International Journal of Industrial Engineering Computations 2 (2011) 1–18

Contents lists available at GrowingScience

International Journal of Industrial Engineering Computations

homepage: www.GrowingScience.com/ijiec

# CONWIP card setting in a flow-shop system with a batch production machine

**Marcello Braglia[a], Marco Frosolini[a], Roberto Gabbrielli[b*], Francesco Zammori[a]**

[a]*Dipartimento di Ingegneria Meccanica, Nucleare e della Produzione, Università di Pisa, via Bonanno Pisano 25/b, Pisa, 56126, Italy*
[b]*Dipartimento di Energetica, Università di Pisa, Largo L. Lazzarino, 1, Pisa, 56126, Italy*

**A R T I C L E I N F O**

*Article history*:
Received 10 August 2010
Received in revised form
15 October 2010
Accepted 16 October 2010
Available online 16 October 2010

Keywords:
CONWIP
pull systems
flow-shop system
batch production
card setting

**A B S T R A C T**

This paper presents an analytical technique to determine the optimum number of cards to control material release in a CONWIP system. The work focuses on the card setting problem for a flow-shop system characterised by the presence of a batch processing machine (e.g. a kiln for long heat treatment). To control production, two different static approaches are developed: the first one is used when the bottleneck coincides with the batch processing machine and the second one is proposed when the bottleneck is another machine of the flow shop. In both contexts, by means of the appropriate model, one can optimize the performance of the flow-shop by maximizing the throughput and keeping the work in process at a minimum level. Numerical examples are also included in the paper to confirm the validity of the models and to demonstrate their practical utility.



## Nomenclature

B = maximum batch size

$B_R$ = actual batch size when the batch machine is not the bottleneck

K = index for the batch machine

M = index for the critical machine. In model #1 the critical machine is the one with the longest processing time apart from that of the batch machine. In model #2 the critical machine coincides with the bottleneck machine.

$N_C$ = optimum card number

N = number of machines in the line

S = overall cycle time per item except the batch process and the sub-line bottleneck

* Corresponding author. Tel.:Fax: + 39-050-2217138, fax: + 39-050-2217150
E-mail: r.gabbrielli@ing.unipi.it (R. Gabbrielli)


doi: 10.5267/j.ijiec.2010.07.004

$S^*$ = critical value of S

$T_i$ = processing time of the *i*-th machine

$T_I$ = idle time of the batch machine when this is not the bottleneck

$T^*$ = time necessary for the sub-line to produce B items

$T_K$ = batch machine processing time

$T_M$ = processing time of the critical machine

## 1. Introduction

CONWIP stands for *Constant Work-In-Process* (Spearman *et al.*, 1990) and designates a pull control strategy that limits the total number of jobs allowed in the production system at the same time. This is usually obtained with the use of cards that are attached to the jobs that are queuing at the first station of the process. After a job has been processed in the last station, the card is detached and sent back to the beginning of the system, where it is attached to the first job of the queue lacking of a card. No job is allowed in the system without a card, therefore the overall amount of work in process (WIP) in the production line is equal to the number of available cards. In other words a CONWIP system can be seen as a whole processing system enveloped in a single kanban cell: as soon as the end consumer collects an item from the finished goods inventory, the first machine of the chain is authorized to produce another part.

Sometimes, to further reduce the amount of WIP that accumulates in the system, it can be advisable to release cards immediately after the bottleneck machine (BNM). This strategy is particularly useful when there is a single and stable BNM and most of the downstream machines are subjected to long downtimes. This approach avoids the problem of cards piling up in front of the failed machine and the consequent starvation of the bottleneck (Spearman *et al.*, 1990; Hoop & Spearman, 2000).

The card setting problem (i.e. the definition of the optimal number of card to be used) is a fundamental, yet critical activity that one has to face to develop an efficient CONWIP control system. Indeed, since the optimal condition is reached when the throughput is at maximum level and the WIP is kept at minimum level, the actual number of cards largely affects the performance of a CONWIP system.

Literature in the subject matter is extensive and a comprehensive review concerning the card setting problem can be found in the work by Framinan *et al.* (2003). Specifically the card setting problem is generally tackled using mathematical models that can be broadly classified as static or dynamic approaches.

Following a static approach, the hypothesis is made that the number of cards cannot be changed during production and, given a certain throughput to be obtained, WIP minimization is considered as the main performance parameter. Several static models for the optimization of the flow-shop performance were developed in technical literature. Each model addresses a specific configuration of flow-shop systems, such as the presence of “machine outages” (Hoop & Spearman, 1991; Lavoie *et al.*, 2010), “set-up times” (Chang & Yih, 1994) and “reworks” (Duri *et al.*, 2000). The optimal solution is generally obtained using three basic approaches: queuing models (Duri *et al.*, 2000; Ryan *et al.*, 2000; Al-Tahat & Rawabdeh, 2008; Li *et al.*, 2010), linear programming (Framinan *et al.*, 2001; Cao & Chen, 2005), simulation analyses (Bonvik *et al.*, 1997; Yang *et al.*, 2007; Sharmaa & Agrawalb, 2009; Lavoie *et al.*, 2010).

Conversely in the dynamic approach, one assumes that the number of cards does not remain fixed, but can shift inside a specified range of variation. Interesting dynamic models are reported in the papers of Hoop and Roof (1998), Tardiff and Maaseidvaag (2001) and Framinan *et al.* (2006). The first paper proposes a statistical throughput control procedure (STC), that permits to dynamically adjust the number of cards in a make-to-order CONWIP system. The second paper introduces an interesting procedure to control the release of additional card in a make-to-stock CONWIP system. In this case, once the level of the buffers drops below an admissible threshold value, extra cards are temporally released into the production system, until this emergency situation is solved. In a similar way the third paper proposes to add/remove cards from the system if the throughput rate is lower or higher than a target value, respectively.

An alternative field of research consists in the application of CONWIP to different productive layouts. As well known, the CONWIP system was originally designed for flow-shop only (Spearman *et al.*, 1990) and the implementation of the CONWIP for other production system may not be easy in practice. Several examples can be found in the literature, including the application of CONWIP to job shop facilities (Ryan *et al.*, 2000), to systems characterized by alternative production routes (Golany *et al.*, 1999), to independent lines that share common machines (Huang *et al.*, 1998) and to merging/assembly lines (Duenyas, 1994; Cao & Chen, 2005). Hence, the CONWIP system can be considered as an effective order release strategy capable to assure good operating performances in terms of throughput and WIP in many production contexts (Huang *et al.*, 1998; Spearman *et al.*, 1990).

The present paper positions itself among the literature contributions that face the card setting problem with a static approach. Specifically the object is to determine the optimal number of card to optimize the performance of a particular flow-shop system characterized by the presence of an un-interruptible batch processing machine (BPM) with a long cycle time (i.e. kilns for long heat treatments), where more than one item can be processed, simultaneously.

In literature, the application of the CONWIP control system to such a productive environment has been scarcely considered. In Fowler *et al.* (1997), to evaluate the operating practices of a high-volume, multiple-product semiconductor fab with CONWIP as order release policy, a simulation based analysis is used. The process is characterized by a BPM (with a batch size of three lots) and results demonstrate how, in this particular instance, releasing lots in groups of three at constant time intervals is much better than a standard CONWIP policy. In a similar way Tay *et al.* (2002) used a simulative approach to compare alternative lot release rules for a micro electromechanical systems manufacturing company, characterized by two parallel BPMs. Also in this case the CONWIP did not result the best policy for that kind of production process.

