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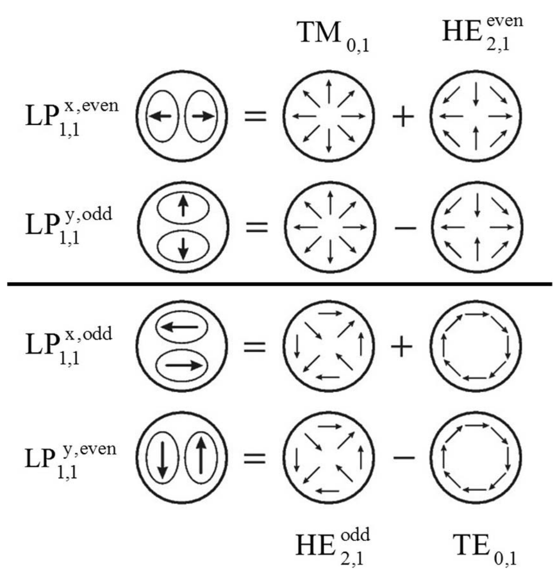

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Coupling Effects Among Degenerate Modes in Multimode Optical Fibers

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Abstract: Multimode optical fibers have recently received revived attention in the framework of space-division multiplexed systems, where the spatial diversity of fiber modes is exploited to increase transmission capacity. The complexity of these systems strongly depends on the coupling characteristic of the fiber. Therefore, a better description of coupling effects may lead to the more accurate modeling of the system and to an optimized design of multimode fibers. In this paper, we analyze coupling among (quasi) degenerate modes as a consequence of different kinds of coupling sources.

Index Terms: Multi-mode fibers, few-mode fibers, mode coupling, birefringence, twist, elliptic core, bending, Faraday rotation.

1. Introduction

Over the past few years, concerns about the capacity limits of worldwide optical communication systems due to nonlinear effects—the so called "capacity crunch" [1], [2], have spurred research on space-division multiplexing (SDM) schemes [3], where the spatial diversity of fiber modes is exploited together with MIMO receivers to increase the transmission capacity [4]–[9]. The characteristics of these SDM systems are strictly affected by mode coupling properties of the fiber, and MIMO receivers have to be used to cope with coupling and modal dispersion.

It is worthwhile recalling that modes in weakly guiding multi-mode fibers are combined in groups of (quasi) degenerate modes, which have (almost) equal propagation constants [10]. For the sake of clarity, from now on, we will call these groups of (quasi) degenerate modes "manifolds." In an ideal fiber with a perfect cylindrical symmetry, modes do not couple to each other. Differently, in real fibers any perturbation that breaks the ideal symmetry (such as bending, twist, etc.) causes modes to couple to each other. In general, coupling is stronger between modes with similar propagation constant; therefore, in general, coupling is much stronger between modes of the same manifold. This mechanism leads, for example, to polarization-mode dispersion in single-mode fibers [11]. On the contrary, coupling among modes of different manifolds is in general weaker, unless longitudinally varying perturbations act on the fiber with proper spatial frequencies so to enhance coupling by phase matching.

The theoretical models adopted so far to describe propagation in multi-mode fibers accounts for coupling in a statistical way, assuming that coupling is "statistically isotropic" [12]–[15]. This assumption is made only for mathematical convenience, and it is not based on physical arguments. As a consequence, these simplified models provide sound results for sufficiently long fibers, but their applicability in describing local effects is questionable. A better understanding of coupling mechanisms is therefore important for a more accurate description of multi-mode propagation.

The general analysis of coupling in multi-mode fibers is quite cumbersome. Therefore, in this paper we analyze only the effects that the most common sources of perturbation have on (quasi) degenerate modes of the same manifold. The effects of perturbations on intra-manifold coupling have been preliminary discussed in [16] and will be described in more detail elsewhere. The results reported in this paper can be used to study possible mechanisms to enhance mode coupling, such as spin, for example. Furthermore, they provide indications for more realistic numerical models.

2. Analysis of Mode Coupling

According to the classical coupled mode theory [17], the propagation of light in a perturbed multi-mode fiber can be described by the N-dimensional vector $\boldsymbol{c}(z)$, made by the complex amplitudes of each of the N ideal modes propagating in the fiber. The evolution of $\boldsymbol{c}$ with the longitudinal coordinate z is given by

$$\frac{d\boldsymbol{c}}{dz} = -j(\beta + \boldsymbol{K})\boldsymbol{c} \tag{1}$$

where $\beta = \mathrm{diag}(\beta_1, \ldots, \beta_N)$ is a diagonal matrix with the propagation constants β_i of the ideal modes, and the $N \times N$ matrix $\boldsymbol{K}$ represents the coupling per unit length (or equivalently the coupling rate) due to perturbations acting on the fiber. Its elements can be approximated as [17]

