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Internal Structure of the Pion Inspired by the AdS/QCD Correspondence

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Abstract We present a study of the pion structure in the context of the AdS/QCD soft-wall model. This approach provides the light-front wave function of the pion in terms of a set of parameters that we fit to available experimental information on the electromagnetic form factor and parton distribution of the pion. We discuss the corresponding predictions for the unpolarized transverse momentum dependent parton distribution of the pion.

1 Introduction

AdS/QCD correspondence with soft-wall holographic models has been employed in a number of recent works to get new insights on hadronic physics [1–4]. Remarkably, light-front holographic QCD methods (see [5] and references therein for a complete review on the topic) have provided expressions for the light-front wave function (LFWF) of a meson valence Fock state [6,7]. This allows one to obtain direct information about many hadronic observables, which can be expressed in terms of overlaps of LFWFs. The outline of this work is as follows: In Sect. 2 we consider the expressions for the pion LFWF derived in [6,7]. We introduce the quark masses in a Lorentz invariant way and derive analytical expressions for the pion valence Parton Distribution Function (PDF) and electromagnetic Form Factor. We determine the free parameters of the LFWFs by fitting simultaneously the available experimental information on form factor and valence PDF. In Sect. 3, we discuss the corresponding predictions for the pion unpolarized valence Transverse Momentum Dependent parton distribution function (TMD). In the final section, we summarize the results.

2 The Pion LFWF

The LFWFs in the soft-wall model of the AdS/QCD correspondence were originally derived in two different matching procedures [6,7].

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Table 1 Results from the fit of the pure- and effective-valence LFWFs in different quark mass scenarios

LFWF	m (GeV)	κ (GeV)	Q_0 (GeV)	$\chi^2_{\text{d.o.f.}}$ (FF)	$\chi^2_{\text{d.o.f.}}$ (PDF)	$\chi^2_{\text{d.o.f.}}$ (Tot)
	0.005 (fixed)	0.397 ± 0.003	0.500 ± 0.003	1.96	1.19	3.15
$\psi^V_{q\bar{q}/\pi}$	0.200 (fixed)	0.351 ± 0.003	0.491 ± 0.003	9.10	2.66	11.76
	0.0500 ± 0.00004	0.371 ± 0.002	0.498 ± 0.002	1.84	0.41	2.25
	0.005 (fixed)	0.261 ± 0.002	0.498 ± 0.003	4.25	1.19	5.44
$\psi^E_{q\bar{q}/\pi}$	0.200 (fixed)	0.322 ± 0.002	0.630 ± 0.008	11.5	1.43	12.97
	0. (fixed)	0.262 ± 0.002	0.498 ± 0.003	4.17	1.21	5.38

From the matching procedure carried out in [6] the valence quark LFWF for the pion follows immediately and reads:

$$\psi^V_{q\bar{q}/\pi}(x,\ \boldsymbol{k}_\perp) \sim \frac{1}{\kappa\sqrt{(1-x)\,x}} e^{-\frac{1}{2}\frac{k_\perp^2}{\kappa^2 x(1-x)}}. \tag{1}$$

The superscript (V) indicates that we are considering the LFWF for the "pure-valence" state for the pion including only the valence Fock state. The quark masses are included by completing the invariant mass of the system in the argument of the exponential as [8]:

$$M^2 = \sum_i \frac{m_i^2 + \boldsymbol{k}_{\perp i}^2}{x_i} = \frac{m^2 + \boldsymbol{k}_\perp^2}{x(1-x)}, \tag{2}$$

where $m = m_1 = m_2$ and, from momentum conservation, $\boldsymbol{k}_\perp = \boldsymbol{k}_{\perp 1} = -\boldsymbol{k}_{\perp 2}$ and $x = x_1 = 1 - x_2$. As a result, the expression in Eq. (1) becomes (see also [9,10])

