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Proton emission, gamma deformation, and the spin of the isomeric state of ^{141}Ho

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ABSTRACT

The nonadiabatic quasiparticle model for triaxial shapes is used to perform calculations for decay of ^{141}Ho, the only known odd-Z even-N deformed nucleus for which fine structure in proton emission from both ground and isomeric states was observed. All experimental data corresponding to this unique case namely, the rotational spectra of parent and daughter nuclei, decay widths and branching ratios for ground and isomeric states, could be well explained with a strong triaxial deformation $\gamma \sim 20^\circ$. The recent experimental observation of fine structure decay from the isomeric state, can be explained only with an assignment of $I = 3/2^+$ as the decaying state, in contradiction with the previous assignment, of $I = 1/2^+$, based on adiabatic calculations. This study reveals that proton emission measurements could be a precise tool to probe triaxial deformations and other structural properties of exotic nuclei beyond the proton drip-line.

Studies at the extreme limits of the nuclear chart have gained momentum in the last decade with the advent of new experimental facilities [1]. The production of new exotic nuclei lead to the discovery of several exciting new phenomena like for example, neutron skins and halos [2] in neutron-rich nuclei, and the novel decay mechanism of proton radioactivity in proton rich nuclei around the proton drip line. On theoretical grounds, the description of these new phenomena, apart from validating existing models in regions far from stability, requires the formulation of new theories.

Fine structure in proton radioactivity, complemented by the decay widths and spectra built on the decaying states, is a perfect example in this context. The high sensitivity of the decay properties to details of the nuclear wave function provides a way to extract rich information about the structure of the nuclei involved [3–6]. Exploiting this fact, we have analyzed the role of triaxial deformation in some proton emitting nuclei. In the case of ^{161}Re [7] the inclusion of triaxial deformation could explain the experimental data, assigning angular momentum $1/2^+$ to the proton emitting state. For the nucleus ^{145}Tm [8] the analysis of the energy spectra of parent and daughter nuclei, half-life and fine structure confirmed a large triaxiality with a high degree of confidence.

In this work we discuss the case of ^{141}Ho which is very interesting since it is the only known odd-Z even-N deformed nucleus where both ground and isomeric states display fine structure, i.e. decay by emission of protons feeding both ground (0^+) and first excited (2^+) states in the daughter nucleus [9]. It is quite challenging to interpret this phenomenon, since decay to the ground state depends only on the component with the lowest angular momentum of the Nilsson wave function, while fine structure might depend on the relative weight of the other components.

Furthermore, there are hints of departure from axial deformation as suggested in Ref. [10] where, using recoil decay tagging the rotational spectrum of ^{141}Ho was established.

Previous calculations for this proton emitter, that took into account the non-axial deformation degree of freedom, were performed within the adiabatic treatment [11], and predicted a high sensitivity of the branching ratio to the triaxial deformation, while a calculation [12] within the nonadiabatic weak coupling model with coupling to γ vibrations predicted a branching ratio almost independent on the non-axial coupling parameter. In both works only the decay from the negative parity ground state was studied, since fine structure of the isomeric state had not yet been discovered. In the only work [9] where this decay was studied, an axially symmetric coupled channel model was used.

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All these works do not take into account correctly the pairing interaction, since it is introduced only as a spectroscopic factor multiplying the decay width. Previous studies [6] have shown that in the nonadiabatic approach, the pairing interaction cannot be treated at the BCS level after the Coriolis interaction has been already diagonalized, since at that stage the system is described by a many-body wave-function and a Bogoliubov transformation cannot be performed anymore. Therefore the BCS transformation has to be carried out before taking into account nonadiabatic effects. Furthermore, omitting the pairing interaction in the Hamiltonian has two main consequences: the components of the wave function have wrong phases and therefore wrong coherence properties [6], and the energy eigenvalues, since the pairing interaction has not been taken into account in the Hamiltonian, have no relation at all with the actual excitation spectra, making the choice of a level as the ground state quite uncertain.

