Available online at www.sciencedirect.com

SciVerse ScienceDirect

Energy Procedia 14 (2012) 1380 – 1385

Energy Procedia

2nd International Conference on Advances in Energy Engineering (ICAEE 2011)

Multi-commodity network flow models for dynamic energy management – Mathematical formulation

M. Manfren*

Building Environment Science & Technology Dept. (BEST), Politecnico di Milano, Via Bonardi 3, 20133 Milano, Italy

Abstract

The evolution of energy infrastructures towards a more distributed, adaptive, predictive and market-based paradigm implies an effort on combining communication protocols and energy transmission and distribution systems in a common architecture. This architecture should allow decentralized control in order to be able to manage efficiently distributed generation, storage and exchange of energy between sources and sinks. Dynamic energy management models are a part of this "systems thinking" vision that aims to create a new field of applications that is at the intersection of computing science and energy technology. The broader implications associated with them are related with the possibility of creating communities that integrate energy supply and demand within a given region, in order to limit their impact. In order to push intelligence to the energy networks' edges, up to individual sources and sinks, scalable and flexible distributed systems will have to be build. In this sense, data mining techniques and multi-commodity network flow models can be combined for pattern detection, forecasting and optimization, which are essential features of dynamic energy management.

*Keywords:*Dynamic energy management; Smart Grid; Multi-commodity network flow models; Linear programming; Convex programming

* Corresponding author. Tel.: +39 0223999469; fax: +39 0223999469.
E-mail address: massimiliano.manfren@gmail.com; massimiliano.manfren@mail.polimi.it

doi:10.1016/j.egypro.2011.12.1105

1. Introduction

The increased awareness about energy efficiency, carbon and pollutant emissions reduction are priorities and key challenges for the 21st century and a "systems thinking" vision, that is necessary to produce a real paradigm shift in energy systems, has determined an increasing interest for adaptive and predictive distributed control systems in recent years. On the other hand, the Smart Grid concept [1] synthesizes the confluence of issues and the convergence of objectives, which include national energy security, climate change, pollution reduction, grid reliability, etc.

Nomenclature

Indices

i,j	nodes
e	emission type
k,r	commodities
t	time-step

Sets

A	arcs
A', A''	arcs with binary decision variables (subset of A)
E	emissions
N	nodes
M	transhipment nodes
R	conversion nodes
S	storage nodes
T	time period

Variables

x^t_{ijk}	continuous decision variable, flow on arc (i,j) of commodity k at time t
y^t_{ijk}	binary decision variable on arc (i,j) of commodity k at time t (connection status)
z^t_{ik}	continuous decision variable, stored quantity on node i of commodity k at time t
δ_{ijk}	binary decision variable on arc (i,j) of commodity k (investment)

Parameters/coefficients

b^t_{ik}	external flow on node i of commodity k at time t
c^t_{ijk}	variable cost of flow (per unit) on arc (i,j) of commodity k at time t

d_{ijk}	fixed cost of connection on arc (i,j) of commodity k
f_{eijk}	factor for type of emission e on arc (i,j) of commodity k
l^t_{ijk}	lower bound of flow on arc (i,j) of commodity k at time t
u^t_{ijk}	upper bound of flow on arc (i,j) of commodity k at time t
μ^t_{ijk}	gain factor of flow on arc (i,j) of commodity k at time t
η^t_{ikr}	conversion factor of flow on arc (i,j) of commodity k at time t
Δt	time-step
Acronyms	
NPV	net present value
DPP	discounted payback period
IRR	internal rate of return

Dynamic energy management systems capable of integrating data mining/machine learning and optimization techniques will play an increasingly important role in the distributed control of future decentralized energy networks. In this context, multi-commodity network flow models flexibility, scalability and efficiency can be exploited for optimization tasks [2,3].

