Available online at www.sciencedirect.com

ScienceDirect

Procedia Engineering 199 (2017) 3404–3409

Procedia Engineering

www.elsevier.com/locate/procedia

X International Conference on Structural Dynamics, EURODYN 2017

# The role of the Modal Assurance Criterion in the interpretation and validation of models for seismic analysis of architectural complexes

Daniele Brigante[a], Carlo Rainieri[b]*, Giovanni Fabbrocino[c]

[a] Ph.D. Candidate, StreGa Lab., DiBT Dept., University of Molise, Campobasso, Italy
[b] Assistant Professor, StreGa Lab., DiBT Dept., University of Molise, Campobasso, Italy
[c] Full Professor, StreGa Lab., DiBT Dept., University of Molise, Campobasso, Italy

**Abstract**

The seismic vulnerability assessment of historical buildings is usually a very complex task because of the large number of uncertainties in the characterization of their structural behaviour. The unique structural configurations, the adoption of old construction techniques and the presence of stratified structural modifications occurred over the centuries make the definition of an appropriate and reliable numerical model very challenging. The available analysis approaches distinguish the local response of selected macro elements from the global response of the structure. The problem of discriminating if the investigated structure shows a global behaviour or a local response has been often reported in the literature. However, the definition of a quantitative measure to discriminate the global modes from the local ones and, in the latter case, the associated macro elements, is currently missing.
The Modal Assurance Criterion is a vector correlation index frequently used in experimental dynamics to quantify the similarity of mode shapes. In the present paper, it is used to define an original and quantitative approach to the discrimination between local and global modes. Results of application of the proposed procedure to an explanatory case study are reported, pointing out how the proposed method can guide towards the selection of the most appropriate analysis method.



*Keywords:* Historical buildings; Macro-elements; Modal Assurance Criterion; Global and Local modes

* Corresponding author. Tel.: +39 0874404959; fax: +39 0874404952.
*E-mail address:* carlo.rainieri@unimol.it

1877-7058 © 2017 The Authors. Published by Elsevier Ltd.
Peer-review under responsibility of the organizing committee of EURODYN 2017.
10.1016/j.proeng.2017.09.484

## 1. Introduction

Heritage Constructions (HC) are exposed to natural and anthropic hazards and need a careful consideration from the technical point of view whenever their preservation and protection are concerned [1]. The task is very complex for all the competences involved in the process and above all for structural engineers that handle the safety of structure and users.

This circumstance is by far more relevant in many European Countries located in seismic areas, where degradation and damage induced by material aging, soil conditions, unique structural configurations are only one facet of the activity required to structural engineers [2]. In fact, distribution of seismic actions strongly depends on the dynamic properties of the architectural asset and, therefore, on its geometrical and mechanical configuration [3]. This is particularly true in the case of old heritage masonry structures located in historical centers, like those affected by the recent earthquake sequence in Central Italy (2016), Emilia (2012), and L'Aquila (2009). This is the reason why it seems appropriate to refer to HCs as Architectural Complexes or Aggregates. The former definition particularly applies to parts of historical urban districts that include not only buildings, but also other constructions such as churches, cloisters and so on; the latter seems to be more appropriate in the case of buildings and/or groups of buildings characterized by the absence of solution of structural continuity.

Such concepts can be found not only in the technical literature [4, 5], but also in relevant National and International Codes of Practice in the field of structural and earthquake engineering with reference to existing structures. As HCs are concerned, it is worth noting that one of the most advanced documents appears to be the Directive 2011 [6] released by the MiBACT (Italian Ministry of Culture and Tourism) as a guide to engineers in managing the problem of the structural and seismic assessment of this specific type of structures.

The core of the guidelines is represented by the process of knowledge aimed at characterizing the main features of the construction in terms of constitutive materials, structural patterns, condition assessment (degradation, damage pattern and so on [2]) in view of a quantitative evaluation of the performance of the construction under seismic actions. In such a context, the role of the dynamic characterization of the asset is clearly identified from the experimental [4, 5] as well as theoretical standpoint. In fact, the base of any seismic assessment is the definition of Structural Units (SU) or relevant components of the construction (macro-elements) that affect the response of the structure enabling a solution of the global problem as a group of separated ones (sub-structuring, [7]).

