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Mode II fracture toughness for non-planar frictional cracks

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Abstract

Traction-free and planar cracks represent a rather idealized picture of the physical reality, commonly used in fracture mechanics problems. In the present paper, the influence of roughness and friction of crack surfaces is examined in relation to both the resulting near-tip stress field and the fracture resistance under monotonic mixed-mode loading. A two-dimensional model is presented where an elastic-plastic-like constitutive interface law is adopted to describe the Mode I/II coupling between displacements and tractions along the crack surfaces. The solution is obtained using the Distributed Dislocation Technique (DDT). By considering a linear piecewise periodic profile of the crack, the present model is employed to quantify the mode II fracture toughness of different types of natural stones under varying mode I compressive load.

Keywords: friction, crack roughness, distributed dislocation technique, crack shielding.

1. Introduction

Observations on the failure of brittle materials, such as concrete, rocks and glass, reveal that crack growth of pre-existing microcracks does not proceed collinearly; on the contrary, cracks tend to kink. This behaviour may be ascribed to the combination of different factors, such as the presence of far field multi-axial stresses, residual stresses, microstructural inhomogeneities, material properties dispersion. As was recognised by several authors (Kitagawa et al., 1975; Evans and Hutchinson, 1989; Gates and Fatemi, 2016), the fracture strength and the crack propagation are strongly influenced by such effects.

Traction-free and planar cracks represent a rather idealized model of the physical reality, commonly used in fracture mechanics problems. For instance, in order to account for the actual features of crack surfaces, the first two authors have recently explored the influence of crack path meandering on fatigue propagation, by modelling the crack profile as a piecewise linear curve in two dimensions (Spagnoli et al., 2015; Brighenti et al., 2014) (the same type of model was initially conceived in the realm of fractal geometry, Carpinteri et al. (2008)).

Many attempts have been made to analytically study the behaviour of cracks in real materials, and here we mention just a few of them, which are consistent with the purpose of our study. The case of a rough and frictional crack in

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an infinite medium, subjected to a remote mode II stress field, has been considered by Ballarini and Plesha (1987), who have used an analytical method to compute the crack tip Stress Intensity Factors (SIFs) and the direction of crack propagation under monotonic loading. A similar approach has also been adopted by Tong et al. (1995), but here, differently, the object of the investigation was the behaviour of cracks under cyclic loading. In particular, their purpose was to quantify the sliding mode crack closure which occurs under shear fatigue loading and is responsible for a reduction of the mode II SIF. An intriguing study on the effect of mixed mode and partial crack closure due to sinusoidal crack profile can be found in Xiaoping and Comninou (1989).

The aforementioned models often differ in the method of solution and computation of the stress intensity factors. In general, the geometrical complexity of real cracks and material models would require a purely numerical technique. However, we would like to remark the importance of analytical methods in fracture mechanics, since they provide the correct form of singularities and serve as a benchmark for the numerical procedures (Erdogan, 2000). In this spirit, we turned our attention to analytical methods, particularly those derived from the application of the complex function theory; among them, a very flexible technique for the solution of crack problems in different geometric configurations is the Distributed Dislocation Technique (DDT) (Hills et al., 2013a).

In the present paper, the model presented in Spagnoli et al. (2018) is applied to analyse the experimental results related to the shear mode fracture toughness of different types of natural stones (Backers et al., 2002). In particular, the effect of crack shielding due to roughness and frictional interface along the crack path is taken into account to predict the fracture resistance for different relative crack sizes and various levels of confinement pressure.

2. Description of the model

In order to consider the effects of friction and roughness, we make use of an interface model formulated as a constitutive relationship between opposing points along the crack. Globally, the crack surfaces are smooth and frictionless, a hypothesis which allows us to obtain a straightforward implementation within the technique of the distributed dislocation. The surface interference is modelled by means of bridging stresses, which are added to the stress distribution determined by the remote loads, and computed at a discrete number of points. Let us define the relative displacement increments between two opposing points as additively composed of a recoverable elastic part dw_i^e and a non-recoverable plastic part dw_i^p, which accounts for frictional sliding and dilatancy. The stresses on the crack interface are related to the displacement increments by means of interface stiffnesses E_{ij}^{EP}:

$$\sigma_i = E_{ij}^{EP}\, dw_j \quad i, j = t, n \tag{1}$$

where t, n denote, respectively, the tangential and normal directions with respect to the nominally flat crack surface (Fig. 1a). The interface stiffnesses are dependent on a slip function F, a slip potential G and their derivative, that is:

$$E_{ij}^{EP} = E_{ij} \quad \text{if} \quad F < 0 \quad \text{or} \quad dF = 0, \qquad E_{ij}^{EP} = E_{ij} - \frac{\frac{\partial F}{\partial \sigma_p} E_{iq} E_{pj} \frac{\partial G}{\partial \sigma_q}}{\frac{\partial F}{\partial \sigma_p} E_{pq} \frac{\partial G}{\partial \sigma_q}} \quad \text{if} \quad F = dF = 0 \tag{2}$$

E_{ij} is the elastic interface stiffness, which has to be calibrated to avoid interpenetrability between the crack surfaces.