In the present work we apply the nonadiabatic quasiparticle method [6], that treats correctly the pairing interaction, to study fine structure in proton emission from both positive and negative parity states of ^{141}Ho, including triaxial deformation [7].

The total Hamiltonian of the system can be written as the intrinsic Hamiltonian of the valence proton plus a collective part representing the core, $H = H_{\text{in}} + H_{\text{col}}$, where H_{in} is the triaxial Nilsson Hamiltonian including a deformed spin–orbit term and the residual pairing interaction. To obtain the single-particle energies and wave functions corresponding to H_{in} we use a potential of the Wood–Saxon type with the parameters from Ref. [13]. The nuclear potential is evaluated in a similar way as in Ref. [14] but including triaxial deformation. The residual pairing interaction is treated within the BCS approach employing a constant gap approximation. The collective Hamiltonian H_{col} describes the rotations of the triaxial core with respect to the body-fixed axis and can be written as

$$H_{\text{col}} = \sum_{\nu=1,2,3} \frac{\hbar^2}{2\mathcal{I}_\nu} \boldsymbol{R}_\nu^2 \tag{1}$$

with $\boldsymbol{R}$ representing the angular momentum of the core which is related to the angular momenta of the nucleus ($\boldsymbol{I}$) and of the proton ($\boldsymbol{j}$) by $\boldsymbol{I} = \boldsymbol{R} + \boldsymbol{j}$. The quantities $\mathcal{I}_\nu$ are the moments of inertia given by the hydrodynamical relation

$$\mathcal{I}_\nu = \frac{4}{3}\mathcal{I}_0(I)\sin^2\left(\gamma - \frac{2\pi\nu}{3}\right), \tag{2}$$

where $0^\circ \leqslant \gamma \leqslant 60^\circ$ is the asymmetry parameter. The moments of inertia may depend on the angular momentum, and hence in the spirit of the variable moment of inertia (VMI) model, one has [15] $\mathcal{I}_0(I) = \mathcal{I}_0\sqrt{1 + bI(I+1)}$, where b is the VMI parameter and the constant $\mathcal{I}_0$ is evaluated by fitting the energy of the first excited 2^+ state [16]. All details regarding the wave functions of the parent and daughter nuclei can be found in Ref. [7].

The decay width, corresponding to the outgoing proton with a given spin j and orbital angular momentum l, is obtained [3] from the overlap of the initial parent state, and the final one which is a coupling between the daughter and emitted proton wave functions. This yields [7] the expression

$$\Gamma_{lj}^{IR} = \frac{\hbar^2 k}{\mu} \frac{2(2R+1)}{2I+1} \left| \sum_{\sigma,K,\Omega,K_R}' u_\sigma^f \langle j\Omega R K_R | IK\rangle \times g_{RK_R}^\tau a_{\sigma,K}^I \frac{\phi_{lj}^{\sigma\Omega}(r)}{G_l(kr) + iF_l(kr)} \right|^2. \tag{3}$$

In the equation above, the symbol σ specifies the single-particle basis states considered, and the prime in the summation stands for the constraint that imposes $K - \Omega$ to be an even integer. F and G are the regular and irregular Coulomb functions, respectively. The coefficients $a_{\sigma K}^I$ are the components of the eigenvectors of the Coriolis interaction between quasiparticles and $\phi_{lj}^{\Omega\sigma}(r)$ are the radial components of the eigenfunctions of the Nilsson Hamiltonian. The quantity $|u_\sigma^f|^2$ gives the probability that the proton single-particle level in the daughter nucleus is empty and is obtained from the BCS calculation. In the case of adiabatic calculations the values of $a_{\sigma K}^I$ are Kronecker symbols since the Coriolis matrix is diagonal. For decay to the ground state ($R = 0$) of the daughter nucleus, angular momentum conservation imposes that the angular momentum of the escaping proton (j) has to be equal to the angular momentum of the decaying nucleus (I), while if $R \neq 0$ then different values of j are allowed and the total width is calculated as

$$\Gamma^{IR} = \sum_{j=|R-I|}^{R+I} \Gamma_{lj}^{IR}. \tag{4}$$

The branching ratio for decay to the first excited 2^+ state is obtained from the relation $\Gamma^{I2}/(\Gamma^{I2} + \Gamma^{I0})$.