This is the background of the present paper, which describes some results of the numerical analyses carried out on a very simple building with the objectives of defining a rational approach to sub-structuring of architectural complexes, and supporting the operator towards the selection of the most appropriate seismic analysis model.

**Nomenclature**

| | |
|---|---|
| HC | Heritage Construction |
| MAC | Modal Assurance Criterion |
| MPMR | Modal Participating Mass Ratio |
| SU | Structural Units |
| FEM | Finite Element Model |

## 2. Methodology

The distribution of inertia forces due to the earthquake ground motion depends on the dynamic properties of the structure of interest, and particularly on the fundamental periods of oscillation and the associated mode shapes. In addition, another relevant parameter is the Modal Participating Mass Ratio (MPMR) of the individual mode, whose magnitude can be addressed as a measure of the regularity of the structure [3]. Mode shapes and MPMRs are not exhaustive tools to classify the structural response and discriminate between global behavior and local response associated to the excitation of limited parts of the structure, the so-called macro-elements [4]. Thus, the opportunity of using a well-known tool of experimental modal analysis, the Modal Assurance Criterion (MAC) [8], to define the nature of the mode of interest is explored. From the mathematical standpoint, given two vectors $\{\phi_i\}$ and $\{\phi_j\}$ representative of the mode shapes under comparison, the MAC is expressed as follows:

$$MAC(\{\emptyset_i\},\{\emptyset_j\}) = \left|\{\emptyset_i\}^T\{\emptyset_j\}\right|^2 / \left[(\{\emptyset_i\}^T\{\emptyset_i\})\left(\{\emptyset_j\}^T\{\emptyset_j\}\right)\right] \quad (1)$$

under the assumption of normal mode, like those provided by numerical modal analyses. The MAC index can be addressed as a squared, linear regression correlation coefficient providing a measure of the consistency (degree of linearity) between two mode shape vectors. It ranges between 1 and 0: the higher the MAC, the higher the correlation between the vectors. It is commonly accepted the that MAC values exceeding the thresholds of 0.8÷0.9 are indicators of good consistency, while values lower than 0.1÷0.2 are considered as indicators of absence of consistency. The MAC index between partial mode shapes is herein computed and used as a tool for discriminating between global and local modes. The process for mode discrimination is hereafter illustrated with reference to the simple masonry building sketched in Fig. 1 in plan and front view. Red circles in Fig. 1 identify nodes assumed to be virtual sensors applied to walls representing relevant structural components of the building. The role of the axial stiffness of floors is assessed by considering two conditions, rigid diaphragm and flexible floor; in the latter case, it is worth noting that there is a 0.02 m gap between the floors and the walls (Fig. 1) in order to simulate the absence of connections between floors and vertical walls in the direction parallel to the floor beams. A Finite Element Model (FEM) made of 3D elements has been set to analyze the modal response under different structural configurations (rigid diaphragm vs. flexible floor) and different alignments and number per alignment of virtual sensors. MAC is then computed between all couples of alignements. Table 1 reports the matrix of the numerical tests performed in this study.