$$\psi^V_{q\bar{q}/\pi}(x,\ \boldsymbol{k}_\perp) = A\frac{4\pi}{\kappa\sqrt{(1-x)\,x}} e^{-\frac{1}{2\kappa^2}\left(\frac{m^2}{x}+\frac{m^2}{1-x}+\frac{k_\perp^2}{x(1-x)}\right)}. \tag{3}$$

where A is a normalization constant fixed by the conditions

$$\int_0^1 dx f(x; Q_0) = F(Q^2 = 0) = 1, \tag{4}$$

Using the LFWF overlap representation formulae [11], the PDF and the form factor read

$$f(x; Q_0) = A^2 e^{\left(-\frac{m^2}{\kappa^2 x(1-x)}\right)}; \quad F(Q^2) = A^2 \int_0^1 dx e^{\left(-\frac{m^2}{\kappa^2 x(1-x)} - \frac{Q^2(1-x)}{4\kappa^2 x}\right)}. \tag{5}$$

In Eq. (3) κ is a free parameter corresponding to the physical mass scale which breaks the conformal symmetry. We fix this parameter along with the quark mass m using the experimental data on the electromagnetic form factor of the pion [12–15] and the parametrization of the PDF in [16]. The latter comparison has to be done after evolving the PDF from the scale of the model Q_0 to the scale $Q = 5$ GeV of the parametrization. We do so by using the Hoppet evolution code [17] inside the fitting procedure, leaving the initial scale Q_0 of the model as an additional free parameter of the fit. For parameter m, we consider three scenarios: either we fix it to a value of 5 MeV (current quark mass), or to a value of 200 MeV (constituent quark mass), or we leave it as a free parameter in the fit.

Our results for the set of parameters in the different quark-mass scenarios are listed in the upper part of Table 1. The value of κ used in Refs. [6,18] is compatible with our results, while a larger value of $\kappa = 0.54$ GeV is needed to describe the hadronic masses and the Regge trajectories [10,19–21].

An alternative expression for the LFWF has been derived in [7], considering the matching with a dressed current. In this case, one obtains a LFWF which incorporates effects due to Fock states beyond the valence sector, and therefore represents an "effective" two-parton state of the pion. Introducing the quark-mass dependence as outlined above, the effective LFWF reads

$$\psi^E_{q\bar{q}/\pi}(x,\ \boldsymbol{k}_\perp) = 4\pi A \frac{\sqrt{\log\left(\frac{1}{x}\right)}}{\kappa\,(1-x)} e^{-\frac{k_\perp^2+m^2}{2\kappa^2}\frac{\log\left(\frac{1}{x}\right)}{(1-x)^2}}. \tag{6}$$

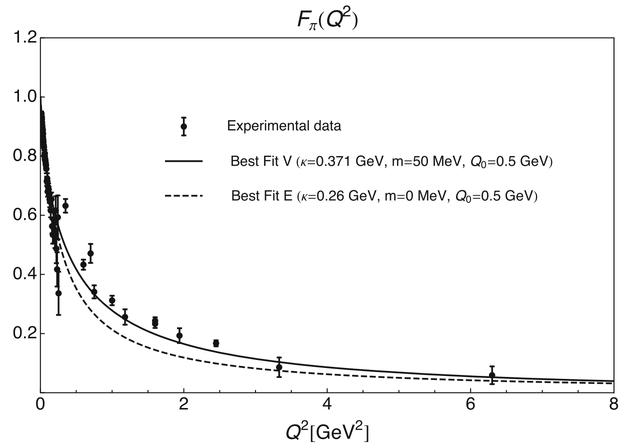

Fig. 1 Results for the electromagnetic form factor from the pure-valence LFWF (*solid curve*) and the effective LFWF (*dashed curve*) with the best-fit parameters. The experimental data are from Refs. [12–15]