The surface roughness is described through a saw-tooth model, characterised by a constant angle α, mean length of the asperities equal to $2L$ and height h (equal to twice the arithmetical mean deviation, which is a widely used roughness parameter of the crack profile). The coefficient of Coulomb friction f is constant everywhere. With the previous assumptions, the functions F and G have the following formulations:

$$F = |\sigma_1| + f\sigma_2 = |\sigma_n \sin\alpha + \sigma_t \cos\alpha| + f(\sigma_n \cos\alpha - \sigma_t \sin\alpha), \qquad G = |\sigma_1| = |\sigma_n \sin\alpha + \sigma_t \cos\alpha| \tag{3}$$

We can define a crack size parameter by considering the ratio c/L, where $c/2L$ approaching the unity ideally identifies the case of a short crack. The parameters used to describe the surface roughness are somehow related to a specific material, and the scale length might differ of several orders of magnitude.

The stress state in an elastic body with a crack can be determined by introducing a suitable distribution of dislocations along the crack line (dislocations are commonly used in the theory of elasticity as kernel of integral equations,

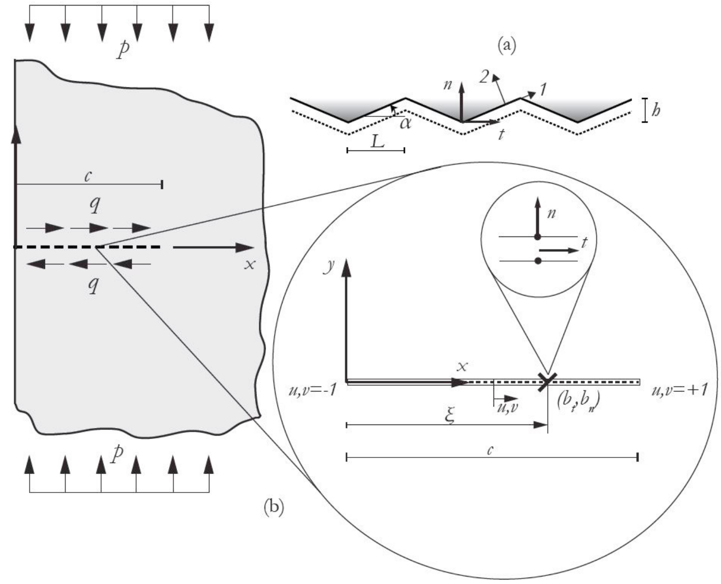

Fig. 1. (a) The saw-tooth asperity model. (b) Schematic model of the geometry with an edge crack of length c.

to describe the singular stress state occurring near a source of discontinuity). The stresses thereby obtained, known as the *corrective term*, assume the following expression (refer to Fig. 1b for an explanation of the variables):

$$\bar{\sigma}_i(x) = \frac{2\mu}{\pi(\kappa+1)} \int_0^c \frac{B_i(\xi)}{x-\xi} + B_j(\xi)F_{ij}(x,\xi)d\xi \tag{4}$$

where μ is the elastic shear modulus and κ is the Kolosov constant of the material, $F_{ij}(x,\xi)$ are influence functions, whose expression for the case considered here can be found in the literature (Hills et al., 2013b) and $B_i(\xi)$ is the dislocation density, which yields the relative displacement between the crack surfaces by integration. Applying the superposition principle, we obtain the stress state along the crack surface, adding the stresses generated by the remote loads $\sigma^\infty(x)$ to the corrective term in (4). With the ideal picture of a traction-free crack, the overall stress state on the surfaces needs to be null; on the contrary, surface interaction adds bridging stresses $\sigma^b(x)$ along the crack, so that the integral formulation is the following

$$\sigma^\infty(x) + \frac{2\mu}{\pi(\kappa+1)} \int_0^c \frac{B_i(\xi)}{x-\xi} + B_j(\xi)F_{ij}(x,\xi)d\xi = \sigma^b(x) \tag{5}$$

Finally, we approximate (5) with a Gauss-Chebyshev numerical quadrature, so that we obtain a set of non-linear algebraic equations:

$$\sigma_i^\infty(v_l) + \frac{2\mu}{\pi(\kappa+1)} \sum_{k=1}^{N} W(u_k)\left[\frac{\phi_i(u_k)}{v_l-u_k} + \phi_j(u_k)F_{ij}(u_k,v_l)\right] = \sigma_i^b(v_l) \tag{6}$$

where $\phi_i(u)$, being $B_i(u) = \phi_i(u)\omega(u)$ ($\omega(u)$ = fundamental singular function), are the unknowns.