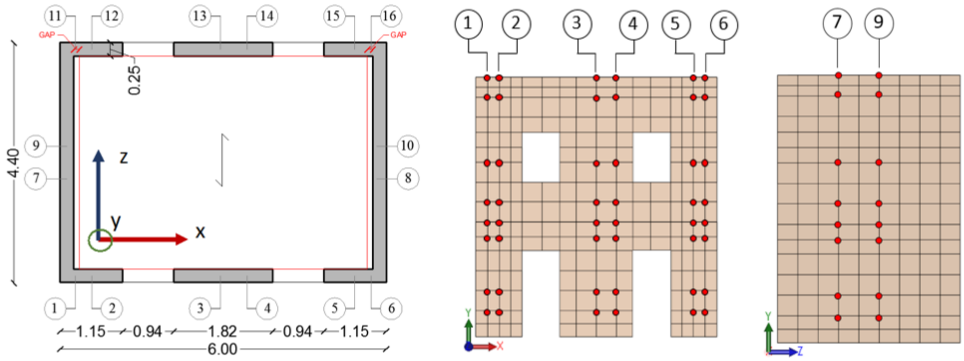


Fig. 1. (left) Geometry of the model; (right) location of virtual sensors

Table 1. Numerical simulations

| Number of alignments | Number of sensors | | | Model | |
|---|---|---|---|---|---|
| 8 | 4 | 8 | 16 | Rigid floor | Flexible floor |
| 16 | 4 | 8 | 16 | Rigid floor | Flexible floor |
| 40 | 4 | 8 | 16 | Rigid floor | Flexible floor |

The main characteristics of the masonry, assumed to be linear elastic according to a macro-modelling approach based on the homogenization of the constituent materials, are: E = 2400 MPa, G = 780 MPa, specific weight of 22 kN/m$^3$. The masses have been defined according to the dimensions of the structural elements and the physical properties of materials (reinforced concrete for the floors, masonry for the walls).

## 3. Results and discussion

The main outcomes of the modal analyses are summarized in the following figures. In particular, Fig. 2, 3 and 4 report the main results of the modal analyses carried out on the building characterized by the rigid diaphragm. Fig.

5, instead, reports explanatory results of the analysis carried out on the building characterized by the flexible floor and it shows the sensitivity of the results to the number of sensors installed along the verticals.

| FLEXURAL MODE 1 (Rigid diaphragm) | | | | | | | |
|---|---|---|---|---|---|---|---|
| Direction | X | Frequency (Hz) | 9.18 | Modal Participating Mass Ratio (%) | 83 | Number of sensors | 4 |

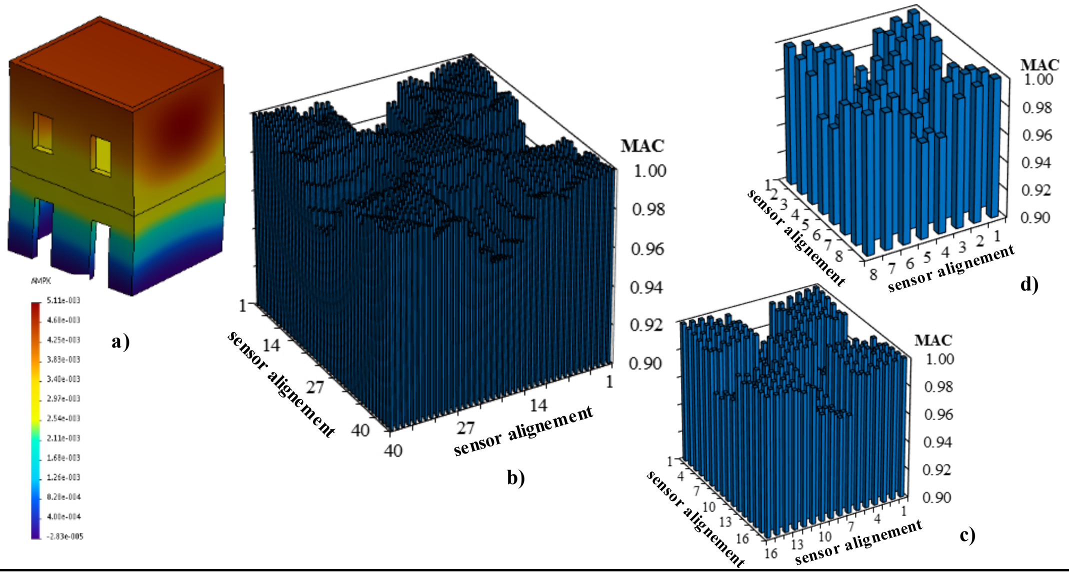


Fig. 2. Global flexural mode discrimination in the case of rigid floor (table on the top provides relevant data). Mode shape (a) and MAC plots in the case of 40 (b), 16 (c) and 8 vertical alignments of virtual sensors (d) are given.

| TORSIONAL MODE 3 (Rigid diaphragm) | | | | | | | |
|---|---|---|---|---|---|---|---|
| Rotation axis | Y | Frequency (Hz) | 15.80 | Modal Participating Mass Ratio (%) | 81 | Number of sensors | 4 |