Before applying our formalism to the calculation of proton emission observables, we should be aware of several parameters involved. The most important ones are the deformation parameters. ^{141}Ho is predicted by several theoretical works [17] to be highly deformed with axial deformation $0.29 \lesssim \beta_2 \lesssim 0.35$ and $-0.06 \lesssim \beta_4 \lesssim -0.04$. However this large deformation would imply a high moment of inertia and therefore a low excitation energy of the 2^+ state in the daughter nucleus ^{140}Dy. This is not the case, since the experimental value of this energy was found [18,19] to be 202.2(1) keV. From the Grodzins empirical formula [20,21] that relates the latter quantity to the quadrupole deformation we deduce a value of $\beta_2 = 0.244$. This value was already adopted in the works of Refs. [10,11], together with a $\beta_4 = -0.046$.

Since ^{141}Ho might have a γ deformation, we start our analysis of this parameter by determining the rotational spectrum of ^{140}Dy, which is known, and depends only on the variable momentum of inertia, and non-axial deformation. The calculation of this spectrum also introduces the possibility of fixing the VMI coefficient b. Assuming that the daughter nucleus is a triaxial rotor, the spectrum is calculated with different values for the VMI parameter and the results are given in Fig. 1. These results clearly show that γ deformation is quite important in order to reproduce the experimental level scheme. Using a constant moment of inertia, we see that the triaxial deformation could be $\gamma \sim 30^\circ$. However within the VMI approach, the nucleus could have a lower γ deformation around 20°, assuming $b = 0.01$. In the case of ^{142}Dy [8], it was clear that only $b = 0.01$ could reproduce the experimental level scheme, and hence one does not expect it to be much different in the nearest even–even nucleus. The quasiparticle plus triaxial rotor model (QPTRM) calculations for rotational bands in ^{141}Ho are also consistent with the analysis of the spectrum of ^{140}Dy in fixing the triaxial deformation. These calculations have been reported in Ref. [10], suggesting a triaxial deformation for the ground state between $\gamma = 10^\circ$ and 20°, and hence are not presented here.

It has to be mentioned that, in spite of the dependence of the QPTRM results on parameters besides deformations, each one of them should fulfill a specific constraint. They are the rotational energy E_{2+} of the first excited 2^+ state in the rotor (daughter nucleus), the VMI parameter b, the Coriolis attenuation factor ρ and the pairing gap Δ. The value of E_{2+} of ^{142}Dy was measured [18,19] precisely as 202.2(1) keV, and the parameter b is fixed at 0.01 as we learn from the spectra of ^{140}Dy displayed in Fig. 1, ^{142}Dy and

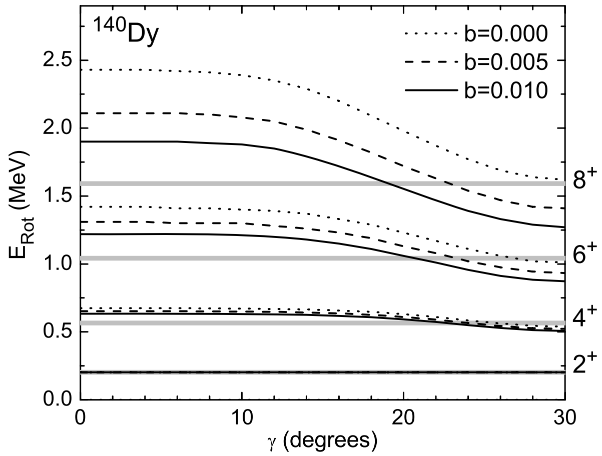

Fig. 1. Level scheme of ^{140}Dy compared with the rotational spectrum calculated using the triaxial rotor model. The thick grey lines represent the experimental levels for which the spins and parities are indicated at the extreme right. The black solid, dashed and dotted lines correspond to the calculations with different values for the variable moment of inertia (VMI) parameter.