Note that in the above mentioned papers the CONWIP configuration was not optimized and the authors did not present any mathematical methods to define the correct number of cards. Hence, the obtained results cannot be generalized and they rather justify the need for a more comprehensive investigation concerning the efficiency of a CONWIP strategy for lot order release in systems with BPMs.

It is also important to note that the presence of a BPM makes the card control a non standard procedure, because a careful scheduling of the arrivals of the items at the BPM buffer is required. This scheduling must ensure that at the planned start of the BPM, the number of items queuing in front of it is not lower than the optimum batch size needed to maximize the throughput of the system. Otherwise, this would lead to severe and unacceptable loss of productivity.

Scheduling and production control of a system with BPMs have been largely studied by many authors and some meaningful contributions are briefly reviewed in the following. In Luhl *et al*. (1997) a mathematical optimization model for a process with machines requiring significant setup times was presented, and a solution methodology based on a combination of Lagrangian relaxation, dynamic programming, and heuristics was developed. In Kim *et al.* (1998), a simulation approach for lot release control, mask scheduling, and batch scheduling in semiconductor wafer fabrication facilities was proposed. Neale & Duenyas (2000) proposed algorithms to determine the optimal policies with the minimization of the average time that jobs spend in a system consisting of a BPM and one or more unit-capacity machines in tandem. In Monch *et al.* (2005), scheduling jobs with incompatible families and unequal ready times on parallel batch machines has been solved via a genetic algorithm. In Monch *et al.* (2006) the same problem has been faced using inductive decision trees and neural networks from machine learning. A multi-objective genetic algorithm was used by Kashan *et al.* (2010) for the scheduling of jobs with non-identical sizes on a single BPM with the simultaneous minimization of makespan and maximization of tardiness.

However, none of these contributions analyzed scheduling and production control of a CONWIP system. This topic is addressed in the present paper, which presents two static analytical models to define an effective way to optimise the number of cards in a flow-shop with a single BPM. The first model can be used when the bottleneck coincides with the BPM and the second one is proposed when the bottleneck is another machine of the flow shop.

## 2. The analytical model for the card number setting

The production plant considered here consists of a flow-shop system characterised by the presence of a single BPM. Each machine in the line processes one item at a time with the exception of one station that can load and process a batch larger than one. A classical example is a production line including a kiln needed to anneal materials in special atmosphere after a cold drawing process.

The best operative conditions are reached when the BNM works at full capacity and the WIP is kept at minimum level. The different sizes of batches in the machines can promote problems in the correct synchronization of production flow. In order to balance the workloads and to optimize a similar production system, the implementation of the CONWIP can be considered as effective approach. Therefore, the primary task is to set the optimum card number, which corresponds to the minimum WIP with the maximum throughput.

In this paper, two analytical models to evaluate the optimum card number for this kind of production plant are developed. The proposed models investigate the card setting when the BPM is the bottleneck or not, respectively.

### *2.1 Case 1: the card setting model when the batch machine is the bottleneck*

Most batch processes, as annealing in kilns with a controlled atmosphere, are characterised by long processing time, even more than two days. Moreover, to avoid a modification of the controlled atmosphere, this type of operation cannot be interrupted to add further items in the kiln. Therefore, in order to reach the maximum throughput it is necessary to schedule production in a way that the BPM can always be loaded with the maximum batch. Thus, a whole batch must be ready in the buffer upstream the BPM just before it completes its current working. Otherwise, if BPM loads a batch smaller than the maximum one, the performance rate of the process is affected by a consistent capacity loss. Specifically this loss equals the product between the processing time and the difference between the maximum and the actual batch. Furthermore, due to the long processing time of the BPM, generally it is not convenient to delay the start of the process waiting for late items in order to start at full capacity load. It may transpire that a whole process should be removed from the weekly scheduling if the scheduled starts are excessively delayed.

Owing to these issues, the hypotheses of the first model to set the optimum card number are as follows:

(1) the batch machine has a capacity load of $B$ items and it is the bottleneck;

(2) according to the standard CONWIP implementation, cards are released at the end of the process in order to allow the entrance of a new item into the line (Fig. 1);

(3) the production mix consists of only one kind of item;

(4) the batch operation can be placed anywhere in the production schedule;

(5) there is no limit to the number of machines in the flow-shop production system;

(6) the conwip card number is constant;

(7) the market for the item is unlimited, so that all products can be sold and more throughput is desirable for the system (Hoop and Spearman, 2000);

(8) all processing times are assumed to be deterministic;

(9) all machines are assumed to work at their full efficiency (without any breakdowns);

(10) the time for material handling is neglected.

The last three assumptions are made for the sake of simplicity.

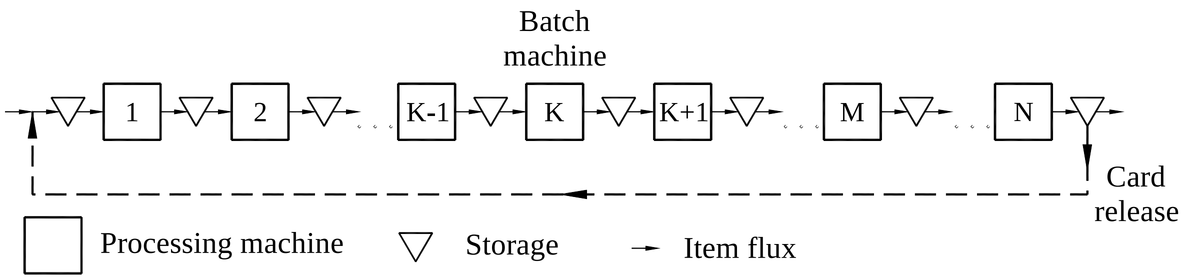


**Fig. 1.** CONWIP system in the flow-shop with *N* machines, where batch machine is labelled with *K*

As clearly shown in Fig. 1, without loss of generality, the BPM, which is also the BN of the system, has been located in the *K*-th position of the flow shop. Therefore, after an operation is performed at the *K*-th machine (i.e. the BPM), *B* items are released to the following operations ((k+1)-th, (k+2)-th, ... N-th) (Fig. 1). To assure the availability of *B* items in the buffer upstream the *K*-th machine, an equal number of items must leave the system. In this way, *B* cards are released and *B* items are allowed into the productive line and can be processed by the equipment installed upstream of the BPM. Note that, since the number of items is equal to the number of cards, it is admissible to talk in terms of cards instead of items and, doing so, the system can be considered as a closed queue.

Now, let us consider the card linked to the *B*-th item of the batch released by the BPM. This card will reach the buffer upstream of the BPM when a time interval ($T^*$) is equal to (Fig. 2):

$$T^* = B \cdot T_M + S, \tag{1}$$

where:

$$S = \sum_{\substack{i=1 \\ i \neq M, K}}^{N} T_i \ ,$$

Eq. (1) can be proved by considering that (Fig. 2) once the batch is released by $K$-th machine, it is possible to assimilate the production line to a flow shop composed by only ($n$-1) machines with a specific bottleneck (i.e. machine $M$). Therefore the production rate depends exclusively on machine $M$ and it is equal to $1/T_M$. As a consequence the time ($T^*$) required by this sub flow-line to manufacture $B$ items is given by the summation of the following four terms:

- the time needed by the machine $M$ to manufacture $B$ items, that is equal to $B \cdot T_M$,
- the time needed by the first item of the batch to reach the machine $M$, that is equal to $\sum_{i=K+1}^{M-1} T_i$,
- the time needed by the last item of the batch to exit the system after being manufactured by the machine $M$, that is equal to $\sum_{i=M+1}^{N} T_i$,
- the time needed by the last item of the new batch to reach the buffer of the BPM ($K$) once entered the system (thanks to the card release), that is equal to $\sum_{i=1}^{K-1} T_i$.

