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Notch Stress Intensity Factors Applied to U and V-Shaped Radiused Notches under In-plane Shear Loading

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Abstract

The present work provides an application of the mode II Generalised Notch Stress Intensity Factor (NSIFs) for U-shaped and V-shaped radiused notches, varying the notch radius as well as the notch opening angle. In the absence of an exact mode II stress field solution for this kind of notches, the Generalised NSIFs are determined using a solution recently developed for V-notches with end-holes. The approximate nature of this solution makes the Generalised NSIFs sensitive to the finiteness of the notch radius and un-constant along the notch bisector line.

Moreover, when the notch radius tends to zero the mode II Generalised NSIF does not converge to the NSIF as defined for pointed V-notches and the discrepancies between the two parameters strongly depend on the notch shape.

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1. Introduction

The intensity of asymptotic stress fields ahead of sharp, zero radius, V-notches under linear elastic conditions is commonly quantified by the Notch Stress Intensity factors, NSIFs. The factors can be thought of as the extension of the conventional stress intensity factors used for cracked components, their units depending on the V-notch opening angle [1]. After Gross and Mendelson [2], values of NSIFs are reported in a number of papers (see, among others, [3-8]). With reference to the crack case, Irwin [9] was the first to give a relationship between the mode I SIF and the maximum stress at the blunt crack tip. Afterwards, dealing with notches under mode I loading, Glinka [10] introduced Irwin's definition into Creager-Paris' solution [11] removing the limit condition $\rho \rightarrow 0$ and making stronger the bridging between LEFM and Linear Elastic Notch Mechanics. Since in the presence of large opening angles, the

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degree of singularity is no longer that of the crack, the link between sharp and blunt V-notches but should include the exponent of Williams' solution for re-entrant corners. An attempt to provide a unified approach to the analysis of cracks, sharp and blunt V-notches was proposed by Lazzarin and Tovo [12]. The analytical solutions by Westergaard [13], Williams [1] and Creager-Paris [11] could be easily derived as special cases of that more general frame. Based on a more flexible version of that analytical frame [14], some relationships linking the Mode I NSIF of a blunt V-notch to the NSIF of the corresponding sharp notch case were provided by Lazzarin and Filippi [15]. By proceeding on parallel tracks, an analytical frame has been proposed for semi-elliptic notches, parabolic or hyperbolic notches, and U- or V-blunt notches under torsion [16-18]. With reference to the mode I loading, an accurate review of the NSIF problem applied to blunt notches has been presented by Savruk and Kazberuck [19], while an extensive analysis of the NSIF problem applied to blunt notches under mode II is due to the present authors [20]. The main aim of the present work is to discuss the application of the mode II GNSIF for U-shaped and V-shaped radiused notches, which are very common in the engineering practice, varying the notch radius as well as the notch opening angle. In the absence of an exact analytical solution for the stress fields of this kind of notches it is proposed to use in engineering sense the frame for V-notches with end holes, showing the degree of accuracy of this simplified proposal.

2. Mode II stress fields solution for V-notches with end holes

In-plane stress distributions for a V-notch with an end hole (see Fig. 1a) have been determined in [21] using the complex potential approach. By using the set of equations given in [21] it is immediate to obtain the shear stress distribution along the notch bisector line:

$$\tau_{r\theta} = \frac{K_{2\rho} r^{\lambda_2-1}}{\sqrt{2\pi}} \left[1 + h_1 \left(\frac{\rho}{r} \right)^{2\lambda_2} + h_2 \left(\frac{\rho}{r} \right)^{2\lambda_2+1} + h_3 \left(\frac{\rho}{r} \right)^{2\lambda_2+2} \right] \quad (1)$$

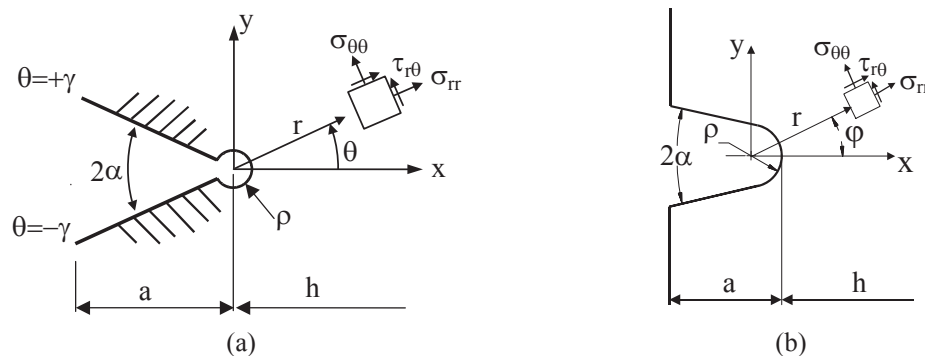


Fig. 1. (a) V-notch with end hole and polar coordinate system with the origin in correspondence to the hole centre; (b) V-shaped radiused notch and polar coordinate system with the origin in correspondence to the radius centre.