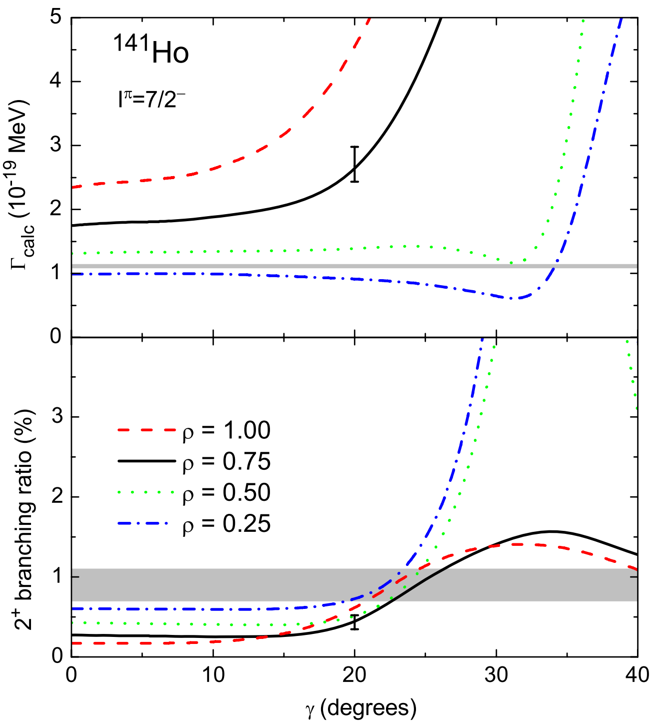

Fig. 2. *Upper panel*: Decay width of proton emission from the ground state of ^{141}Ho as a function of triaxial deformation. *Lower panel*: Branching ratio for proton emission from the $7/2^-$ state to the first excited 2^+ state in the daughter nucleus. In both panels, different curves correspond to calculations with different attenuation factors for the Coriolis interaction labelled in the lower panel. The areas shaded in grey correspond to the experimental values including uncertainties in the data. The error bars on the theoretical curves at $\gamma = 20°$ represents the typical uncertainty in the calculation due to the experimental error in the Q_p value.

^{145}Tm [8]. A pairing strength with $G_\pi = 0.136$ MeV and a Coriolis attenuation of $\rho = 0.85$ are reported [10] to reproduce the ground state band of ^{141}Ho. In order to see the dependence on pairing gap and Coriolis attenuation factor, we have performed calculations varying these parameters.

The decay width and branching ratio for proton emission from the ground state ($7/2^-$) of ^{141}Ho, for different Coriolis attenuation factors are given in Fig. 2 as a function of γ deformation. If we