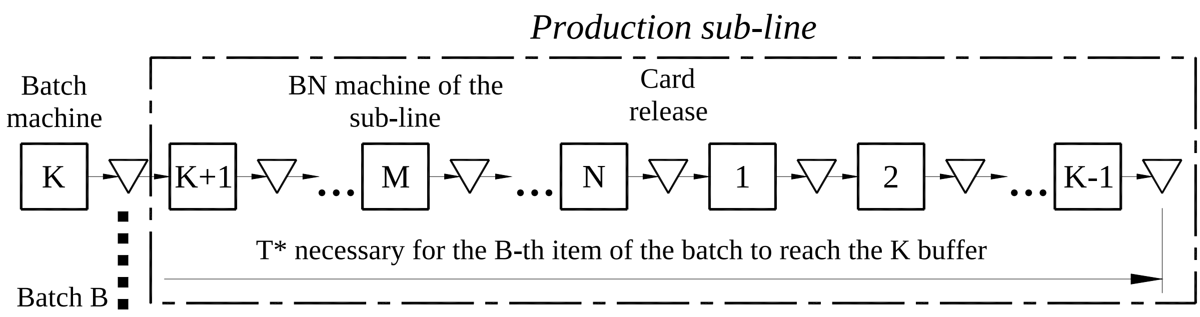


**Fig. 2.** Production sub-line with its specific bottleneck when the batch releases

It is evident that the minimum number of cards needed to assure the full utilisation of $K$-th machine can never be lower than twice the batch size (i.e. $2B$). In this case, just before the end of the batch process, $B$ items are being processed by $K$-th machine and other $B$ items are stored in the buffer of the $K$-th machine and it is ready to be processed. This condition can be satisfied only if the time taken by $B$ cards to close the loop and reach the $K$–th buffer, is lower than $T_K$. From Eq. (1) we have

$$T^* = B \cdot T_M + S < T_K. \tag{2}$$

If the Eq. (2) is not satisfied it will be necessary to add one or more cards in order to assure the maximum production rate of the system. In particular, the limit condition is equal to:

$$T^* = B \cdot T_M + S = T_K, \tag{3}$$

$$S = S^* = T_K - B \cdot T_M. \tag{4}$$

Note that $S^*$ represents the capacity gap between the production capacity of the BPM and that of machine $M$. Therefore, Eq. (4) defines an important threshold value for the overall cycle time of the other equipment. When Eq. (2) is not satisfied, one additional card will be sufficient to assure the maximum throughput of the line if the time taken by the ($B$-1)-th card to close the loop is lower than $T_K$ (Fig. 3).

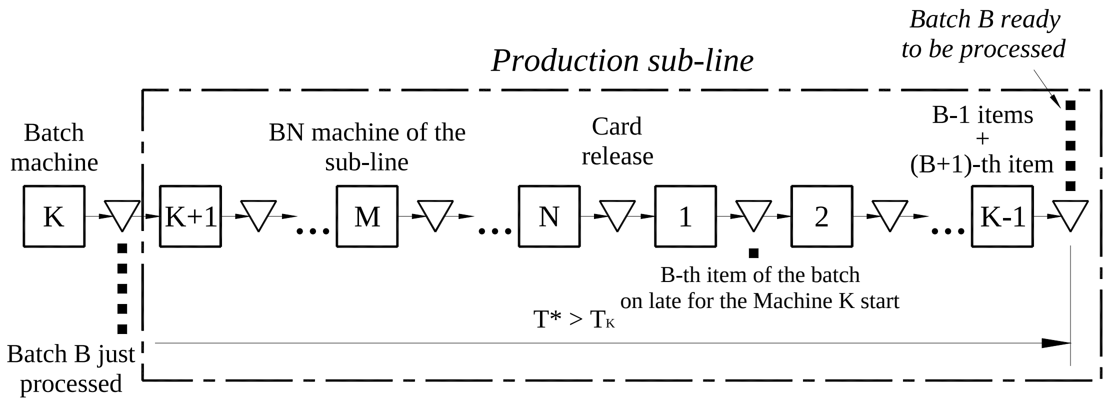


**Fig. 3.** Production sub-line with its specific bottleneck when the optimum card number is equal to 2·B +1

This is because the last *B*-th card takes more than $T_K$ to close the loop and it is still on the line when the BPM starts a new production cycle. Therefore, the addition of one card assures that the buffer contains the required *B* items at starting time of the BPM.

From Eq. (1), Eq. (2) and Eq. (4), we have:

$$S < T_K - (B\text{-}1)\cdot T_M = S^* + T_M \tag{5}$$

As a consequence, 2·*B*+1 cards are sufficient if *S* belongs to the following interval:

$$[S^*; S^* + T_M[ \tag{6}$$

To generalize the previous result it is necessary to add *X* cards (*X* < *B*), when the (*B*-*X*)-th card is able to reach the BPM buffer in a time interval lower than $T_K$. From Eq. (1) and Eq. (2), we have:

$$S < T_K - (B\text{-}X)\cdot T_M = S^* + X\cdot T_M. \tag{7}$$

Thus, $2\cdot B+X$ cards are sufficient if *S* belongs to the following interval:

$$[S^* + (X-1)\cdot T_M; S^* + X\cdot T_M[ \tag{8}$$

Note that one additional card is sufficient to cover an increment of *S* to be equal to $T_M$. This condition does not hold if *B* cards, or an integer multiple of *B*, have to be added. In this case, the WIP is equal to 3*B* items/cards. Thus, just before the operation of the BPM ends, *B* items are processed on BPM; *B* items are stored in the buffer of the BPM, while a complete batch of *B* items is still along the “sub-line” (Fig. 4).

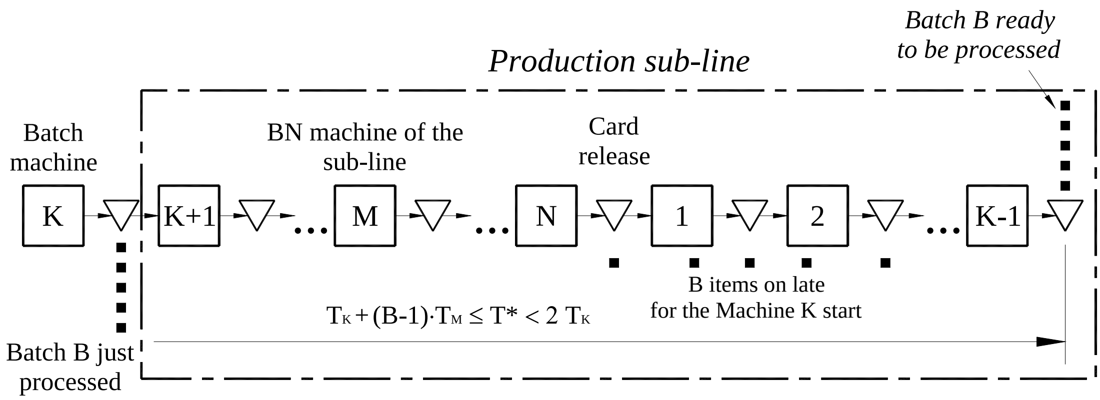


**Fig. 4.** Production sub-line with its specific bottleneck when the optimum card number is equal to 3·B

Therefore to assure the full capacity of the system, the *B*-th card of the batch still on the “sub-line” must reach the BPM in a time interval lower than $2 \cdot T_K$. Consequently the following condition must be satisfied:

