Hindawi Publishing Corporation
Advances in Mechanical Engineering
Volume 2014, Article ID 514639, 5 pages
http://dx.doi.org/10.1155/2014/514639

Research Article

Local Fractional Fourier Series Solutions for Nonhomogeneous Heat Equations Arising in Fractal Heat Flow with Local Fractional Derivative

Ai-Min Yang,[1,2] Carlo Cattani,[3] Ce Zhang,[4] Gong-Nan Xie,[5] and Xiao-Jun Yang[6]

[1] *College of Science, Hebei United University, Tangshan, China*
[2] *College of Mechanical Engineering, Yanshan University, Qinhuangdao, China*
[3] *Department of Mathematics, University of Salerno, Via Ponte don Melillo, Fisciano, 84084 Salerno, Italy*
[4] *School of Computer Science and Technology, Harbin Institute of Technology, Harbin 150001, China*
[5] *School of Mechanical Engineering, Northwestern Polytechnical University, Xi'an, Shaanxi, China*
[6] *Department of Mathematics and Mechanics, China University of Mining and Technology, Xuzhou, Jiangsu 221008, China*

Correspondence should be addressed to Xiao-Jun Yang; dyangxiaojun@163.com

Received 22 December 2013; Accepted 29 January 2014; Published 2 March 2014

Academic Editor: Yonghui Xie

The fractal heat flow within local fractional derivative is investigated. The nonhomogeneous heat equations arising in fractal heat flow are discussed. The local fractional Fourier series solutions for one-dimensional nonhomogeneous heat equations are obtained. The nondifferentiable series solutions are given to show the efficiency and implementation of the present method.

1. Introduction

In porous media, there are fractals [1], which are applied to describe the heat transfer [2–12]. For example, Yu and Cheng considered the fractal characteristics of effective thermal conductivity in porous media [3]. Xiao and coauthors studied the fractal behaviors of the nucleate pool boiling heat transfer of nanofluids [4]. Le Mehaute and Crepy investigated the geometry of kinetics arising in fractal transfer [5]. Ghodoossi analyzed the thermal behavior for fractal microchannel network [6]. Lahey Jr. suggested the application of heat transfer in fractal geometry theory [7]. Pence proposed the effective heat transport in fractal-like flow networks [8]. Hu and coauthors reported the discontinuous heat transfer in fractal one-phase flows [9]. Liu and coauthors presented fractal heat transfer in microchannel heat sink for cooling of electronic chips [10]. Cai and Huai used the lattice Boltzmann method to model the fluid-solid coupling heat transfer in fractal porous medium [11]. Blyth and Pozrikidis discussed the heat conduction in fractal-like surfaces [12].

There are some models for the differential equation arising in heat transfer [13–16] in fractal media by using fractional calculus [17]. Recently, the local fractional model for heat conduction arising in fractal heat transfer was considered in [18–21]. In [18], the local fractional variational iteration method was applied to solve the heat conduction equations with local fractional derivative. Local fractional Adomian decomposition method was used to obtain the solution for the one-dimensional heat equations with the heat generation arising in fractal transient conduction in [20]. The heat conduction equation with local fractional derivative by using the Cantor-type cylindrical-coordinate method was presented [21].

The aim of this paper is to investigate the nonhomogeneous heat equations arising in fractal heat flow with local fractional derivative and to deal with the one-dimension nonhomogeneous heat equation by using the local fractional Fourier series method. This paper is structured as follows. In Section 2, we investigate the fractal heat flow via local fractional derivative. In Section 3, the notion for local fractional Fourier series method is presented. The solution for the one-dimensional nonhomogeneous heat equation arising fractal heat flow is shown in Section 4. Finally, Section 5 is conclusions.