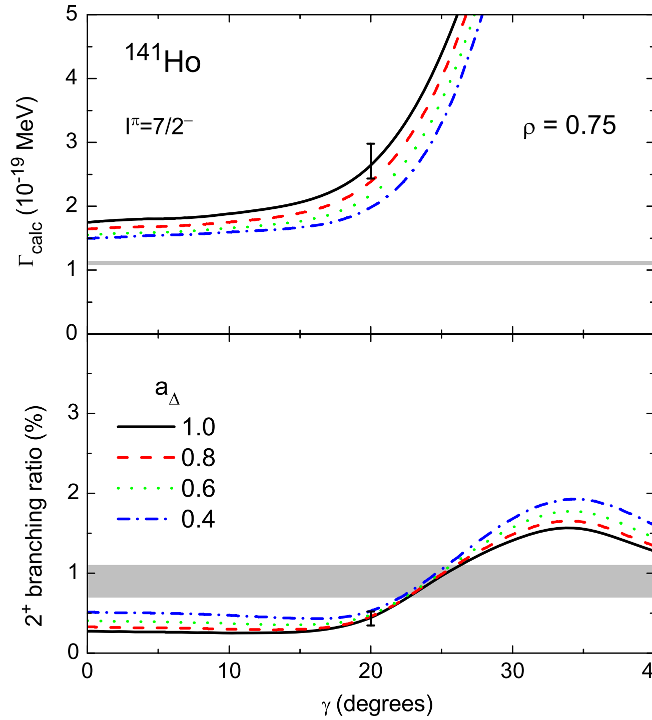

Fig. 3. Same as Fig. 2 but the different curves correspond to different values of the pairing gap parameter denoted by a_Δ, defined as $\Delta = a_\Delta \times 12/\sqrt{A}$.

assume $\rho = 0.75$, which is common in literature, we see that the decay widths are slightly overestimated. This discrepancy is less if we consider the uncertainty in the Q-value represented by the error bar at $\gamma = 20°$. Very strong Coriolis attenuation, $\rho < 0.5$, could explain the decay width but such a scenario is not very realistic. In the high-j orbitals the Coriolis interaction is supposed to be strong and since we have negative parity states arising from the $h_{11/2}$ shell, our results are quite sensitive to ρ. Since the probability for finding a $f_{7/2}$ component in the Nilsson wave-function increases quadratically with β_2 deformation, as can be seen using simple perturbative arguments, a slight decrease in β_2 would imply a large decrease in the decay width, while it would not change appreciably the excitation spectrum. In any case the decay width is not very useful in determining triaxial deformation.

While decay widths might depend strongly on the different parameters entering into the calculation, like for instance deformation, radius of the potential and Q-value, branching ratios are practically insensitive [14] to variations of these quantities, but depend strongly only on the difference of Q-values of the two transitions, which is known experimentally with a very good accuracy, since it is equal to the energy of the gamma transition between the 2^+ and the ground state of the daughter nucleus.

Our results for the branching ratio, with reasonable Coriolis attenuation, show clearly that with an axially symmetric deformation it is impossible to reproduce the experimental value, while for a strong triaxial deformation of $\gamma \sim 20°$ it crosses the data, independently of the attenuation factor. We can also observe that the uncertainty in the Q-value does not affect significantly the calculated branching ratio.

In the above calculations we have used a pairing gap corresponding to $a_\Delta = 1.0$ where a_Δ is defined such that $\Delta = a_\Delta \times 12/\sqrt{A}$. However, in the present case, the choice of pairing gap has a small effect on the decay width and practically does not influence the branching ratio as illustrated in Fig. 3. For ^{141}Ho, the value of $a_\Delta = 1.0$ corresponds to a pairing gap $\Delta = 1.0$ MeV. This is not much different than the choices made in Ref. [11] (0.9 MeV) and Ref. [10] (0.8 MeV), for this parameter.

In Fig. 4, the results for proton emission from the isomeric $(1/2^+)$ state of ^{141}Ho are presented. The decay width and branching ratio are shown as a function of triaxial deformation and for different Coriolis attenuation factors. For a $1/2^+$ state the Coriolis effect on the wave-functions is weaker and hence our results are almost independent of the choice of ρ. The decay width is at least a factor 4 larger than the experimental value and this discrepancy increases for γ greater than $\sim 16°$. The branching ratio is at least a factor of three smaller than the experimental value, and becomes almost zero above $\gamma \sim 25°$. This disagreement with the experimental value is quite surprising since, at least the branching ratio are quite unsensitive to the model parameters.