$$S < 2 \cdot T_K - B \cdot T_M = S^* + T_K \tag{9}$$

As a result, $3 \cdot B$ cards are sufficient if $S$ belongs to the following interval:

$$[S^* + (B-1) \cdot T_M; S^* + T_K [ \tag{10}$$

It is important to note that, in this case, the actual increment of $S$ is equal to $S^* + T_M$. In summary, the general formula for the evaluation of the optimum card number ($N_C$) is as follows:

$$\begin{cases} N_C = 2 \cdot B & \text{if} \quad S \in [0; S^*[, \\ N_C = 2 \cdot B + R \cdot B + X & \text{if} \quad S \in [S^* + R \cdot T_K; S^* + R \cdot T_K + (B-1) \cdot T_M[, \\ N_C = 2 \cdot B + (R+1) \cdot B & \text{if} \quad S \in [S^* + R \cdot T_K + (B-1) \cdot T_M; S^* + (R+1) \cdot T_K[ , \end{cases} \tag{11}$$

where:

$$R = \left\lfloor \frac{S - S^*}{T_K} \right\rfloor,$$

$$X = \left\lfloor \frac{S - S^* - R \cdot T_K}{T_M} + 1 \right\rfloor.$$

The correspondent intervals are as follows:

$N_C = 2 \cdot B$ if $S \in [0; S^*[$,

$N_C = 2 \cdot B + 1$ if $S \in [S^*; S^* + T_M[$,

$N_C = 2 \cdot B + 2$ if $S \in [S^* + T_M; S^* + 2 \cdot T_M[$ ,

…,

$N_C = 2 \cdot B + (B-1)$ if $S \in [S^* + (B-2) \cdot T_M; S^* + (B-1) \cdot T_M[$,

$N_C = 3 \cdot B$ if $S \in [S^* + (B-1) \cdot T_M; S^* + T_K[$,

$N_C = 3 \cdot B + 1$ if $S \in [S^* + T_K; S^* + T_K + T_M[$,

…,

$N_C = Y \cdot B + Z$ if $S \in [S^* + (Y-1) \cdot T_K + (B-Z) \cdot T_M; S^* + (Y-1) \cdot T_K + (B-Z+1) \cdot T_M[$,

…,

$N_C = L \cdot B$ if $S \in [S^* + (L-1) \cdot T_K + (B-1) \cdot T_M; S^* + L \cdot T_K[$, where $Y \in N$, $L \in N$ and $Z \in [1, B\text{-}1]$ are generic numbers of cards.

When the maximum batch is reduced to a single item ($B=1$), the production line is simplified into a traditional flow-shop system and each machine of the line processes one item at a time. Since no hypotheses are made regarding the minimal size of the batch, all the previous considerations are valid. It is important to note that in this case the addition of one card is always sufficient to cover an increment of $S$ to be equal to $T_K$. Thus, the width of the interval, to which $S$ belongs, remains constant. This is due to the fact that the card number is always an integer multiple of the batch, which is equal to one. On the basis of this consideration the analytical formula to define the optimum card number for a standard flow-shop can be re-arranged in the following way:

$$\begin{cases} N_C = 2 & \text{if} \quad S \in [\,0; S^*[, \\ N_C = 2 + \left\lfloor \dfrac{S - S^*}{T_K} \right\rfloor + 1 & \text{if} \quad S \geq S^*. \end{cases} \tag{12}$$

The correspondent intervals are as follows:

$N_C = 2$ if $S \in [0; S^*[$,

$N_C = 3$ if $S \in [S^*; S^* + T_K[$,

…,

$N_C = Z$ if $S \in [S^*+(Z-1) \cdot T_K; S^* + Z \cdot T_K[$.

Note that the optimal card number obtained with equation (12) coincides exactly with the critical WIP as the well-known Little's law for a zero-variability flow-shop system (Hoop and Spearman, 2000):

$WIP = TH \cdot CT,$

where $TH$ and $CT$ are the throughput of the bottleneck machine and the overall cycle time, respectively. This fact confirms the validity of the method of this paper.

*2.2 Case 2: the card setting model when the batch processing machine is not the bottleneck*

In the second model, the production capacity of the BPM (which is still in position $K$) exceeds the production capacity of one or more machines in the productive process. With respect to the previous model, the following hypotheses are added: (*i*) the bottleneck does not coincide with the BPM; (*ii*) the BNM can be placed anywhere in the production process (i.e. upstream or downstream the batch operation).

In this case, the following equation is satisfied:

$$T_K / B < T_M, \tag{13}$$

where the notation is remained the same as the previous model, with the exception of $T_M$, which now denotes the bottleneck of the whole production line.

Although the production capacity of the BPM exceeds that of the machine $M$ (i.e., the actual bottleneck), this extra capacity can be removed by reducing to $B_R$ (with $B_R < B$) the size of the fixed batch processed by the BPM. To this aim one has to set $B_R$ to the following value:

$$B_R = \left\lceil \frac{T_K}{T_M} \right\rceil. \tag{14}$$

According to Eq. (14), to align its throughput with that of the machine $M$, the BPM manufactures a constant number $B_R$ of items (lower than its maximum capacity $B$) and remains idle for a time ($T_I$) for each productive cycle. In particular from Eq. (13) and (14) it follows that $T_I$ is equal to:

$$T_I = B_R \cdot T_M - T_K \tag{15}$$

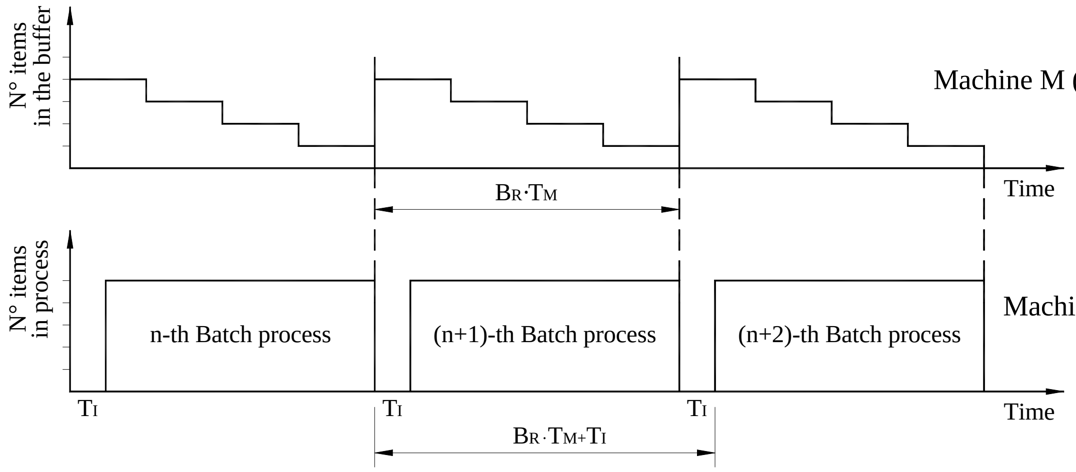


**Fig. 5.** Balance of work capacity between machine $M$ and machine $K$ (supposing that machine K is just upstream the machine $M$