2. Fractal Heat Flow via Local Fractional Derivative

In order to investigate the governing equations for fractal heat flow, we consider that $u(x, y, z, t)$ is the temperature at the point $(x, y, z) \in \Omega$, time $t \in T$, and $H(t)$ is the total amount of heat contained in Ω. We have

$$H(t) = \iiint c_\alpha \rho_\alpha u(x, y, z, t)\, d\Omega^{(\gamma)}, \tag{1}$$

where the local fractional volume integral of the function $\mathbf{u}$ is defined by [19]

$$\iiint \mathbf{u}(r_P)\, d\Omega^{(\gamma)} = \lim_{N\to\infty} \sum_{P=1}^{N} \mathbf{u}(r_P)\, \Delta\Omega_P^{(\gamma)}, \tag{2}$$

with the elements of volume $\Delta\Omega_P^{(\gamma)} \to 0$ as $N \to \infty$ and fractal dimension of volume γ, c_α is the special heat of the fractal material and ρ_α is the density of the fractal material.

From (1) the change in heat is given by

$$\frac{d^\alpha}{dt^\alpha} H(t) = \iiint c_\alpha \rho_\alpha u_t^{(\alpha)}(x, y, z, t)\, d\Omega^{(\gamma)}. \tag{3}$$

Local fractional Fourier's law of the material in fractal media gives [19]

$$\frac{d^\alpha}{dt^\alpha} H(t) = \oiint_{\partial\Omega^{(\beta)}} k^{2\alpha} \nabla^\alpha u(x, y, z, t) \cdot d\mathbf{S}^{(\beta)}, \tag{4}$$

where $\partial\Omega^{(\beta)}$ is the boundary of $\Omega^{(\gamma)}$, $d\mathbf{S}^{(\beta)}$ is the fractal surface measure over $\Omega^{(\gamma)}$, and $k^{2\alpha}$ is the thermal conductivity of the fractal material, and the local fractional surface integral is defined by [19, 22]

$$\iint \mathbf{u}(r_P) \cdot d\mathbf{S}^{(\beta)} = \lim_{N\to\infty} \sum_{P=1}^{N} \mathbf{u}(r_P) \cdot \mathbf{n}_P \Delta S_P^{(\beta)} \tag{5}$$

with N elements of area with a unit normal local fractional vector $\mathbf{n}_P$, $\Delta S_P^{(\beta)} \to 0$ as $N \to \infty$ for $\gamma = (3/2)\beta = 3\alpha$.

Making use of (3) and (4), we obtain

$$\begin{aligned} &\iiint c_\alpha \rho_\alpha u_t^{(\alpha)}(x, y, z, t)\, d\Omega^{(\gamma)} \\ &\quad = \oiint_{\partial\Omega^{(\beta)}} k^{2\alpha} \nabla^\alpha u(x, y, z, t) \cdot d\mathbf{S}^{(\beta)}. \end{aligned} \tag{6}$$

Using the local fractional divergence theorem [19, 22], from (6) we get

$$\begin{aligned} &\oiint_{\partial\Omega^{(\beta)}} k^{2\alpha} \nabla^\alpha u(x, y, z, t) \cdot d\mathbf{S}^{(\beta)} \\ &\quad = \iiint \nabla^\alpha \cdot \left[k^{2\alpha} \nabla^\alpha u(x, y, z, t)\right] d\Omega^{(\gamma)}, \end{aligned} \tag{7}$$

which leads to

$$\iiint \left\{c_\alpha \rho_\alpha u_t^{(\alpha)}(x, y, z, t) - \nabla^\alpha \cdot \left[k^{2\alpha} \nabla^\alpha u(x, y, z, t)\right]\right\} d\Omega^{(\gamma)} = 0. \tag{8}$$

In view of (8), we arrive at

$$c_\alpha \rho_\alpha u_t^{(\alpha)}(x, y, z, t) - \nabla^\alpha \cdot \left[k^{2\alpha} \nabla^\alpha u(x, y, z, t)\right] = 0. \tag{9}$$

Hence, (9) can be rewritten as follows:

$$u_t^{(\alpha)}(x, y, z, t) - D\nabla^{2\alpha} u(x, y, z, t) = 0, \tag{10}$$

where $D = k^{2\alpha}/c_\alpha \rho_\alpha$ and $k^{2\alpha}$ is a constant. For (10), also see [19].