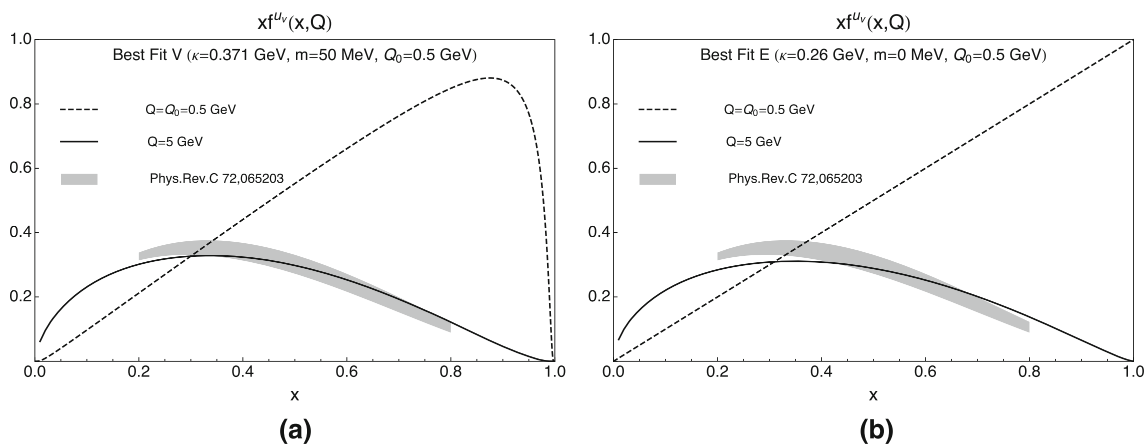

Fig. 2 Results for the quark PDF of the pion from the pure-valence LFWF (**a**) and the effective LFWF (**b**) with the best-fit parameters. *Dashed curves* at the initial scale of the model. *Solid curves* LO evolved to $Q = 5\,\text{GeV}$. *Dashed band* LO parametrization at $Q = 5\,\text{GeV}$ from Ref. [16]

The explicit expression for the form factor and PDF from the effective LFWF will be shown elsewhere [22]. Following the same fitting procedure as for the valence LFWF, with the quark mass fixed to three different values ($m = 0$, 5, and 200 MeV), we obtain the parameter values quoted in the lower part of Table 1.

In Fig. 1 we show the results for the pion form factor obtained from the pure-valence (solid curve) and effective (dashed curve) LFWFs, with the best-fit parameter set, i.e. the values with the best total $\chi^2_{\text{d.o.f}}$ in Table 1. The corresponding results for the PDF are shown in Fig. 2a, b, respectively. The dashed curves show the results at the hadronic scale, and the solid curves are obtained after leading-order (LO) evolution to $Q = 5\,\text{GeV}$. The shaded band corresponds to the results from the parametrization at $Q = 5\,\text{GeV}$ of Ref. [16] (shaded band). The results from the pure-valence LFWF are in good agreement with the available experimental information, while a worse comparison, especially for the form factor, is obtained in the case of the effective LFWF. Our results can be compared with those obtained in, e.g., Refs. [9,21,23,24].

3 TMD Analysis

The unpolarized TMD $f_1(x, \boldsymbol{k}_\perp^2)$ can be obtained from the following overlap of LFWF [25]

$$f_1\left(x, \boldsymbol{k}_\perp^2; Q_0^2\right) = \frac{1}{16\pi^3} |\psi_{q\bar{q}/\pi}\left(x,\ \boldsymbol{k}_\perp\right)|^2. \tag{7}$$