As for the previous case, the minimum number of cards is equal to $2 \cdot B_R$ and one can get the maximum throughput using $2 \cdot B_R$ cards provided that the following condition is satisfied:

$$S < T_I. \tag{16}$$

Indeed, the time taken by the last item/card of the batch (released by BPM) to close the loop and reach the $K$-th buffer is equal to:

$$B_R \cdot T_M + S. \tag{17}$$

Moreover, the time interval from the end of the $n$-th batch process to the beginning of the ($n$+2)-th batch process (Fig. 5) is equal to:

$$B_R \cdot T_M + T_I. \tag{18}$$

Consequently, by reducing the number of items that can be possibly loaded by the BPM, the resulting flow-shop system can be seen as characterized by two bottleneck machines. These are the BNM (i.e. the machine in $M$-th position*)* and the BPM (i.e. the machine in the $K$-th position): the first one has a nominal processing time which is equal to $T_M$ and the second one has a fictional processing time which is equal to:

$$B_R \cdot T_M = T_K + T_I. \tag{19}$$

Thanks to this analogy, the set of equations needed to optimize the system can be directly derived from the Case 1, by simply considering the extra time (i.e. $T_I$ ) allowed to the last item of the batch $B_R$ to close the loop. On the basis of these considerations, the corresponding intervals of card number $N_C$ are as follows:

$N_C = 2 \cdot B_R$ if $S \in [0; T_I[$,

$N_C = 2 \cdot B_R + 1$ if $S \in [T_I; T_I + T_M[$,

…,

$N_C = 2 \cdot B_R + X$ if $S \in [T_I + (X\text{-}1) \cdot T_M; T_I + X \cdot T_M[$.

where $X$ is a generic number of added cards and belongs, obviously, to $N$.

In a generic case, the formula for the optimum card number $N_C$ evaluation is as follows:

$$\begin{cases} N_C = 2 \cdot B_R & if \quad S < T_I , \\ N_C = 2 \cdot B_R + \left\lfloor \dfrac{S - T_I}{T_M} + 1 \right\rfloor & if \quad S \geq T_I . \end{cases} \tag{20}$$

It is interesting to note how the main difference of this model with respect to the previous one is that the rate at which cards are released does not depend on the BPM. Indeed this machine does not process a full batch and can stay idle to align its production capacity to that of the BN of the line.

Also note that when the batch $B$ is equal to one this model is equivalent to the model of Eq. (12).

## 3. Numerical example

In order to validate the proposed models and to outline the performance in controlling the material flow in a flow-shop with a batch machine, numerical tests are performed in this section.

### *3.1 The batch processing machine K is the bottleneck*

In this section the analytical model reported in section 2, when the BPM is the bottleneck, is numerically tested. The flow shop production system considered here is constituted by four machines as shown by Fig. 6.

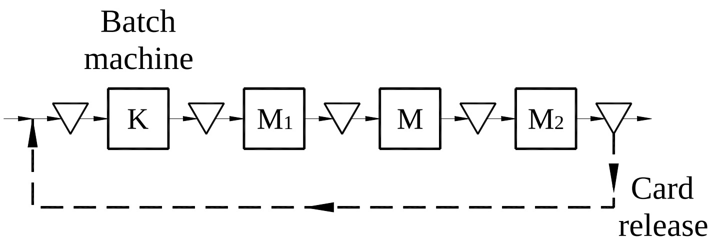


**Fig. 6.** Flow-shop with the CONWIP system considered in the numerical example

Two configurations of the production system are taken into account to show the behaviour of the system when the card number is equal to $2 \cdot B$ and $(2 \cdot B+1)$, respectively.

#### *3.1.1 Card number equal to 2·B*

The processing times of the four machines are as follows:

$T_K$: 100 min $T_{M1}$: 4.5 min $T_M$: 30 min $T_{M2}$: 5 min

The batch size $B$ is equal to 3 items.

In accordance with Eq. (11), the minimum number of cards required to maximize the throughput of the system is equal to 6.

$S^*$ is equal to:

$$S^* = T_K - B \cdot T_M = 10\ min.$$

The actual value of $S$ is equal to:

$$S = T_{M1} + T_{M2} = 9.5\ min.$$

S results evidently lower than $S^*$ thus $2 \cdot B$ cards are the minimum required.

The Gantt diagram of the production system together with the stock level of the machine $K$ buffer is summarized in Fig. 7.

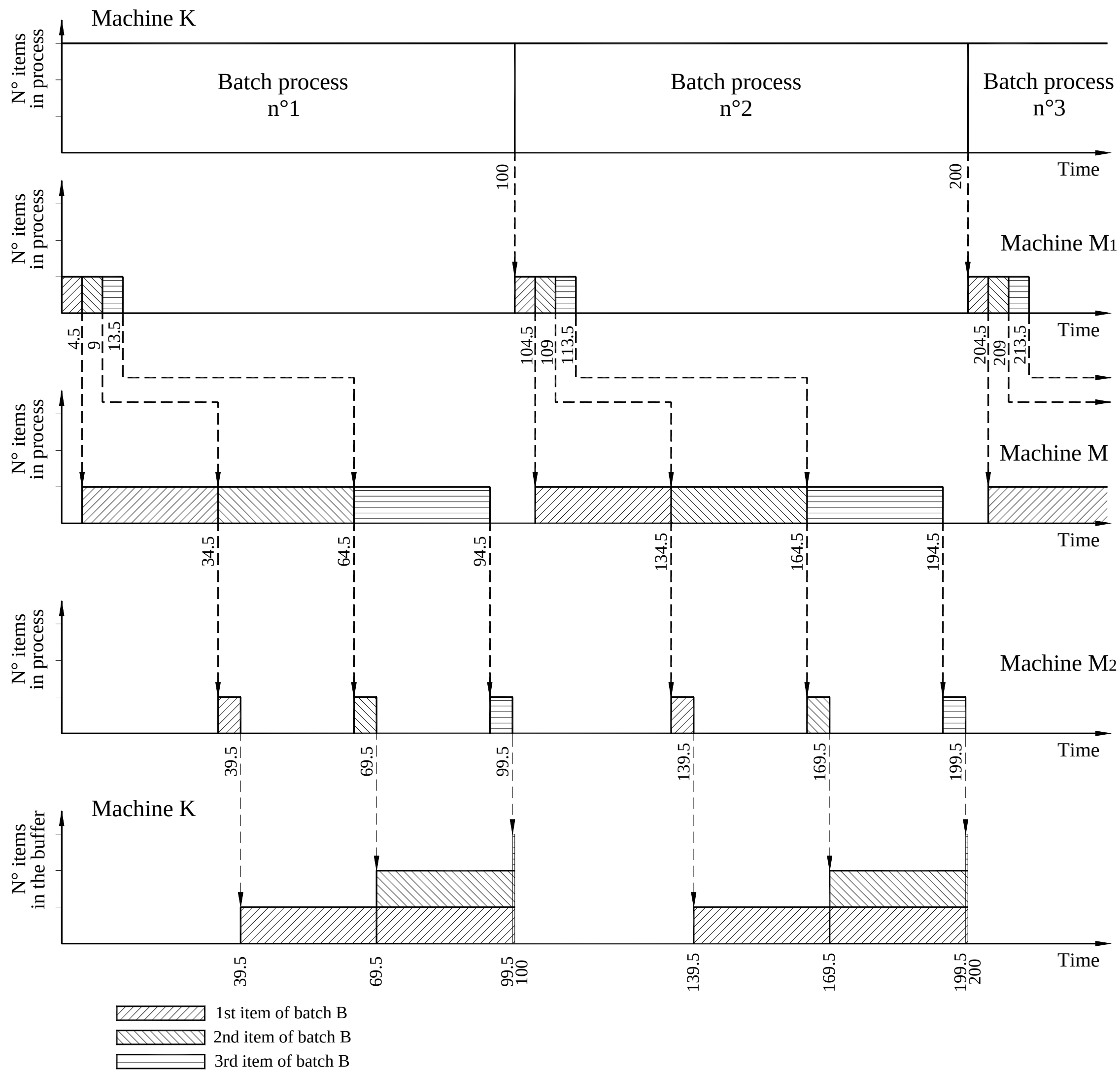


**Fig. 7.** The Gantt diagram of the production system with the stock level of the machine K buffer when the card number is equal to 2·B

It is interesting to note that the third item of each batch always reaches the *K-th* buffer just 0.5 min before the starting time of the BPM.