Therefore, from (10) we obtain the one-dimensional heat equation for fractal heat flow in the following form:

$$u_t^{(\alpha)}(x, t) - D u_x^{(2\alpha)}(x, t) = 0. \tag{11}$$

The above result is presented in [18, 19].

When $t = l\tau$, $x = m\eta$, $y = s\zeta$ and $z = j\xi$, (10) changes into

$$U_t^{(\alpha)}(\eta, \zeta, \xi, \tau) - \kappa \nabla^{2\alpha} U(\eta, \zeta, \xi, \tau) = 0, \tag{12}$$

where $\kappa = Dl^\alpha (1/m^{2\alpha} + 1/s^{2\alpha} + 1/j^{2\alpha})$.

When $\kappa = 1$, we have

$$U_t^{(\alpha)}(\eta, \zeta, \xi, \tau) - \nabla^{2\alpha} U(\eta, \zeta, \xi, \tau) = 0. \tag{13}$$

From (13) the one-dimensional heat equation for fractal heat flow reads as

$$U_t^{(\alpha)}(\eta, \tau) - U_\eta^{(2\alpha)}(\eta, \tau) = 0. \tag{14}$$

Meanwhile, an initial value condition of (14) is presented as follows:

$$U(\eta, 0) = \phi(\eta). \tag{15}$$

The Dirichlet boundary value conductions of (14) are considered as follows:

$$U(0, \tau) = 0, \qquad U(L, \tau) = 0. \tag{16}$$

The Neumann boundary value conductions of (14) read as follows:

$$U_\eta^{(\alpha)}(0, \tau) = 0, \qquad U_\eta^{(\alpha)}(L, \tau) = 0. \tag{17}$$

The Robin boundary value conductions of (14) are suggested as follows:

$$U_\eta^{(\alpha)}(0, \tau) - a_0 U(0, \tau) = 0, \qquad U_\eta^{(\alpha)}(L, \tau) + a_L U(L, \tau) = 0. \tag{18}$$

The periodic boundary value conductions of (14) are given by

$$U(-L, \tau) = U(L, \tau), \qquad U_\eta^{(\alpha)}(-L, \tau) = U_\eta^{(\alpha)}(L, \tau). \tag{19}$$

If there is a heat source term $g(\eta, \tau)$, (14) can be rewritten as

$$U_t^{(\alpha)}(\eta, \tau) - U_\eta^{(2\alpha)}(\eta, \tau) = g(\eta, \tau) \tag{20}$$

and the initial and Dirichlet boundary value conductions of (14) are considered as follows:

$$U(\eta, 0) = \phi(\eta), \qquad U(0, \tau) = 0, \qquad U(L, \tau) = 0. \tag{21}$$

3. Local Fractional Fourier Series Method

In this section, the local fractional Fourier series is a generalized Fourier series based upon the local fractional calculus. We introduce the basic concept of local fractional Fourier series [23, 24].

Suppose that the quantity $f(x)$ is a local fractional continuous function on the interval (a,b); we denote it as follows [18–20]:

$$f(x) \in C_\alpha(a,b). \tag{22}$$

Definition 1. Let $f(x) \in C_\alpha(a,b)$. Local fractional derivative of $f(x)$ of order α at $x = x_0$ is given by [18–22]

$$\begin{aligned} D_x^{(\alpha)} f(x_0) = f^{(\alpha)}(x_0) &= \left.\frac{d^\alpha f(x)}{dx^\alpha}\right|_{x=x_0} \\ &= \lim_{x \to x_0} \frac{\Delta^\alpha (f(x) - f(x_0))}{(x - x_0)^\alpha}, \end{aligned} \tag{23}$$

where $\Delta^\alpha(f(x) - f(x_0)) \cong \Gamma(1+\alpha)\Delta(f(x) - f(x_0))$.