$$K_{\mu,\nu} = \frac{\omega}{4}\int_0^{\infty}\int_0^{2\pi} r\boldsymbol{E}_{\mu}^{*}\tilde{\boldsymbol{\epsilon}}\boldsymbol{E}_{\nu}\,d\phi\,dr = K_{\nu,\mu}^{*} \tag{2}$$

where r and ϕ are the polar coordinates on the fiber section and $\tilde{\boldsymbol{\epsilon}}$ is the 3×3 matrix representing the additive dielectric tensor of perturbations; since we are assuming lossless media, $\tilde{\boldsymbol{\epsilon}}$ is Hermitian [10] and the last equality holds. The 3-D vectors $\boldsymbol{E}_{\mu}$ and $\boldsymbol{E}_{\nu}$ are the complete electric fields (i.e., both transverse and longitudinal components) of the ideal modes μ and ν of an isotropic and symmetrical step-index fiber. We approximate these field expressions with the well known set of linearly polarized LP modes [10], [18], defined in such a way that

$$\frac{\beta_{\mu}}{2\omega\mu_0}\int_0^{\infty}\int_0^{2\pi} r\boldsymbol{E}_{\mu}^{(t)*}\boldsymbol{E}_{\nu}^{(t)}\,d\phi\,dr = \delta_{\mu,\nu} \tag{3}$$

where the superscript (t) indicates the transverse component of the field. The transverse electric field of mode $\mathrm{LP_{n,p}}$ reads $\boldsymbol{E}_{n,p}^{(t)} = f_{n,p}(r)g_n(\phi)\boldsymbol{e}$, where $n = 0, 1, \ldots$ is the azimuthal order of the mode, $f_{n,p}(r)$ is a proper Bessel function, $g_n(\phi)$ can be set equal to either $\cos n\phi$ or $\sin n\phi$, and the unit vector $\boldsymbol{e}$ represents the mode polarization and can be either $\boldsymbol{x}$ or $\boldsymbol{y}$. Similarly, the longitudinal component can be evaluated as $E_{n,p}^{(z)} = -(j/\beta_{n,p})\nabla \cdot \boldsymbol{E}_{n,p}^{(t)}$ [19], and it can be represented in general as a linear combination of terms like $u_{n,p}(r)v_n(\phi)$, where $u_{n,p}(r)$ is a Bessel functions and v_n is either $\cos(n \pm 1)\phi$ or $\sin(n \pm 1)\phi$ [10].

The evaluation of the integrals (2) in the most general case is quite cumbersome, therefore here we limit the analysis to groups of (quasi) degenerate modes. Two cases have to be considered. The first one is that of $\mathrm{LP_{0,p}}$ modes that are 2-fold degenerate; the matrix describing coupling for each manifold is therefore 2×2. We may anticipate that the results about this case are qualitatively equal to those already obtained for the $\mathrm{LP_{0,1}}$ mode [20]. The second case is that of $LP_{n,p}$ modes (with $n > 0$) which make a 4-D manifold of quasi-degenerate modes. In this case, results can be more involved and quite different from those of the $LP_{0,1}$ mode.

Before analyzing the effects of each perturbation separately, note that since we limit the analysis (quasi) degenerate manifolds, the effect of β on mode coupling properties can be neglected, since modes of the same manifold have the same propagation constant. Moreover, hereinafter we order modes by alternating x and y polarizations of even $(\cos n\phi)$ modes and then x and y polarizations of odd $(\sin n\phi)$ modes.

2.1. Effect of Perturbation Orientation

The expression of the coupling matrix depends also on the orientation of the perturbation with respect to the reference frame. This aspect would add to the analysis further difficulties, but it can be discussed in general once and for all.

Suppose the coupling matrix $\boldsymbol{K}'$ has been evaluated with respect to a reference frame $\{\hat{x}', \hat{y}'\}$, which is rotated by an angle θ (positive in counterclockwise direction) with respect to second reference frame $\{\hat{x}, \hat{y}\}$. The relationship between $\boldsymbol{K}'$ and the coupling matrix $\boldsymbol{K}$ evaluated with respect to the frame $\{\hat{x}, \hat{y}\}$ can be calculated as follows. First the electromagnetic field is expressed as a linear combination of LP modes defined with respect to the frame $\{\hat{x}', \hat{y}'\}$; let $\boldsymbol{c}'$ be the vector of the coefficients of such combination. Then the field is projected on the LP modes expressed with respect to the frame $\{\hat{x}, \hat{y}\}$, determining the new coefficients $\boldsymbol{c}$. In doing this projection we should notice that if r' and ϕ' are the polar coordinate associated with $\{\hat{x}', \hat{y}'\}$, then we have $r' = r$ and $\phi' = \phi - \theta$. Performing explicit calculations, in the new reference frame the coefficients of the modes in the $LP_{n,p}$ manifold reads in general $\boldsymbol{c}_{n,p} = \boldsymbol{R}_n(\theta)\boldsymbol{c}'_{n,p}$. More specifically, for the 2-fold-degenerate modes of azimuthal order $n = 0$ the matrix $\boldsymbol{R}_0$ reads