### *3.1.2 Card number equal to (2·B+1)*

The processing time of $M_2$ is increased up to 20 min. $S^*$ obviously maintains the same value as before, which is equal to 10 min. From Eq. (11), the minimum number of cards required to maximize the throughput of the system is equal to 7.

The actual value of $S$ is equal to:

$S = T_{M1} + T_{M2} = 24.5\ min.$

S is evidently larger than $S^*$ so that $2{\cdot}B$ cards are no longer sufficient. The upper limit of the interval corresponding to ($2{\cdot}B+1$) cards is equal to:

$S^* + T_M = 40\ min.$

Thus, it is necessary to add only one card to assure the use of the BPM at its full rate. The Gantt diagram of the production system is summarized in Fig. 8. It is interesting to note that only two items (i.e. cards) of the batch released by BPM at the *n*-th process return to the *K*-th buffer before the (*n*+2)-th process start. With references to Fig. 8, item *C* returns to the *K*-th buffer 14.5 min after its start.

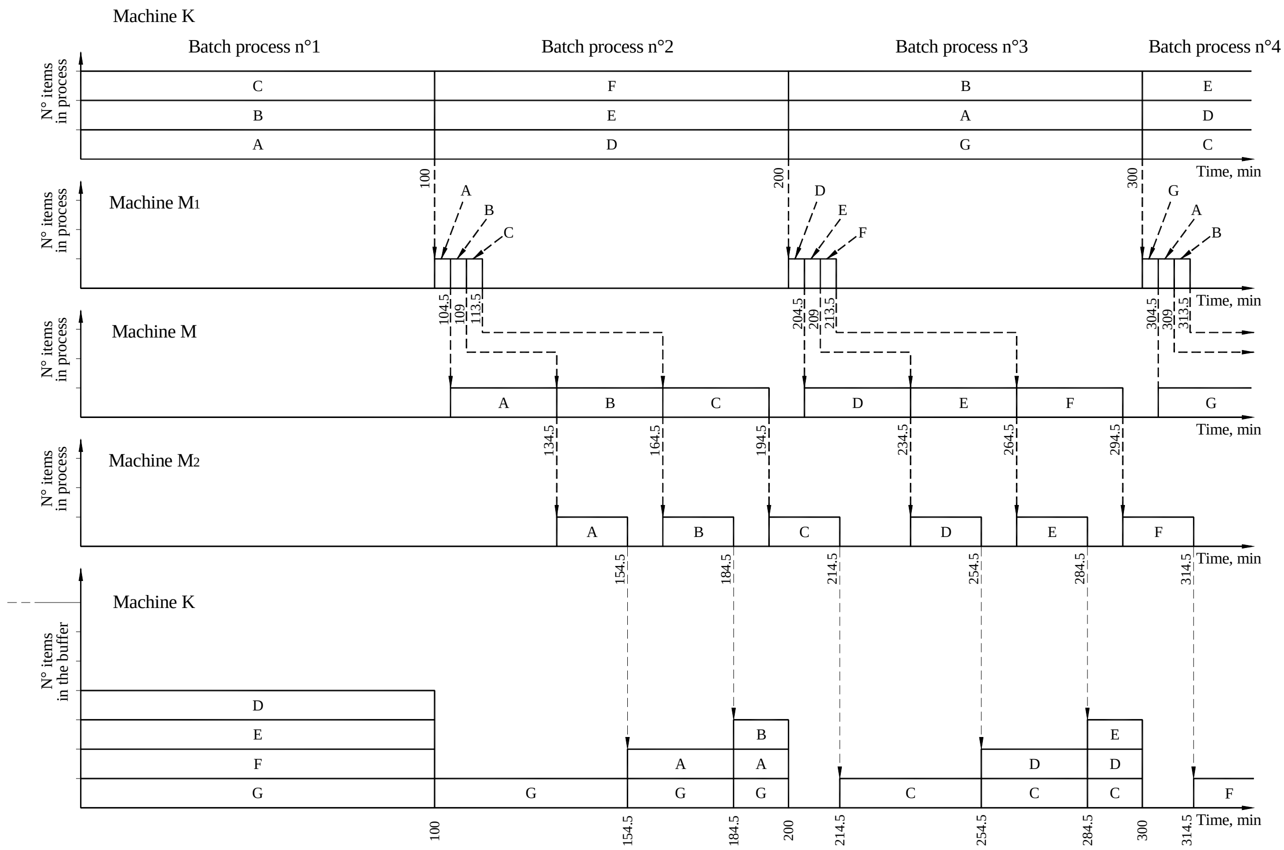


**Fig. 8.** The Gantt diagram of the production system with the stock level of the machine K buffer when the card number is equal to 2·B+1 (the seven items are labelled with the letters A to G), assuming a production start time of 0

### **3.2** *The batch processing machine K is not the bottleneck*

The flow-shop production system considered here is constituted by four machines as in the previous case (Fig. 6).

Two configurations of the production system are taken into account to show its behaviour when the card number is equal to $2 \cdot B_R$ and $(2 \cdot B_R+1)$, respectively.

*3.2.1 Card number equal to $2 \cdot B_R$*

The processing time of the four machines are as follows:

$T_K$: 100 min $T_{M1}$: 10 min $T_M$: 60 min $T_{M2}$: 9 min.

The batch size B is equal to 3 items.

In order to balance the capacity of the BPM to that of machine $M$, the actual size of the batch $B_R$ is given by:

$$B_R = \left\lceil \frac{T_K}{T_M} \right\rceil = 2.$$

From Eq. (20), the minimum card number is equal to 4. Indeed, the idle time $T_I$ is equal to

$$T_I = B_R \cdot T_M - T_K = 20 \text{ min},$$

with a value of $S$ lower than $T_I$:

$$S = T_{M1} + T_{M2} = 19 \text{ min}.$$

The Gantt diagram of the production system is summarized in Fig. 9. It is interesting to note that the last item of the batch reaches the *K-th* buffer just 1 minute before the start time of the BPM.