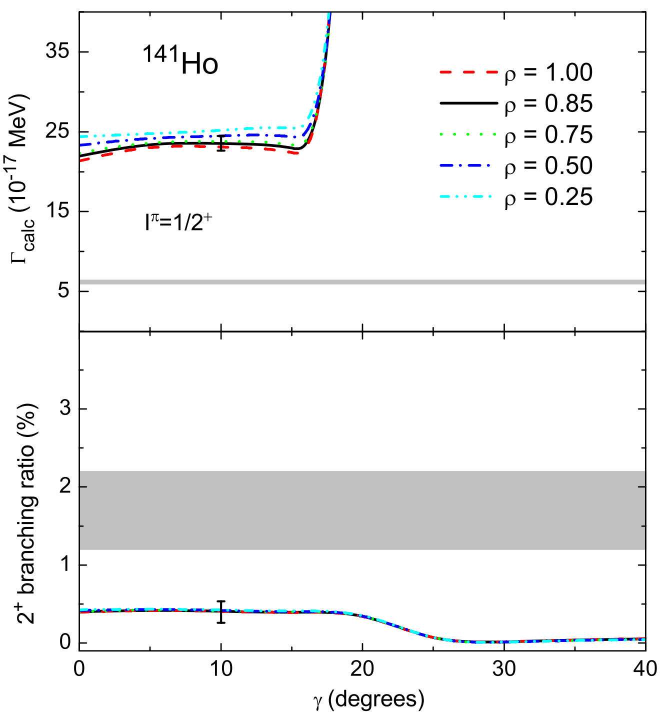

Fig. 4. Same as Fig. 2 but for decay from the isomeric state having spin and parity $I^\pi = 1/2^+$.

The ordering in energy of levels for bands based on $K = 1/2$ Nilsson states, depends on the decoupling parameter [22], therefore the $I = 1/2$ state might not be the lowest one. The wave-functions of these bands are a superposition of $K = 1/2$ and $K = -1/2$ basis states, and the Coriolis interaction acting on them has a diagonal term not present in all other states, which alters the rotational energy spectrum. Therefore, we have calculated the energies of the positive parity states according to the QPTRM model, displayed in Fig. 5.

In fact the $I = 1/2^+$ is always the bandhead, but the difference in energy from the $I = 3/2^+$ is quite small, as supported by the large signature splitting observed in the experimental band [10]. Given this small energy difference, one might thus think that the half-life for gamma decay of the $I = 3/2^+$ state might be quite long compared to the one for proton emission. Thus, we present in Fig. 6 the half-life and branching ratio for the $I = 3/2^+$ state. The agreement with the experimental values for both half-life and branching ratio is excellent, giving support to our interpretation.

For positive parity, in contrast to what happened for the negative parity state, it is not possible to single out one definite value for γ but we can say that it can lie between zero and 20 degrees.

Inspired from the suggestion of Ref. [9] that ^{141}Ho could have a very large deformation, we have repeated the calculation for $\beta_2 = 0.35$. The results are very similar to the ones already presented above for smaller deformation, with the exception that now the $1/2^+$ state is always above the $3/2^+$, and that the latter one reproduces the experimental half-life and branching ratio only for $\gamma \lesssim 10$ degrees. So, if we request that the ground and isomeric state have the same beta and gamma deformation, then the smaller beta deformation is the only able to reproduce all the experimental data.