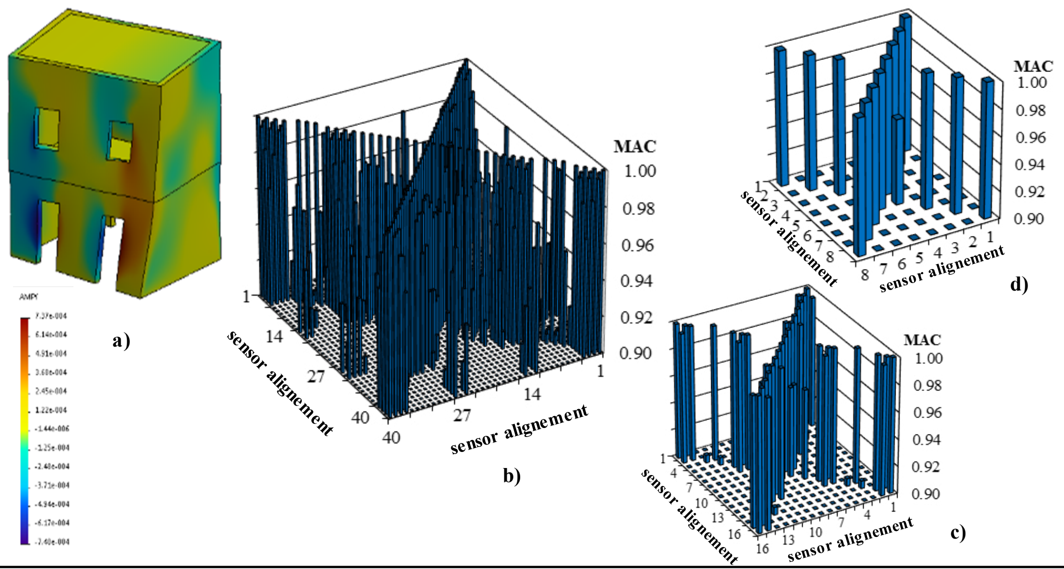


Fig. 3. Global torsional mode discrimination in the case of rigid floor (table on the top provides relevant data). Mode shape (a) and MAC plots in the case of 40 (b), 16 (c) and 8 vertical alignments of virtual sensors (d) are given

| LOCAL MODE 4 (Rigid diaphragm) | | | | | | | |
|---|---|---|---|---|---|---|---|
| Direction | Z | Frequency (Hz) | 16.90 | Modal Participating Mass Ratio (%) | 10e-06 | Number of sensors | 4 |