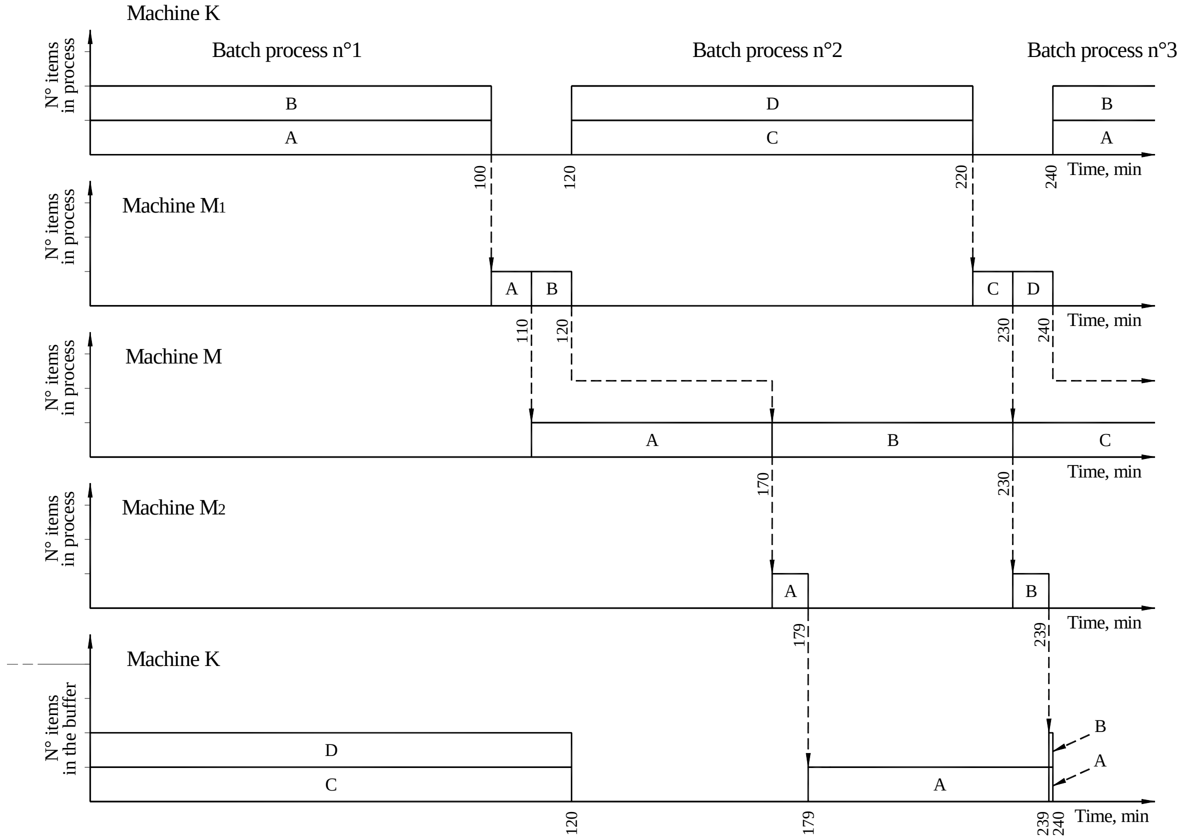


**Fig. 9.** The Gantt diagram of the production system with the stock level of the machine K buffer when the card number is equal to $2 \cdot B_R$ (the four items are labelled with the letters A to D), assuming a production start time of 0

### *3.2.2 Card number equal to ($2{\cdot}B_R + 1$)*

In this case, the processing time of $M_2$ is increased to 11 min. The other data remains unchanged. In this case, $S$ is equal to 21 min and it is larger than $T_I$, which remains to 20 minutes. Thus, in accordance with Eq. (20), $2{\cdot}B_R$ cards are no longer sufficient and an additional card must be added. Indeed, the upper limit of the interval corresponding to $2{\cdot}B_R + 1$ cards is equal to:

$$T_I + T_M = 80\ min.$$

The Gantt diagram of the production system is summarized in Fig. 10. It is interesting to note that in this case only one item of the batch released by the BPM at the $n$-th process returns to the $K$-th buffer before the ($n$+2)-th process start.

According to Fig. 10 item $B$ returns to the machine $K$ buffer 1 min after its start.

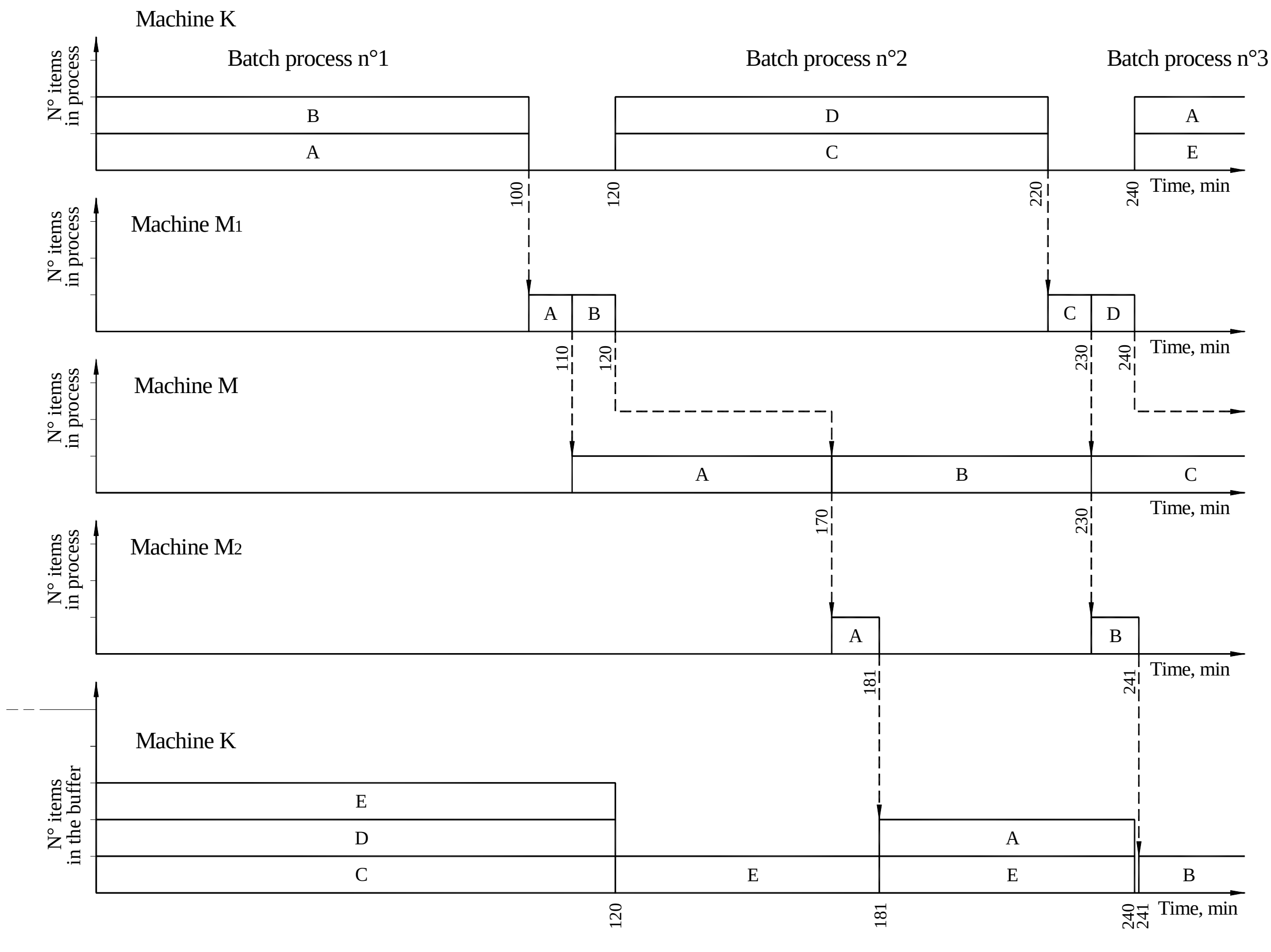


**Fig. 10.** The Gantt diagram of the production system with the stock level of the machine K buffer when the card number is equal to $2{\cdot}B_R{+}1$ (the five items are labelled with the letters A to E), assuming a production start time of 0

## 4. Conclusions

In this paper, the card setting problem in a flow shop environment characterized by the presence of a BPM has been addressed. Two innovative analytical models to define the optimum card number with a deterministic approach have been presented.

