20], *SWT* [25] and Fatemi-Socie (*FS*) models [26]. Model comparison results are shown in Figure 2. Note from the results obtained by using the weight function form 1 that the proposed model gives accurate predictions under the plastic strain energy dominated conditions. However, it gives conservative predictions under small total strain energy conditions. The results obtained by using the weight function form 2 are basically coincident and relatively stable, within the range of ± 2 life factors. Thus, the weight function 2 is chosen as the weight function

$$\varphi_1(\chi, r) = 1 - |\chi|(r/R) \tag{19}$$

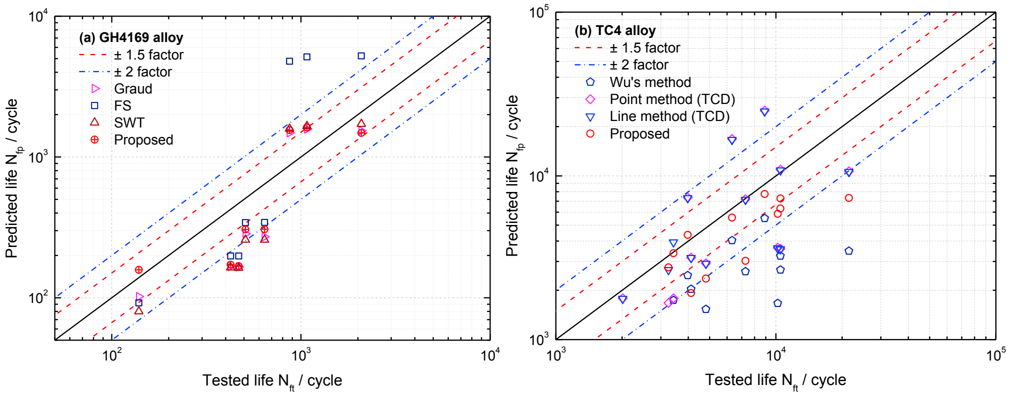

Figure 2 Life prediction and experimental life of (a) GH4169 and (b) TC4 alloys

For FS criterion, it should be pointed out that it predicts the life with large scatter when the plastic strain energy is high or low due to the changing of effective fatigue damage zone radius. This radius relates to the strain energy or life according to [27]. In order to prove the generality of the proposed method, experiments data of disc alloy TC4 [28] is also introduced for model validation and comparison, most of the proposed model prediction results are within the region of ± 2 life factors as shown in Figure 2(b). Though the proposed energy gradient-based model has been verified by GH4169 and TC4 notched specimens, more experimental data from different notch shapes, materials, load conditions are expected for further model validation.

4 Conclusions

The present work was performed to investigate an energy gradient based LCF assessment of turbine discs. The following conclusions can be drawn as

(1) A concept of energy gradient is proposed to explore the notch effect under complex loadings, which reflects the effects of notch and stress gradient on fatigue life.

(2) An effective damage zone is presented to define the zone which contributes to the fatigue process. The radius of this effective damage zone depends upon the material, stress ratio and loading conditions.

(3) The proposed energy gradient method can be utilized by three main steps: firstly, obtain the energy distribution by FE analysis, then calculate the energy gradient and the weighted energy value, and finally, combining with the critical distance theory to predict the fatigue life.

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