All parameters of Eq. (1) are listed in Table 1. The mode II NSIF, $K_{2\rho}$, has been defined as a function of the shear stress component ahead of the notch tip:

$$K_{2\rho} = \frac{\sqrt{2\pi} r^{1-\lambda_2} \tau_{r\theta}(r,0)}{\left[1 + h_1 \left(\frac{\rho}{r}\right)^{2\lambda_2} + h_2 \left(\frac{\rho}{r}\right)^{2\lambda_2+1} + h_3 \left(\frac{\rho}{r}\right)^{2\lambda_2+2} \right]} \quad (2)$$

When the notch tip radius is different from zero, the denominator of Eq. (2) quantifies the stress redistribution due to the notch tip, while when $\rho \rightarrow 0$, the denominator tends to the unity and $K_{2\rho}$ tends to the NSIF of the pointed V-notch, K_2 , defined according to Gross and Mendelson [2]:

$$K_2 = \sqrt{2\pi} \lim_{r \rightarrow 0} \tau_{r\theta} r^{1-\lambda_2} \quad (3)$$

The constancy of the Generalised NSIFs, when applied to a V-notch with end-hole with a finite value of the notch root radius, is documented in Fig. 2, which shows the NSIFs, according to Eq. (2), versus the distance from the notch tip for a notch opening angle equal to 30° . The NSIF of the corresponding sharp V-notch case are also reported for comparison.

Table 1. Parameters to describe the stress components along the notch bisector of a V-notch with end hole under in-plane shear (mode II).

2α ($^\circ$)	0°	30°	45°	60°	90°	120°	135°
γ (rad)	3.1416	2.8798	2.7489	2.6180	2.3562	2.0944	1.9635
λ_2	0.5000	0.5982	0.6597	0.7309	0.9085	1.1489	1.3021
h_1	1.6250	1.6422	1.6639	1.6941	1.7782	1.8873	1.9451
h_2	-0.7500	-0.6011	-0.5393	-0.4849	-0.3921	-0.3079	-0.2614
h_3	-1.8750	-2.0411	-2.1246	-2.2092	-2.3861	-2.5794	-2.6837

3. An engineering proposal for U- and blunt V-notches in plates under mode II loading

In the engineering practice U- and V-notches with rectilinear flanks and a constant root radius are very common in the practice (see Fig 1b).

In the absence of an exact solution of the stress field for this type of notches, we force the solution for V-notches with end holes to be applied to this type of notches with the aim to determine an approximate mode II NSIF. As a matter of facts, the real shear stress components, as determined from FE models with U- or V-notches in finite size plates, are introduced into Eq. (2) which is valid, for a theoretical point of view, for V-notches with end hole. The aim is to show which is the degree of accuracy and the range of applicability of this engineering proposal, as a function of the opening angle.

The numerical results are shown in Fig. 3 as obtained from a model with a V-notch with $2\alpha=30^\circ$. It is evident that the Generalised NSIF of the radiused notches, even if rather constant along the bisector line, can be lower or greater than that derived from the sharp notch case. Considering the minimum notch radius of 0.1 mm, the deviation with respect to the sharp case is -9%. Varying the notch opening angle results also in a variation of the error, being it minimum for the U-notch case [20].