Both models are exhaustive and they are intended to minimize WIP levels assuring in all conditions the full rate of the bottleneck. The main feature of the models consists in the determination of the required card number, which is computed through a comparison between the processing time of the batch machine and the time taken by the last item of the batch to close the processing loop. Depending on the value assumed by these times, a set of discrete intervals are determined: each interval is associated with the minimum number of required cards.

The main advantages of both models are their simplicity, since they are based on neither a simulation approach nor on queuing theory, and possibility that the proposed methods can be easily extended to different configurations of the flow-shop system.

Obviously the extension to other production contexts, for example with two kinds of items that cannot be mixed inside the BPMs (due to different process parameters) or with a BPM used more than once in the same work-cycle, should impose conceptual modifications in order to assure the respect of the scheduled start of the BPM.

A further development of the present work could be the introduction of probability density functions to adequately describe the processing times of each machine. In this way, it would be possible to associate a statistical value of the throughput with the optimal number of cards.

The card release after the bottleneck could also be investigated to highlight its performance in comparison with the full release technique analysed here.

## Acknowledgements

The authors are grateful to the reviewers for their useful suggestions, which highly contributed to improve the readability and the value of the paper.

## References

Al-Tahat, M. D., & Rawabdeh, I. A. (2008). Stochastic analysis and design of CONWIP controlled production systems. *Journal of Manufacturing Technology Management*, 19, 253-273.

Bonvik, A. M., Couch, C.E., & Gershwin., S.B. (1997). A comparison of production-line control mechanisms. *International Journal of Production Research*, 25, 789-804.

Cao, D., & Chen, M. (2005). A mixed integer programming model for a two line CONWIP-based production and assembly system. *International Journal of Production Economics*, 95, 317–326.

Chang, T.M., & Yih, Y. (1994). Generic Kanban systems for dynamic environments. *International Journal of Production Research*, 32, 889-902.

Duenyas, I. (1994). Estimating the throughput of a cyclic assembly system. *International Journal of Production Research*, 32, 1403-1419.

Duri, C., Frein, Y., & Lee, H.-S. (2000). Performance evaluation and design of a CONWIP system with inspections. *International Journal of Production Economics*, 64, 219-229.

Framinan, J. M., Ruiz-Usano, R., & Leisten, R. (2001). Sequencing CONWIP flow-shops: analysis and heuristics. *International Journal of Production Research*, 39, 2735-2749.

Framinan, J.M., Gonzàlez, P.L., & Ruiz-Usano, R. (2003). The CONWIP production control system: review and research issues. *Production Planning & Control*, 14, 255-265.

Framinan, J.M., Gonzàlez, P.L., & Ruiz-Usano, R. (2006). Dynamic card controlling in a Conwip system. *International Journal of Production Economics*, 99, 102-116.

Fowler, J.W., Brown, S., Gold, H., & Schoemig A. (1997). Measurable improvements in cycle-time-constrained capacity, Proceedings of the 6th International Symposium on Semiconductor Manufacturing (ISSM), San Francisco, USA.

Golany, B., Dar-El, E., & Zeev, N. (1999). Controlling shop floor operations in a multi-family, multi-cell manufacturing environment through constant work-in-process. *IIE Transactions*, 31, 771-781.

Hopp, W. J., & Spearman, M. L. (1991). Throughput of a constant work in process manufacturing line subject to failures. *International Journal of Production Research*, 29, 635-655.

Hopp, W.J., & Roof, M.L. (1998). Setting WIP level with statistical throughput control (STC) in CONWIP production lines. *International Journal of Production Research*, 36, 867-882.

Hopp, J.H., & Spearman, M.L. (2000). *Factory Physics*. New York: McGraw Hill.

Huang, M., Wang, D., & Ip, W.H. (1998). Simulation study of CONWIP for a cold rolling plant. *International Journal of Production Economics*, 54, 257-266.

Lavoie, P., A.Gharbi, A., & Kenné, J.-P. (2010). A comparative study of pull control mechanisms for unreliable homogenous transfer lines. *International Journal of Production Economics*, 124, 241–251.

Li, N., Yao, S., Liu, G., & Zhuang, C. (2010). Optimization of a multi-Constant Work-in-Process semiconductor assembly and test factory based on performance evaluation. *Computers & Industrial Engineering*, 59, 314–322.

Luhl, P.B., Wangl, J.H., Wangl, J.L., & Tomastikz, R.N. (1997). Near-optimal scheduling of manufacturing systems with presence of batch machines and setup requirements. *Annals of the CIRP*, 46, 397-402.

Kashan, A.H., Karimi, B., & Jolai, F. (2010). An effective hybrid multi-objective genetic algorithm for bi-criteria scheduling on a single batch processing machine with non-identical job sizes. *Engineering Applications of Artificial Intelligence*, 23, 911–922.

Kim, Y.D., Lee, D.H., Kim, J.U., & Roh, H.K. (1998). A simulation study on lot release control, mask scheduling, and batch scheduling in semiconductor wafer fabrication facilities. *Journal of Manufacturing Systems*, 17, 107-117.

Monch, L., Zimmermann, J., & Otto, P. (2006). Machine learning techniques for scheduling jobs with incompatible families and unequal ready times on parallel batch machines. *Engineering Applications of Artificial Intelligence*, 19, 235–245.

Monch, L., Balasubramanian, H., Fowler, J.W., & Pfund, M.E. (2005). Heuristic scheduling of jobs on parallel batch machines with incompatible job families and unequal ready times. *Computers & Operations Research*, 32, 2731–2750.

Neale, J.J., & Duenyas, I. (2000). Control of manufacturing networks which contain a batch processing machine. *IIE Transactions*, 32, 1027–1041.

Ryan, M., Baynat, B., & Choodineh, F. (2000). Determining inventory levels in a CONWIP controlled job shop. *IIE Transactions*, 32, 105-114.

Sharmaa, S., & Agrawalb, N. (2009). Selection of a pull production control policy under different demand situations for a manufacturing system by AHP-algorithm. *Computers & Operations Research*, 36, 1622-1632.

Spearman, M. L., Woodruff, D. L., & Hopp, W. J. (1990). CONWIP: A pull alternative to kanban. *International Journal of Production Research*, 28, 879-894.

Tardiff, V., & Maaseidvaag, L. (2001). An adaptive approach to controlling kanban systems. *European Journal of Operational Research*, 132, 411-424.

Tay, F., Lee, L.H., & Wang, L. (2002). Production Scheduling of a MEMS Manufacturing System with a Wafer Bonding Process. *Journal of Manufacturing Systems*, 21, 287-301.

Yang, T., Fub, H.P., & Yang, K.Y. (2007). An evolutionary-simulation approachfor the optimization of multi-constant work-in-process strategy-A case study. *International Journal of Production Economics*, 107, 104−114.