We recall that, in (6), u are the integration points at which the displacements are computed, whereas v are the collocation points at which we evaluate the stresses. $W(u)$ are weight functions. An efficient technique to solve the non-linear problem is achieved if we introduce a compliance matrix which directly connects stresses and displacements (Ballarini and Plesha, 1987), so that we eliminate the need to integrate the dislocation densities at each step of the incremental solution. For each increment of the external loads, the stiffness matrix E_{ij}^{EP} in (2) needs to be updated,

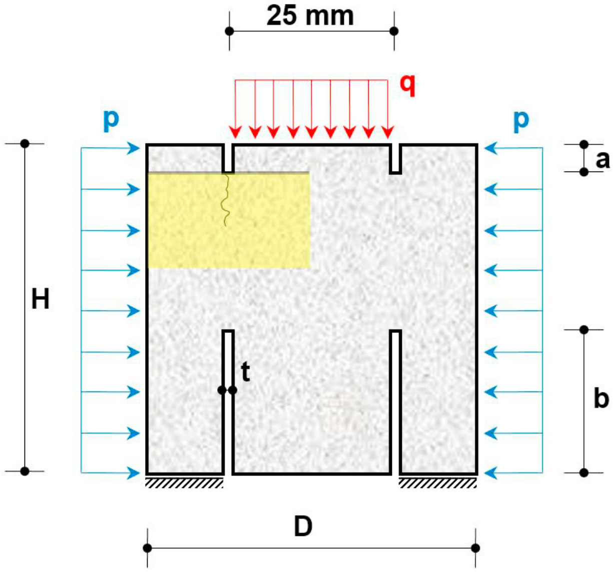

Fig. 2. Geometry of the specimens tested by Backers et al. (2002).

using the configuration of stresses and displacements at the beginning of the increment. We then compute the stress intensity factors at the crack tip from the unknown functions $\phi(j)$, through an extrapolation to the singular point $u = 1$:

$$K_I = \frac{2\mu}{\pi(\kappa+1)} \sqrt{2\pi c}\, \phi_n(+1), \qquad K_{II} = \frac{2\mu}{\pi(\kappa+1)} \sqrt{2\pi c}\, \phi_t(+1) \tag{7}$$

3. Application to experimental tests

The present model is used to estimate the fracture resistance under shear mode of different types of stones, i.e. Aue granite, Carrara marble and Rüdersdorf limestone (Backers et al., 2002). In particular, the effect of varying levels of confinement pressure on mode II fracture toughness is assessed with the present model. The experimental tests of Backers et al. (2002) were carried out on cylindrical specimens of diameter $D = 50$ mm, and height-to-diameter ratio equal to unity ($H/D = 1$). A circular notch of radius 12.5 mm and height $a = 5$ mm was machined on the top base of the cylindrical specimen, and an analogous notch of heigh b was made on the bottom base (different values of b were considered in the tests). The width of the notches is $t = 1.5$ mm. The external ring of the cylinder on the bottom base was supported and a normal compressive traction q was applied on the inner circle of the top base, so as to apply a shear force to the circular ligament of height $H - a - b$. A radial confinement pressure p was applied to the cylinder side, see Fig. 2.

In an engineering attempt to analyse these experimental tests, the axisymmetric specimen is treated as a plane strain crack problem. In particular, the axial cross section of the cylindrical specimen highlighted in Fig. 2 is treated as a semi-infinite space with an edge crack. According to Backers et al. (2002), the initial length of the crack c_0 is deemed to be equal to the notch width t. Then, considering the quasi-brittle nature of the stones, the ratio between the crack length at failure c and its initial length c_0 is assumed to be equal to 2 (Spagnoli et al., 2016), so that $c = 3$ mm. The characteristic material length is taken to be equal to the average grain size d, being $d = 1.4$, 0.4 and 0.3 mm for granite, marble and limestone, respectively (Backers et al., 2002). In the saw-tooth model (see Fig. 1), we assume: semi-length $L = d/2$; inclination angle α calculated by considering a constant asperity height $h = c/100$, namely $\alpha = \arctan(h/L)$. The coefficient of friction is taken to be equal to $f = 0.85$. The other relevant material properties are:

Granite: Young modulus $E = 48$ GPa, Poisson coeff. $\nu = 0.19$, mode I fracture toughness $K_{Ic} = 1.60$ MPa $\sqrt{\text{m}}$;
Marble: $E = 49$ GPa, $\nu = 0.23$, $K_{Ic} = 1.14$ MPa $\sqrt{\text{m}}$;
Limestone: $E = 22$ GPa, $\nu = 0.22$, $K_{Ic} = 1.12$ MPa $\sqrt{\text{m}}$.