Definition 2. Let $f(x) \in C_\alpha(a,b)$. Local fractional integral operator $f(x)$ of order α is given by [18–20, 23, 24]

$$\begin{aligned} {}_aI_b^{(\alpha)} f(x) &= \frac{1}{\Gamma(1+\alpha)} \int_a^b f(t)(dt)^\alpha \\ &= \frac{1}{\Gamma(1+\alpha)} \lim_{\Delta t \to 0} \sum_{j=0}^{j=N-1} f(t_j)(\Delta t_j)^\alpha, \end{aligned} \tag{24}$$

where $\Delta t_j = t_{j+1} - t_j$, $\Delta t = \max\{\Delta t_0, \Delta t_1, \ldots, \Delta t_j, \ldots\}$ and $[t_j, t_{j+1}]$, $j = 0, \ldots, N-1$, $t_0 = a$, $t_N = b$, is a partition of the interval $[a,b]$.

Definition 3. Let $f(x) \in C_\alpha(-\infty, +\infty)$ and $f(x)$ be $2l$-periodic. For $k \in Z$, local fraction Fourier series of $f(x)$ is defined as (see [23, 24])

$$f(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty}\left(a_n \cos_\alpha \frac{\pi^\alpha (kx)^\alpha}{l^\alpha} + b_n \sin_\alpha \frac{\pi^\alpha (kx)^\alpha}{l^\alpha}\right), \tag{25}$$

where the local fraction Fourier coefficients are as follows:

$$\begin{aligned} a_n &= \left(\frac{2}{l}\right)^\alpha \int_0^l f(x)\cos_\alpha \frac{\pi^\alpha (kx)^\alpha}{l^\alpha}(dx)^\alpha, \\ b_n &= \left(\frac{2}{l}\right)^\alpha \int_0^l f(x)\sin_\alpha \frac{\pi^\alpha (kx)^\alpha}{l^\alpha}(dx)^\alpha. \end{aligned} \tag{26}$$

Making use of (26), we have the weight forms of the local fractional Fourier series as follows:

$$\begin{aligned} a_k &= \frac{(1/\Gamma(1+\alpha))\int_0^l f(x)\cos_\alpha(\pi^\alpha (kx)^\alpha / l^\alpha)(dx)^\alpha}{(1/\Gamma(1+\alpha))\int_0^l \cos_\alpha^2(\pi^\alpha (kx)^\alpha / l^\alpha)(dx)^\alpha}, \\ b_k &= \frac{(1/\Gamma(1+\alpha))\int_0^l f(x)\sin_\alpha(\pi^\alpha (kx)^\alpha / l^\alpha)(dx)^\alpha}{(1/\Gamma(1+\alpha))\int_0^l \sin_\alpha^2(\pi^\alpha (kx)^\alpha / l^\alpha)\ (dx)^\alpha}. \end{aligned} \tag{27}$$

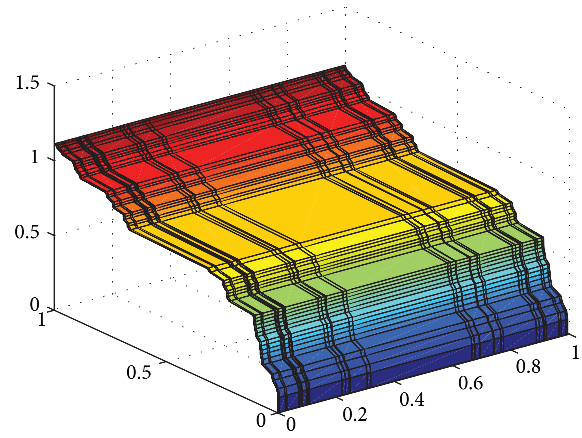

FIGURE 1: The graph of the heat source term $g(\eta,\tau) = \eta^\alpha/\Gamma(1+\alpha)$ with parameter $\alpha = \ln 2/\ln 3$.