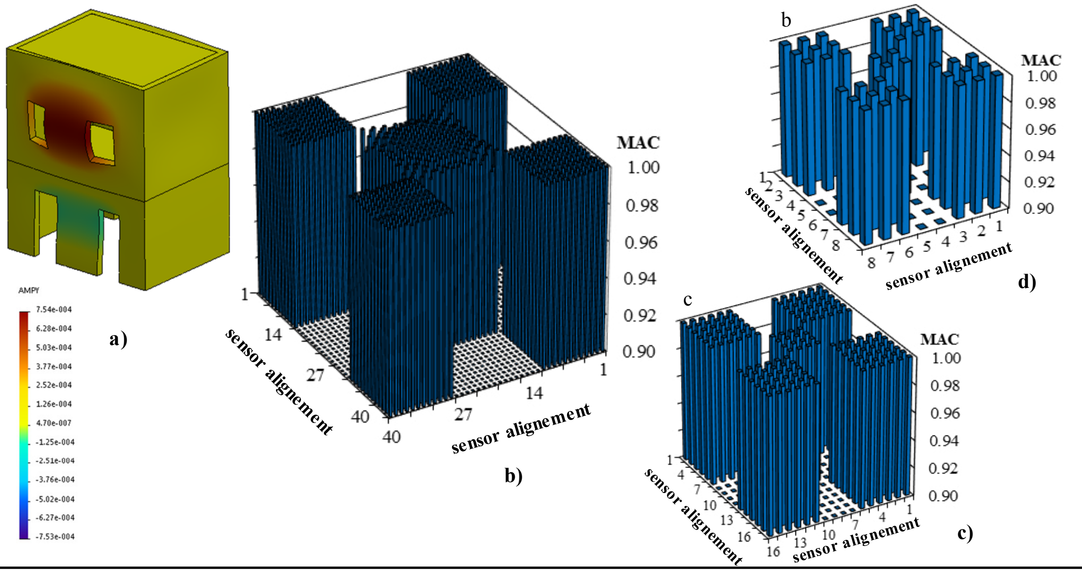


Fig. 4. Local mode discrimination in the case of rigid floor (table on the top provides relevant data). Mode shape (a) and MAC plots in the case of 40 (b), 16 (c) and 8 vertical alignments of virtual sensors (d) are given.

| TORSIONAL MODE 4 (Flexible floor) | | | | | | | |
|---|---|---|---|---|---|---|---|
| Rotation axis | Z | Frequency (Hz) | 15.41 | Modal Participating Mass Ratio (%) | 78 | Number of verticals | 16 |

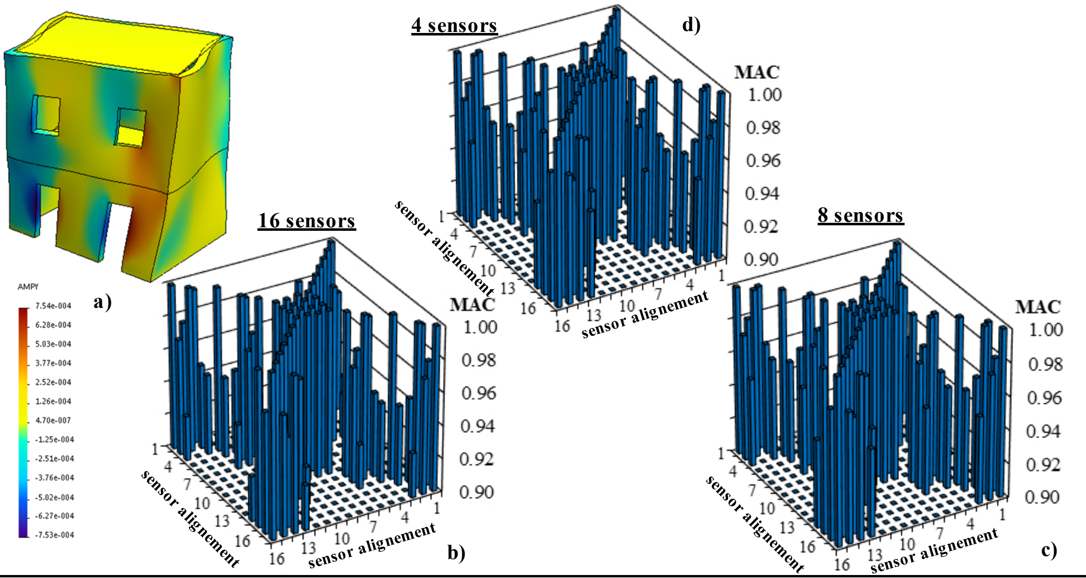


Fig. 5. Torsional mode discrimination in the case of flexible floor (table on the top provides relevant data). Mode shape (a) and MAC plots in the case of 16 (b), 8 (c) and 4 (d) virtual sensors arranged on 16 vertical alignments (d) are given.