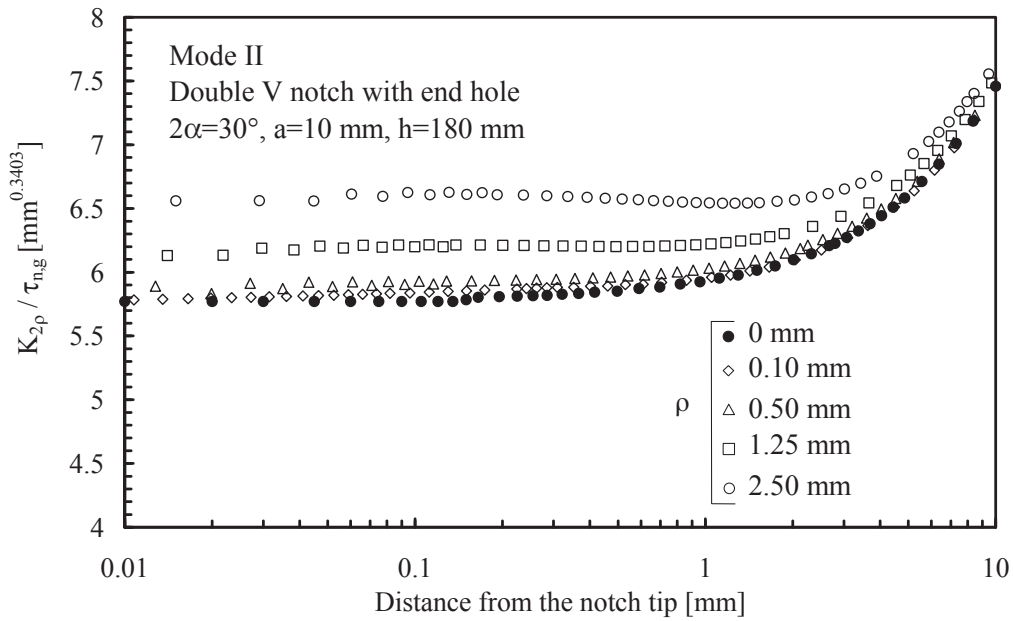


Fig. 2. Notch stress intensity factors $K_{2\rho}$ according to Eq. (2). V notches with end holes ($2\alpha=30^\circ$).

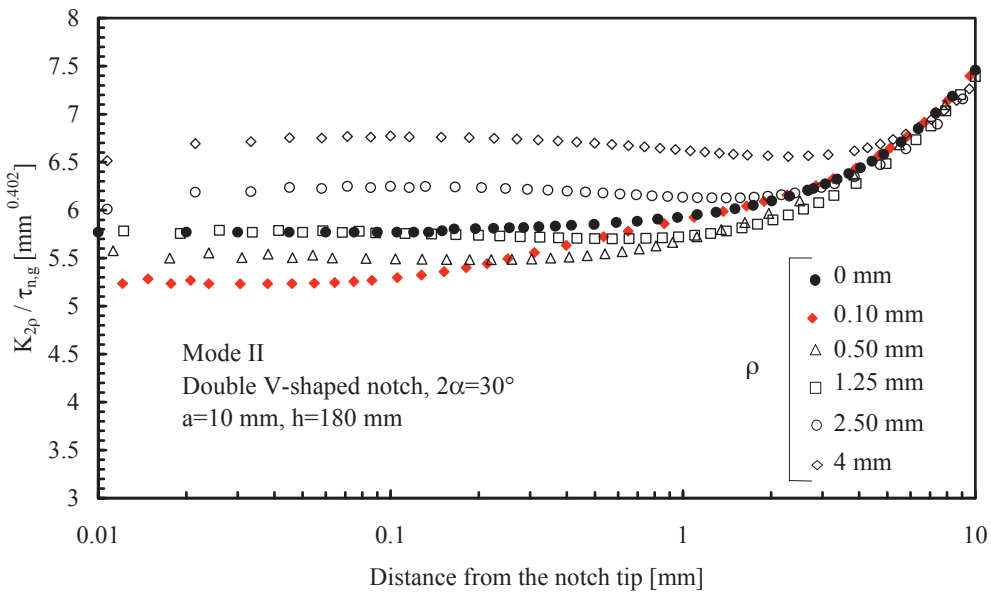


Fig. 3. Plots of the mode II NSIF determined according to Eq. (2). V-shaped notches with $2\alpha=30^\circ$.

A synthesis in terms of $K_{2\rho}/K_2$ is shown in Fig. 4 as a function of the ρ/a ratio, together with the curves determined by a best fit of the numerical data, with ρ ranging from 0.1 mm to 5.0 mm. Note that this ratio does not tend to 1.0 when ρ tends to zero but ranges from 0.8345 ($2\alpha=60^\circ$) to 0.9465 (U-notch,

$2\alpha=60^\circ$).

By using the $K_{2\rho}$ values reported in Fig. 4, one can determine the shear stress distribution along the notch bisector line. A comparison with the numerical data is shown in Fig. 5 considering $2\alpha=30^\circ$ and four values of the notch tip radius. The position of the maximum shear stress component is exactly assessed as well as the local shear stress values preceding the maximum value.