2. Multi-commodity network flow model formulation

Multi-commodity network flow problems involve several flow types or commodities, which simultaneously use the network and are coupled through either arc capacities or through the cost function. At the most general level, the optimal multi-commodity flow formulation takes the form:

$$\begin{aligned} \text{minimize} \quad & \sum_{k\in K}\sum_{(i,j)\in A} C_{ijk}\left(x_{ijk}\right) \\ \text{subject to} \quad & \text{conservation of flow constraints} \\ & \text{side constraints} \end{aligned}$$

where $x_{i,j,k}$ denotes the flow of commodity k on the arc between nodes (i,j), and A is the set of arcs in the network. The objective function $C_{i,j,k}(x_{i,j,k})$ represent the cost of flow in arcs and, in the simplest case, is a convex monotonically increasing function.

The network flow model consists of nodes and arcs, arcs are directed line segments (edges are un-directed line segments, used in graph theory) connecting nodes (origin node, terminal node).

The arc flows are the decision variables for the network flow model. Flow is conserved at the nodes (node balance), implying that the total flow entering a node must equal the total external flow leaving the node; this property is expressed by conservation of flow constraints. It is possible to consider in the balance also external flows b_i (parameters). We will assume as positive the flow entering the node (supply) and negative the flow leaving the node (supply). In a model where decision variables are energy flows, flow conservation constraints are equivalent to impose energy conservation in nodes.

Flow is limited in an arc between nodes (i,j) by the lower and upper bounds l_{ij} and u_{ij}. The term capacity refers to the upper bound on flow. It is possible to model flow variation in an arc with a gain factor μ_{ij}. The flow at the end of arc is obtained by multiplying the flow at the beginning of the arc by the gain factor. If μ_{ij}<1, the arc is losing flow, if μ_{ij}>1, the arc is gaining flow. When all unitary gains are present, we have a pure network; otherwise we have a generalized network model. In generalized network models the solution is usually not constituted by all integers.

Finally, side constraints term identifies all the constraints that cannot be modeled using the network structure, arc parameters (cost, gain) or external flows (general types of constraints).

2.1. Model construction and assumptions

The assumptions used in modeling are here discussed. They derive from the study of models for techno-economic optimization of distributed generation and Smart Grid vision that constitute the essential knowledge base for the research work.

2.1.1. Input/output relationships

The input/output characteristics of model components (nodes, arcs) have to be modelled both with respect to objective functions and to technical side constraints. In the mathematical formulation, we have the necessity to model linear as well as non-linear relationships (cost functions, conversion processes, losses, etc.). We can decide either to assume a linear approximation, thus remaining in the realm of linear programming, to introduce integer variables, creating as a result a mixed integer program, or to introduce a convex relaxation of the non-convex function, thereby creating a convex optimization problem [4,5].

2.1.2. Integer programming

Integer decision variables are introduced in the mathematical formulation in order to model discontinuous function and logical constraints. The choice of using this kind of variables depends on the task to be performed (on-off status of plants, technology/configuration selection and sizing).

In some cases non-linear relationships and logical conditions can be formulated with special ordered sets (SOS) [6], in order to improve solvability with respect to a pure integer formulation. There are two types of SOS; type 1 occurs when, in a set of binary decisions (yes/no), at most one can be nonzero; type 2 occurs when, in a fixed order list of integer nonnegative variables, at most two adjacent variables can be nonzero. This is the case, for example, of non-convex relationship representation via piecewise linear approximation (using multipliers) [7].

2.1.3. Multi-period network flow formulations

Multi-period network flow models are the composition of multiple networks linked by arcs that define the connection between different temporal states. For example, is the case of a storage system (node) the balance is determined from both input/output flows in the current time-step and the amount of energy stored in the previous time-step. The connections between the different networks have to respect and reproduce the chronological order of system's state. The size of a multi-period network is proportional to the number of periods in which we want to perform the calculation. In multi-period network models, the choice of a unique time-step is not forced by the model structure and it is possible to decompose it into subsystem, each one with its distinct dynamics. This choice, nonetheless, reduce the easiness of model assembly, which is one of the advantages related with the use of network models. The time discretization has to be accurate enough to properly describe the fastest dynamics in the system. A dynamic energy management system for Smart Grid and DG applications requires the use of hourly or sub-hourly time-steps [8-10]. On the other hand, a multi-period formulation of the model is decisive for applications aimed at finding economic, technical and environmental solutions with different time horizons [11,12].