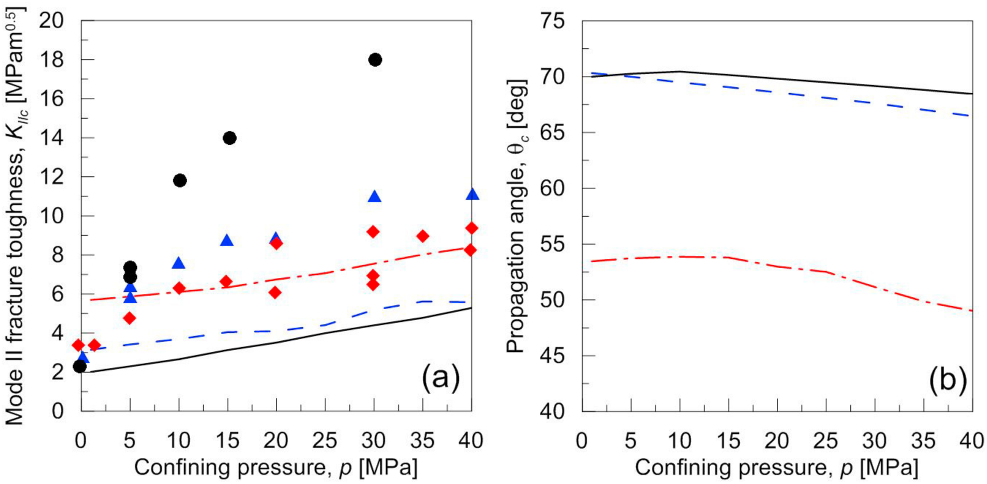

Fig. 3. Comparison between experimental data of Backers et al. (2002) and theoretical results (granite: black continuous lines and black circles; marble: blue dashed lines and blue triangles; linestone: red dash-dot lines and red rhombi): (a) Influence of confining pressure p on mode II fracture toughness K_{IIc}; (b) Influence of confining pressure p on crack propagation angle θ.

In the present model the confined pressure p is applied, and the shear pressure q is linearly increased until a critical condition of fracture is reached. Here, the fracture criterion of the maximum principal stress is applied (Erdogan and Sih, 1963), expressed by

$$K_{Ic} = K_I \cos^3\left(\frac{\theta}{2}\right) - 3K_{II}\cos^2\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right) \tag{8}$$

where θ is the angle of crack propagation with respect to the initial nominal crack plane, given by

$$\tan\left(\frac{\vartheta}{2}\right) = 0.25\frac{K_I}{K_{II}} \pm 0.25\left(\frac{K_I{}^2}{K_{II}{}^2} + 8\right)^{1/2} \tag{9}$$

For each value of the confinement pressure p, from the calculated critical value of the shear stress q_c, the critical value of the nominal mode II SIF can be worked out, namely $K_{IIc} = 1.122 q_c \sqrt{\pi c}$.

A qualitative comparison between experimental results and theoretical calculations is shown in Fig. 3, where the trend of mode II fracture toughness with confining pressure is illustrated along with that of the crack propagation angle. It can be noticed that the experimentally observed increase of K_{IIc} with p, at relatively low values of confining pressure, is captured by the model. On the other hand, the asymptotic trend of K_{IIc} vs p at high values of confining pressure (see also Fig. 10 in Backers et al. (2002)) is not well described by the present model, possibly because the model does not account for an expected smoothening of fracture surfaces at high confining pressure. For a qualitative comparison, even at low confining pressure, further investigation is needed to improve the model representation of the experimental reality, in terms of the description of: specimen geometry, loading conditions, crack roughness, friction, etc.

4. Concluding remarks

In this paper, we have presented an effective strategy to characterise the crack tip stress fields of cracks, including the effect of the surface interference, by means of a simple interface model. According to the model, the effects of friction and surface roughness are included in the formulation through an elastic-plastic type constitutive relationship, while the crack itself is considered smooth and frictionless. We have applied the Distributed Dislocation Technique to compute the crack tip stress intensity factors for different geometries, and possibly with any kind of remote loading. We have investigated the effects of friction and roughness on the crack tip SIFs and on the onset of unstable crack propagation, under monotonic loading. The results clearly show that the influence of friction and roughness is remarkable. Finally, the technique described in this paper is used to qualitatively analyse some experimental tests on the shear mode fracture resistance of natural stones. In particular, the influence of confinement pressure and of microstructurally-related crack morphology on the fracture resistance is highlighted.

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