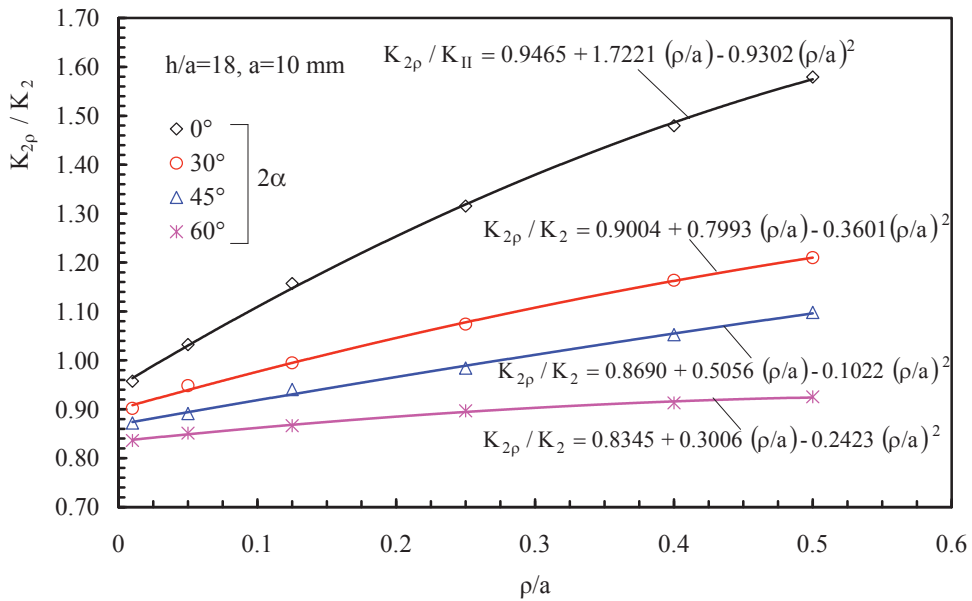


Fig. 4. $K_{2\rho} / K_2$ for plates weakened by faced U- and V-notches by using Eq.(2) for $K_{2\rho}$

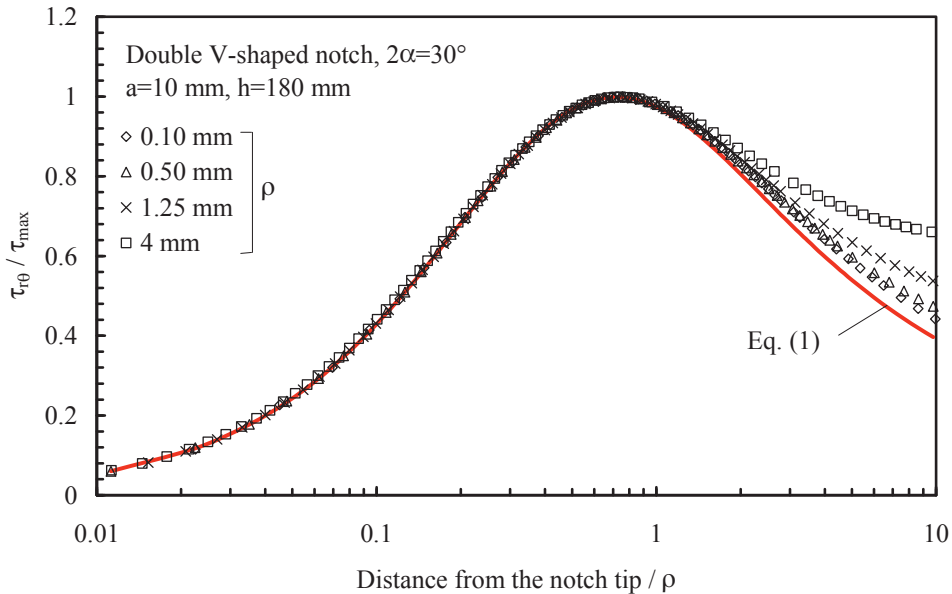


Fig. 5. Plots of the shear stress component normalised to its maximum value as a function of normalised distance from the notch tip.

4. Conclusions

The main aim of the present paper is to provide an application of the mode II Generalised NSIF for U-shaped and V-shaped radiused notches, varying the notch radius as well as the notch opening angle. A recent solution developed for V-notches with end holes has been applied to U- and V-notches having rectilinear flanks and a rounded notch tip, which are common in the practice. The K_{2p}/K_2 ratio has been determined for $0^\circ < 2\alpha < 60^\circ$. The limits of the procedure, which is approximate, have been clarified: when ρ tends to zero, the ratio was found to range between 0.93 ($2\alpha=0^\circ$) to 0.83 ($2\alpha=60^\circ$). By introducing the relevant NSIFs, K_{2p} , onto the analytical frame, the shear stress distribution is accurately predicted up to the point of the maximum shear value.

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