4. Solution for the One-Dimensional Nonhomogeneous Heat Equation Arising Fractal Heat Flow

In this section, we present the local fractional Fourier series solutions for nonhomogeneous heat equation arising in fractal heat flow in one-dimensional case. In order to discuss it, we give the special example as follows.

We take the heat source term,

$$g(\eta,\tau) = \frac{\eta^\alpha}{\Gamma(1+\alpha)}. \tag{28}$$

When the fractal dimension α is equal to $\ln 2/\ln 3$, the graph of the heat source term is shown in Figure 1.

Hence, (20) can be rewritten as follows:

$$U_t^{(\alpha)}(\eta,\tau) - U_\eta^{(2\alpha)}(\eta,\tau) = \frac{\eta^\alpha}{\Gamma(1+\alpha)}, \tag{29}$$

subject to the initial boundary value conditions

$$U(\eta,0) = \frac{\eta^\alpha}{\Gamma(1+\alpha)}, \qquad U(0,\tau) = 0, \qquad U(L,\tau) = 0. \tag{30}$$

When the fractal dimension α is equal to $\ln 2/\ln 3$, the graph of the initial value condition is shown in Figure 2.

We now consider the local fractional Fourier solution in the following form:

$$U(\eta,\tau) = \sum_{n=1}^{\infty} U_n(\tau)\sin_\alpha \frac{\pi^\alpha (n\eta)^\alpha}{L^\alpha}. \tag{31}$$

Here, we have $\{g_n(\tau)\}_{n=1}^{\infty}$ such that

$$g(\eta,\tau) = \sum_{n=1}^{\infty} g_n(\tau)\sin_\alpha \frac{\pi^\alpha (n\eta)^\alpha}{L^\alpha}, \tag{32}$$

and $\{U_n(0)\}_{n=1}^{\infty}$ such that

$$U(\eta,0) = \sum_{n=1}^{\infty} U_n(0)\sin_\alpha \frac{\pi^\alpha (n\eta)^\alpha}{L^\alpha}, \tag{33}$$

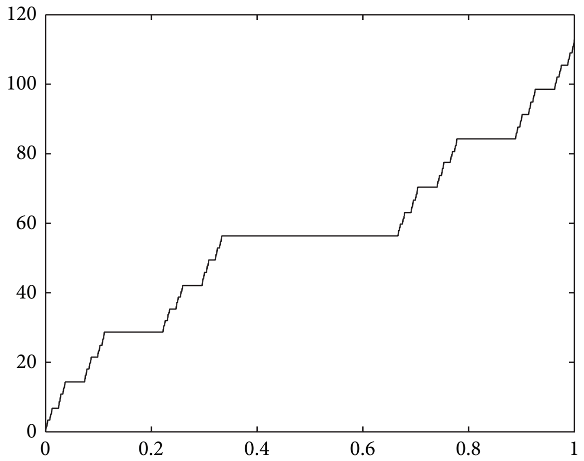

FIGURE 2: The graph of initial value condition $U(\eta, 0) = \eta^\alpha / \Gamma(1+\alpha)$ with parameter $\alpha = \ln 2/\ln 3$.

where

$$\begin{aligned} g_n(\tau) &= \left(\frac{L}{2}\right)^\alpha \int_0^L g(\eta,\tau) \sin_\alpha \frac{\pi^\alpha (n\eta)^\alpha}{L^\alpha} (d\eta)^\alpha \\ &= \left(\frac{L}{2}\right)^\alpha \int_0^L \frac{\eta^\alpha}{\Gamma(1+\alpha)} \sin_\alpha \frac{\pi^\alpha (n\eta)^\alpha}{L^\alpha} (d\eta)^\alpha \\ &= -\frac{L^{3\alpha}}{(2\pi n)^\alpha}, \end{aligned} \tag{34}$$

$$U_n(0) = \left(\frac{L}{2}\right)^\alpha \int_0^L U(\eta,0) \sin_\alpha \frac{\pi^\alpha (n\eta)^\alpha}{L^\alpha} (d\eta)^\alpha. \tag{35}$$