The figures, in particular, show a deformed shape of the selected mode - a) – together with relevant dynamic parameters (primary direction in the case of flexural modes, axis of rotation in the case of torsional modes, frequency of oscillation, MPMR, number of verticals or sensors) reported in the table placed on top of the figures. Plot identified with the letters b), c) and d) in Fig. 2, 3 and 4 show the matrix graph of the MAC computed according to Eq. 1 between all couples of vertical alignments of virtual sensors. The vertical axis of the 3D graphs of MAC is bounded between 0.9 and 1 to highlight the verticals characterized by high correlation of the corresponding modal vectors. Absence of bars denotes low correlation between the selected verticals. Two degrees of freedom at each virtual sensor location are considered. More in detail, Fig. 2 and 3 report two modes that can be classified as 'global' either due to the specific deformed shape or the MPMR magnitude. Fig. 4, instead, reports a local mode of the upper wall – Fig. 4a –; as the model with rigid floor is concerned, it is worth noting that plots b), c), d) report the outcomes of the sensitivity analysis as a function of the number of verticals. It is possible to note that the MAC matrix between all couples of alignments shows a distinct structure depending on the nature of the mode. Fig. 2 shows that, for global bending modes, the MAC matrix is a full matrix with nearly all values very close to 1. Increasing the number of alignments does not change the overall structure of the MAC matrix. In the case of torsion mode, the largest MAC values are located along the main diagonal and the main anti-diagonal when the number of alignments is set to a minimum. When this number increases, the largest MAC values are located not only along the above-mentioned diagonals but also along a few nearby diagonals. As a result, torsion modes are always characterized by a cross structure of the MAC matrix that can be eventually cross banded in the presence of a redundant number of alignments.

Fig. 5 reports the results in the case of flexible floor and an assessment of the influence of the number of sensors on MAC values. Attention is focused on the torsional mode of the structure, whose shape appears to be not so different from the one reported in Fig. 3a (rigid floor) in terms of shape and MPMR. However, in this case, the influence of the flexible floor is evident in the representation of the MAC matrix. This is characterized by a larger number of MAC values exceeding the selected threshold of 0.90 with respect to the case of rigid floor. This is the effect of a higher correlation between adjacent verticals. On the other hand, the MAC matrix does not seem to be significantly influenced by the increase in the number of (bidirectional) sensors along each vertical above the minimum number of four, as clearly shown by Fig. 5b, 5c and 5d.

## 4. Final remarks

The present paper investigated the possibility of using the Modal Assurance Criterion to discriminate between global and local modes of architectural assets characterized by complex geometry and structural configuration. The proposed methodology is fairly simple and it offers some advantages for dynamic substructuring based on the results of numerical modal analyses. Results of mode discrimination are fundamental for accurate assessment of historical structures and design of seismic retrofitting interventions. The herein presented case study, although very basic, is illustrative of the potentialities of the method. Encouraging results have been obtained. However, further analyses and systematic validation of the proposed methodology are needed in view of its extended use in the practice.

## References

[1] VV. AA. (2002). La Carta di Cracovia 2000. Principi per la conservazione e il restauro del patrimonio costruito, Ed. Marsilio, 258 pp. (in Italian).

[2] Marra, A. (2015). Interdisciplinary approach to the conservation of cultural heritage in seismic areas. Ph.D. Thesis, Seismic Risk Doctoral Programme, XVII Cycle, University of Naples Federico II.

[3] Elnashai, A.S., Di Sarno, L. (2015). Fundamentals of Earthquake Engineering: from source to fragility, John Wiley and Sons, New York.

[4] Boscato, G., Dal Cin, A., Ientile, S., Russo, S. (2016). Optimized procedures and strategies for the dynamic monitoring of historical structures. Journal of Civil Structural Health Monitoring, Vol. 6, Issue 2, pp. 265-289.

[5] Rainieri, C., Marra, A., Rainieri, G.M., Gargaro, D., Pepe, M., Fabbrocino, G. (2015). Integrated non-destructive assessment of relevant structural elements of an Italian heritage site: the Carthusian monastery of Trisulti. Journal of Physics: Conference Series 628 (2015) 012018.

[6] Presidenza del Consiglio dei Ministri (2011). Valutazione e riduzione del rischio sismico del patrimonio culturale con riferimento alle norme tecniche per le costruzioni di cui al DM 14 Gennaio 2008. Direttiva P.C.M. 9 febbraio 2011. (in Italian).

[7] De Klerk, D., Rixen, D.J., Voormeeren, S.N. (2008). General Framework for Dynamic Substructuring: History, Review, and Classification of Techniques. AIAA Journal, Vol. 43, No 5, pp. 1169-1181.

[8] Allemang, R.J. & Brown, D.L. (1982). A correlation coefficient for modal vector analysis. Proceedings of The 1st International Modal Analysis Conference, Orlando, FL, USA.