The definition of time discretization implies neglecting all the variability that exists within the chosen time-step. As introduced before, setting up a model unable to capture the dynamics of the faster evolving sub-systems will result in a useless model. Further, when selecting the time-step, we are aggregating the result for each one. This means that energy flows and cost coefficients chosen in the mathematical formulation are represented by a single value over this interval. Because of the generality of the model, the numeric quantities must be redefined depending on the task to be performed.

For example, the cost coefficients have to be weighted (multiplied) for the number of time-step considered; if we consider a daily load profile representative of average behavior over a certain period (e.g. a month), we have to multiply the cost coefficients for the number of days in the selected period. Since operating conditions can be redundant (similar characteristics), simplifications such as the one just depicted can be introduced in long-term planning to reduce model size without losing accuracy.

2.2. Mathematical formulation

The mathematical formulation of the multi-commodity network flow model in mixed integer linear programming form is the subsequent:

$$\text{minimize} \quad \sum_{t\in T}\sum_{k\in K}\sum_{(i,j)\in A} c_{ijk}^{t} x_{ijk}^{t} + \sum_{k\in K}\sum_{(i,j)\in A} d_{ijk}\delta_{ijk} \tag{1}$$

$$\text{subject to} \quad \sum_{\{j\in N:(i,j)\in A\}} x_{ijk}^{t} - \sum_{\{j\in N:(j,i)\in A\}} \mu_{jik}^{t} x_{jik}^{t} = b_{i,k}^{t} \qquad \forall k\in K, \forall i\in M, \forall t\in T \tag{2}$$

$$x_{ijr}^{t} - \eta_{ikr}^{t}\mu_{jik}^{t}x_{jik}^{t} = 0 \qquad \forall k\in K, \forall r\in K, \forall i\in R, \forall t\in T \tag{3}$$

$$z_{ik}^{t} - \mu_{ik}^{t} z_{ik}^{t-1} + \sum_{\{j\in N:(i,j)\in A\}} x_{ijk}^{t}\Delta t - \sum_{\{j\in N:(j,i)\in A\}} \mu_{jik}^{t} x_{jik}^{t}\Delta t = b_{i,k}^{t}\Delta t \qquad \forall k\in K, \forall i\in S, \forall t\in T \tag{4}$$

$$l_{ijk}^{t} \le x_{ij}^{t} \le u_{ijk}^{t} \qquad \forall k\in K, \forall (i,j)\in A, \forall t\in T \tag{5}$$

$$l_{ijk}^{t} y_{ijk}^{t} \le x_{ijk}^{t} \le u_{ijk}^{t} y_{ijk}^{t} \qquad \forall k\in K, \forall (i,j)\in A', \forall t\in T \tag{6}$$

$$y_{ijk}^{t} \le \delta_{ijk} \qquad \forall k\in K, \forall (i,j)\in A'', \forall t\in T \tag{7}$$

$$\sum_{t\in T}\sum_{\{j\in N:(j,i)\in A\}} f_{ejik}\mu_{jik}^{t}x_{jik}^{t} \le U_{ei} \qquad \forall k\in K, \forall i\in R, \forall e\in E, \forall t\in T \tag{8}$$

The constraints of the model are as follows:

1. conservation of flow constraints on energy transshipment nodes, Eq.(2);
2. conservation of flow constraints on energy conversion nodes, Eq.(3);
3. conservation of flow constraints on energy storage nodes, Eq.(4);
4. upper and lower bounds of flow in arcs, Eq.(5);
5. upper and lower bounds of flow in arcs with on/off status, Eq.(6);
6. side constraints related with investment cost, Eq.(7);
7. emission factors, Eq.(8).