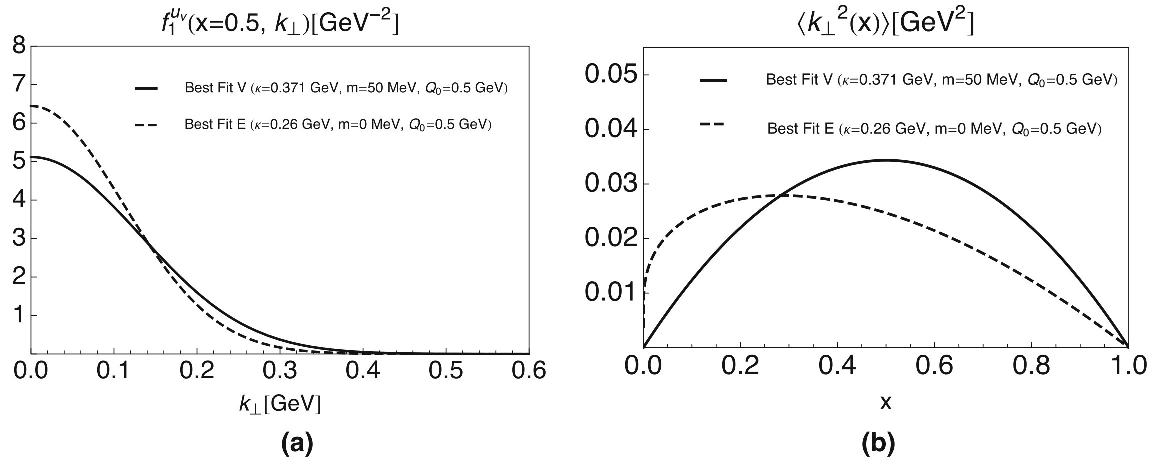

Fig. 3 **a** The unpolarized quark TMD of the pion as function of $k_\perp$ at fixed $x = 0.5$. **b** $\langle \boldsymbol{k}_\perp^2(x)\rangle$ as function of x. *Solid curves* results from the pure-valence LFWF. *Dashed curves* results from the effective LFWF

Using the expressions in Eqs. (3) and (6), one finds that the TMD in both models is a gaussian distribution in $k_\perp = |\boldsymbol{k}_\perp|$ with an x-dependent mean square transverse momenta given by

$$\langle \boldsymbol{k}_\perp^2(x)\rangle^V = \kappa^2 x(x-1), \quad \langle \boldsymbol{k}_\perp^2(x)\rangle^E = \frac{\kappa^2(x-1)^2}{\log(x)}. \tag{8}$$

In the pure-valence case (V), $\langle \boldsymbol{k}_\perp^2(x)\rangle$ is symmetric under exchange of $x \to 1-x$, with a maximum at $x = 0.5$. There is no such a symmetry in the effective case (E) and the maximum is $x = 0.28$ (Fig. 3a). Integration over x produces the values $\langle \boldsymbol{k}_\perp^2\rangle^V = 0.023\,\text{GeV}^2$ and $\langle \boldsymbol{k}_\perp^2\rangle^E = 0.020\,\text{GeV}^2$ at the scale of the model (Fig. 3b).

To compute the pion TMD at some higher scale, closer to experimental data, the effect of TMD evolution has to be included. The TMD is dramatically broadened, particularly at low x. These effects will be explicitly discussed in an upcoming work [22].

4 Conclusions

We presented a study of the pion LFWF obtained from AdS/QCD correspondence with a soft-wall holographic model. We consider two different variants of the LFWF, which we denote with "pure valence" and "effective" ones, already proposed in the literature. We obtained the expressions of the pion valence parton distribution function, the pion form factor and the pion unpolarized TMD. The free parameters of the model are the quark mass m, the scale of the conformal symmetry breaking κ, the initial scale of the model Q_0. We fixed these parameters by reproducing the pion form factor data and the parametrization of the pion valence PDF. The best fits were obtained with values of Q_0 around 0.50 GeV and values of κ below 400 MeV. The best description of the data was obtained with the pure-valence version, with $m = 0.050$ GeV. We studied the pion unpolarized TMD, computing in particular the mean square transverse momentum of the distribution. Comparison with experimental data will be possible only after the application of TMD evolution, which dramatically broadens the TMD when going from the model scale to the experimental one.

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