In summary, with a strong triaxial deformation, we could explain the decay widths and branching ratios of proton emission from both ground and isomeric states of ^{141}Ho. This assignment of deformation is also consistent with the analysis of the rota-

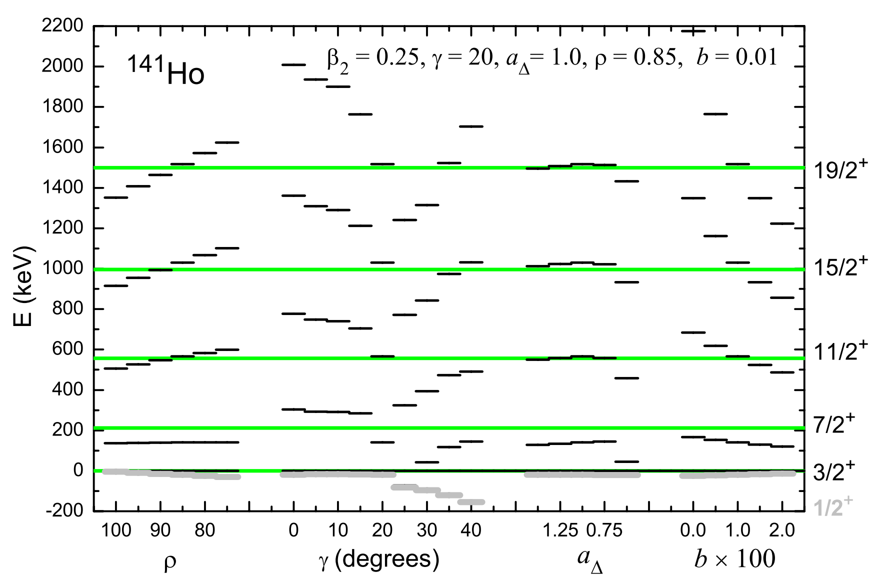

Fig. 5. Positive parity excitation spectrum of ^{141}Ho. The continuous lines spanning the width of the graph correspond to the experimental values [10] and our calculations are represented by the short black lines. The calculations for $1/2^+$ state are represented by thick grey lines. The optimum set of values chosen from the analysis is given in the top of the figure and the variations around these values are indicated in the x-axis.

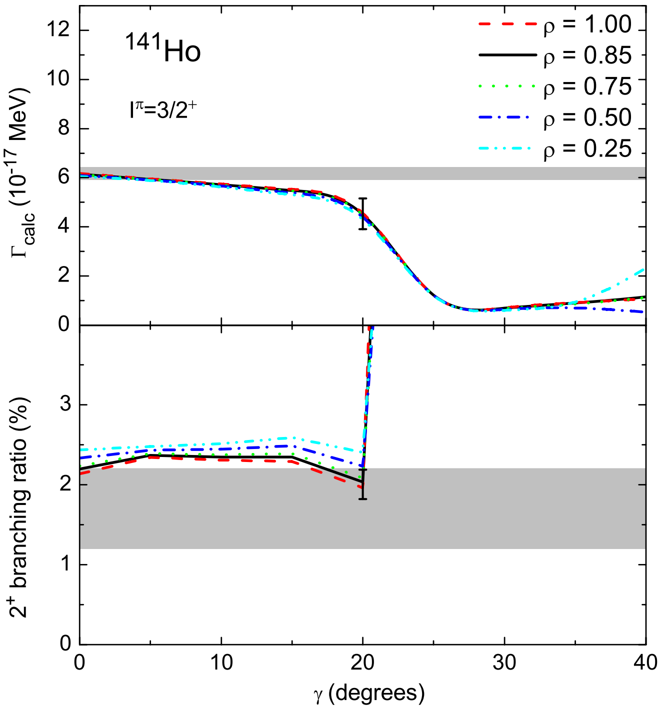

Fig. 6. Same as Fig. 4 but for decay from the isomeric state having spin and parity $I^{\pi} = 3/2^{+}$.

tional spectra of ^{140}Dy and ^{141}Ho. The interpretation of the decay data of the isomeric state lead us to reject the previous assignment of decay from a $I = 1/2^{+}$ with which it is impossible to reproduce the experimental branching ratio. Assuming instead a decaying $I = 3/2^{+}$ state, data are perfectly reproduced. This is further supported by the calculation of the positive parity excitation spectrum that can be done only using the QPTRM model.

The precision of this assignment shows the versatility of proton emission studies.

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