Submitting (31), (32), and (34) into (29) yields

$$\sum_{n=1}^{\infty} \left\{ \frac{d^\alpha U_n(\tau)}{d\tau^\alpha} - \frac{\pi^\alpha k^\alpha}{L^\alpha} U_n(\tau) - g_n(\tau) \right\} \sin_\alpha \frac{(n\eta)^\alpha \pi^\alpha}{L^\alpha} = 0, \tag{36}$$

which leads to

$$\frac{d^\alpha U_n(\tau)}{d\tau^\alpha} - \frac{\pi^\alpha n^\alpha}{L^\alpha} U_n(\tau) - g_n(\tau) = 0. \tag{37}$$

Making use of (30), (33), and (35) with $\tau = 0$, we have

$$\sum_{n=1}^{\infty} \{ U_n(0) - c_n(\tau) \} \sin_\alpha \frac{(n\eta)^\alpha \pi^\alpha}{L^\alpha} = 0 \tag{38}$$

such that

$$U_n(0) - c_n(\tau) = 0, \tag{39}$$

where

$$\begin{aligned} c_n &= \left(\frac{L}{2}\right)^\alpha \int_0^L \frac{\eta^\alpha}{\Gamma(1+\alpha)} \sin_\alpha \frac{\pi^\alpha (n\eta)^\alpha}{L^\alpha} (d\eta)^\alpha \\ &= -\frac{L^{3\alpha}}{(2\pi n)^\alpha}. \end{aligned} \tag{40}$$

In view of (37) and (39), we get the system of boundary value problem in the following form:

$$\begin{gathered} \frac{d^\alpha U_n(\tau)}{d\tau^\alpha} - \frac{\pi^\alpha n^\alpha}{L^\alpha} U_n(\tau) - g_n(\tau) = 0, \\ U_n(0) - c_n = 0, \\ n = 1, 2, \ldots. \end{gathered} \tag{41}$$

So, we obtain the solution of (40) in the following form:

$$\begin{aligned} U_n(\tau) &= c_n E_\alpha \left(\frac{\pi^\alpha n^\alpha \tau^\alpha}{L^\alpha} \right) \\ &\quad + \frac{1}{\Gamma(1+\alpha)} \int_0^\tau E_\alpha \left(\frac{\pi^\alpha n^\alpha (\tau - s)^\alpha}{L^\alpha} \right) g_n(s) (ds)^\alpha. \end{aligned} \tag{42}$$

Hence, the local fractional Fourier series solution reads as follows:

$$\begin{aligned} U(\eta,\tau) &= \sum_{n=1}^{\infty} \left\{ c_n E_\alpha \left(\frac{\pi^\alpha n^\alpha \tau^\alpha}{L^\alpha} \right) + \frac{1}{\Gamma(1+\alpha)} \right. \\ &\qquad \left. \times \int_0^\tau E_\alpha \left(\frac{\pi^\alpha n^\alpha (\tau - s)^\alpha}{L^\alpha} \right) g_n(s) (ds)^\alpha \right\} \\ &\quad \times \sin_\alpha \frac{\pi^\alpha (n\eta)^\alpha}{L^\alpha} \\ &= \sum_{n=1}^{\infty} c_n E_\alpha \left(\frac{\pi^\alpha n^\alpha \tau^\alpha}{L^\alpha} \right) \sin_\alpha \frac{\pi^\alpha (n\eta)^\alpha}{L^\alpha} \\ &\quad + \sum_{n=1}^{\infty} \left\{ \frac{1}{\Gamma(1+\alpha)} \int_0^\tau E_\alpha \left(\frac{\pi^\alpha n^\alpha (\tau - s)^\alpha}{L^\alpha} \right) g_n(s) (ds)^\alpha \right\} \\ &\quad \times \sin_\alpha \frac{\pi^\alpha (n\eta)^\alpha}{L^\alpha} \\ &= -\sum_{n=1}^{\infty} \frac{L^{3\alpha}}{(2\pi n)^\alpha} E_\alpha \left(\frac{\pi^\alpha n^\alpha \tau^\alpha}{L^\alpha} \right) \sin_\alpha \frac{\pi^\alpha (n\eta)^\alpha}{L^\alpha} \\ &\quad - \sum_{n=1}^{\infty} \left\{ \left[\frac{1}{\Gamma(1+\alpha)} \int_0^\tau E_\alpha \left(\frac{\pi^\alpha n^\alpha (\tau - s)^\alpha}{L^\alpha} \right) \right. \right. \\ &\qquad \left. \left. \times \frac{L^{3\alpha}}{(2\pi n)^\alpha} (ds)^\alpha \right] \sin_\alpha \frac{\pi^\alpha (n\eta)^\alpha}{L^\alpha} \right\} \\ &= -\sum_{n=1}^{\infty} \frac{L^{3\alpha}}{(2\pi n)^\alpha} E_\alpha \left(\frac{\pi^\alpha n^\alpha \tau^\alpha}{L^\alpha} \right) \sin_\alpha \frac{\pi^\alpha (n\eta)^\alpha}{L^\alpha} \\ &\quad - \sum_{n=1}^{\infty} \left\{ \left[\frac{L^{4\alpha}}{(2\pi^2 n^2)^\alpha} \left(E_\alpha \left(\frac{\pi^\alpha n^\alpha \tau^\alpha}{L^\alpha} \right) - 1 \right) \right] \right. \\ &\qquad \left. \times \sin_\alpha \frac{\pi^\alpha (n\eta)^\alpha}{L^\alpha} \right\}. \end{aligned} \tag{43}$$

5. Conclusions

In this work we derived the nonhomogeneous heat equations arising in fractal heat flow based upon the local fractional vector calculus. Meanwhile, we presented the different classes of boundary value problems for local fractional heat equations. Finally, the local fractional Fourier solution for nonhomogeneous heat equations arising in fractal heat flow was obtained by using the local fractional series. It is shown that the components of the result are nondifferentiable functions.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

This work was supported by National Scientific and Technological Support Projects (no. 2012BAE09B00), the National Natural Science Foundation of China (nos. 11126213 and 61170317), and the National Natural Science Foundation of Hebei Province (nos. A2012209043 and E2013209215).

References

[1] B. Yu and J. Li, "Some fractal characters of porous media," *Fractals*, vol. 9, no. 3, pp. 365–372, 2001.

[2] G. A. Ledezma, A. Bejan, and M. R. Errera, "Constructal tree networks for heat transfer," *Journal of Applied Physics*, vol. 82, no. 1, pp. 89–100, 1997.

[3] B. Yu and P. Cheng, "Fractal models for the effective thermal conductivity of bidispersed porous media," *Journal of Thermophysics and Heat Transfer*, vol. 16, no. 1, pp. 22–29, 2002.

[4] B. Xiao, G. Jiang, and L. Chen, "A fractal study for nucleate pool boiling heat transfer of nanofluids," *Science China: Physics, Mechanics and Astronomy*, vol. 53, no. 1, pp. 30–37, 2010.

[5] A. Le Mehaute and G. Crepy, "Introduction to transfer and motion in fractal media: the geometry of kinetics," *Solid State Ionics*, vol. 9-10, no. 1, pp. 17–30, 1983.