Additional side constraints that can be added involve: availability constraints, ramp-up/ramp down limits, startup constraints, minimum-up/minimum down time constraints and load scheduling constraints.

The first objective of the dynamic energy management system is to minimize the overall cost of the system under constraints over a certain time interval. This cost is defined as the sum of running costs (operation and maintenance) and the investment costs (technologies, configurations, etc.). Of course, investment costs are taken into account only in long term production planning and business evaluation. As explained before, the introduction of the choice between alternatives requires the use of integer variables, expressing the logical condition of arc existence. In this case, the problem is transformed into a NP-hard mixed integer linear programming problem. The objective function (1) can be expressed in the form of the economic indicator NPV. Other indicators such as DPP, IRR can be calculated consequently. In order to take into account correctly the costs over project life-time (discounted cash flow) it is necessary to multiply costs by annuity factors (annualized cash flow). Environmental objectives (externalities and emissions) can be incorporated in the single objective function via multipliers or consider independently [insert references], (multi-objective optimization). As illustrated before, in short-term planning environmental objectives are incorporated as additional side constraints (e.g. the maximum amount of pollutant emission permitted over a certain period of time).

3. Conclusion

Future energy infrastructures have to push intelligence to the networks' edges, through information and communication technology. The diversity of underlying technologies in energy networks will not represent a limiting factor if appropriate interfaces and protocols for the exchange of energy and information will be present. Dynamic energy management systems represent possible applications for the distributed control of the future decentralized grid, at the intersection of computing science and energy technology. Multi-commodity network flow models are efficient models for the management of multiple commodities over arbitrary networks. The development of reliable, scalable and flexible models that can be used by the professionals', aims to accelerate the transition from present centralized energy systems to adaptive, predictive management systems with a seamless integration with data mining/machine learning techniques for pattern detection and forecasting.

References

[1] G. Chicco, P. Mancarella, Distributed multi-generation: A comprehensive view, Renewable and Sustainable Energy Reviews, 13 (2009) 535-551.

[2] P.A. Jensen, J.F. Bard, Introduction to Operations Research, Wiley, 2003.

[3] D.P. Bertsekas, Network optimization: continuous and discrete methods, Athena Scientific, 1998.

[4] S.P. Boyd, L. Vandenberghe, Convex optimization, Cambridge University Press, 2004.

[5] D.P. Bertsekas, A. Nedić, A.E. Ozdaglar, Convex analysis and optimization, Athena Scientific, 2003.

[6] E.M.L. Beale, The current algorithmic scope of mathematical programming systems

Computational Practice in Mathematical Programming, in: M.L. Balinski, E. Hellerman (Eds.), Springer Berlin Heidelberg, 1975, pp. 1-11.

[7] H.P. Williams, Model building in mathematical programming, Wiley, 1999.

[8] D. Connolly, H. Lund, B.V. Mathiesen, M. Leahy, A review of computer tools for analysing the integration of renewable energy into various energy systems, Applied Energy, 87 (2010) 1059-1082.

[9] M. Manfren, P. Caputo, G. Costa, Paradigm shift in urban energy systems through distributed generation: Methods and models, Applied Energy, 88 (2011) 1032-1048.

[10] H. Lund, Renewable Energy Systems: The Choice and Modeling of 100% Renewable Solutions, Academic, 2009.

[11] A. Rong, R. Lahdelma, CO2 emissions trading planning in combined heat and power production via multi-period stochastic optimization, European Journal of Operational Research, 176 (2007) 1874-1895.

[12] K.C. Kavvadias, A.P. Tosios, Z.B. Maroulis, Design of a combined heating, cooling and power system: Sizing, operation strategy selection and parametric analysis, Energy Conversion and Management, 51 (2010) 833-845.