[6] L. Ghodoossi, "Thermal and hydrodynamic analysis of a fractal microchannel network," *Energy Conversion and Management*, vol. 46, no. 5, pp. 771–788, 2005.

[7] R. T. Lahey Jr., "An application of fractal and chaos theory in the field of two-phase flow & heat transfer," *Wärme- und Stoffübertragung*, vol. 26, no. 6, pp. 351–363, 1991.

[8] D. Pence, "The simplicity of fractal-like flow networks for effective heat and mass transport," *Experimental Thermal and Fluid Science*, vol. 34, no. 4, pp. 474–486, 2010.

[9] M. S. Hu, D. Baleanu, and X. J. Yang, "One-phase problems for discontinuous heat transfer in fractal media," *Mathematical Problems in Engineering*, vol. 2013, Article ID 358473, 3 pages, 2013.

[10] S. Liu, Y. Zhang, and P. Liu, "Heat transfer and pressure drop in fractal microchannel heat sink for cooling of electronic chips," *Heat and Mass Transfer*, vol. 44, no. 2, pp. 221–227, 2007.

[11] J. Cai and X. Huai, "Study on fluid-solid coupling heat transfer in fractal porous medium by lattice Boltzmann method," *Applied Thermal Engineering*, vol. 30, no. 6-7, pp. 715–723, 2010.

[12] M. G. Blyth and C. Pozrikidis, "Heat conduction across irregular and fractal-like surfaces," *International Journal of Heat and Mass Transfer*, vol. 46, no. 8, pp. 1329–1339, 2003.

[13] Y. Z. Povstenko, "Theory of thermoelasticity based on the space-time-fractional heat conduction equation," *Physica Scripta*, vol. 2009, Article ID 014017, 2009.

[14] V. Z. Povstenko, "Fractional heat conduction equation and associated thermal stress," *Journal of Thermal Stresses*, vol. 28, no. 1, pp. 83–102, 2005.

[15] J. Hristov, "Heat-balance integral to fractional (half-time) heat diffusion sub-model," *Thermal Science*, vol. 14, no. 2, pp. 291–316, 2010.

[16] J. Hristov, "Approximate solutions to fractional subdiffusion equations," *The European Physical Journal: Special Topics*, vol. 193, no. 1, pp. 229–243, 2011.

[17] B. J. West, M. Bologna, and P. Grigolini, *Physics of Fractal Operators*, Institute for Nonlinear Science, Springer, New York, NY, USA, 2003.

[18] X.-J. Yang and D. Baleanu, "Fractal heat conduction problem solved by local fractional variation iteration method," *Thermal Science*, vol. 17, no. 2, pp. 625–628, 2013.

[19] X.-J. Yang, *Advanced Local Fractional Calculus and Its Applications*, World Science Publisher, New York, NY, USA, 2012.

[20] A. M. Yang, C. Cattani, H. Jafari, and X. J. Yang, "Analytical solutions of the one-dimensional heat equations arising in fractal transient conduction with local fractional derivative," *Abstract and Applied Analysis*, vol. 2013, Article ID 462535, 5 pages, 2013.

[21] X.-J. Yang, H. M. Srivastava, J.-H. He, and D. Baleanu, "Cantor-type cylindrical-coordinate method for differential equations with local fractional derivatives," *Physics Letters A*, vol. 377, pp. 1696–1700, 2013.

[22] Y. Zhao, D. Baleanu, C. Cattani, D. F. Cheng, and X.-J. Yang, "Maxwell's equations on Cantor Sets: a local fractional approach," *Advances in High Energy Physics*, vol. 2013, Article ID 686371, 6 pages, 2013.

[23] X. J. Yang, *Local Fractional Functional Analysis and Its Applications*, Asian Academic Publisher, Hong Kong, 2011.

[24] G. A. Anastassiou and O. Duman, *Advances in Applied Mathematics and Approximation Theory*, Springer, New York, NY, USA, 2013.