$$\boldsymbol{R}_0(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \tag{4}$$

confirming a well-known result [20], while for the 4-fold-degenerate modes the matrix $\boldsymbol{R}_n$ is more conveniently written as the product $(n \geq 1)$

$$\boldsymbol{R}_n(\theta) = \begin{pmatrix} \boldsymbol{R}_0(\theta) & \mathbf{0} \\ \mathbf{0} & \boldsymbol{R}_0(\theta) \end{pmatrix} \begin{pmatrix} \cos(n\theta)\boldsymbol{\sigma}_0 & -\sin(n\theta)\boldsymbol{\sigma}_0 \\ \sin(n\theta)\boldsymbol{\sigma}_0 & \cos(n\theta)\boldsymbol{\sigma}_0 \end{pmatrix} \tag{5}$$

where $\boldsymbol{\sigma}_0$ is the 2×2 identity matrix and $\mathbf{0}$ is the 2×2 null matrix. This expression can be given a clear physical interpretation. First of all note that the two factors commute. Recalling how modes are ordered, we see also that the first factor represents rotation of polarization within the 2-D sub-manifolds of even and odd modes. Furthermore, although less evident, it can be shown that the second factor describes the rotation of the field pattern with azimuth order n, and as a result the matrix $\boldsymbol{R}_n(\theta)$ represents (in the 4-dimensional space of the manifold) a rigid rotation by an angle θ of the mode.

We can so conclude that the effects on a generic manifold with azimuth order n of a perturbation oriented at an arbitrary angle θ with respect to the reference frame, is represented by

$$\boldsymbol{K}_n = \boldsymbol{R}_n(\theta)\boldsymbol{K}'_n\boldsymbol{R}_n^T(\theta), \qquad (n = 0, 1, 2, \ldots) \tag{6}$$

where $\boldsymbol{K}'_n$ is the coupling matrix of the same perturbation evaluated with respect to a reference frame aligned with the symmetry axes of the perturbation itself.

Owing to this general result, from now on the coupling matrices are evaluate for simplicity with respect to a frame aligned with the symmetry axes of the perturbation.

2.2. Birefringence

Silica fibers may become slightly birefringent as a consequence of residual stress after drawing, or owing to lateral forces acting on the fiber. Assuming that birefringence is aligned with the reference frame and extends to the whole fiber section, the tensor of perturbation reads

$$\tilde{\boldsymbol{\epsilon}} = \frac{\delta\epsilon}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{7}$$

and as a consequence, coupling can occur only among transverse components of the fields and among fields with the same polarization. Explicit calculation of the integral (2) shows that the coupling matrices of 0-order and n-order modes read, respectively

$$\boldsymbol{A}_{0,p} = \frac{\omega^2\mu_0\delta\epsilon}{4\beta_{0,p}}\boldsymbol{\sigma}_1, \qquad \boldsymbol{A}_{n,p} = \frac{\omega^2\mu_0\delta\epsilon}{4\beta_{n,p}}\begin{pmatrix} \boldsymbol{\sigma}_1 & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\sigma}_1 \end{pmatrix}, \qquad \text{where} \quad \boldsymbol{\sigma}_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{8}$$

and where ω is the field angular frequency, μ_0 the magnetic permeability of vacuum, and $\beta_{n,p}$ is the propagation constant of the considered mode.

We see then that the effect of birefringence is just the expected detuning of x- and y-polarized modes, generalizing the result known for the $\mathrm{LP}_{0,1}$ mode. Assuming weak birefringence, the detuning factor in (8) can be approximated as $(n_{\mathrm{av}}/n_{\mathrm{eff}})\pi\Delta n/\lambda$, where λ is the wavelength in vacuum, Δn the difference of refractive indices between the two birefringence axes, n_{av} is the mean refractive index, and n_{eff} the effective refractive index of the mode under consideration. Note that in general $n_{\mathrm{av}}/n_{\mathrm{eff}} \simeq 1$, and therefore, the effect of birefringence is by and large equal on all modes.

2.3. Core Ellipticity

A slightly elliptical core can be represented by the following scalar perturbation [20]

$$\tilde{\boldsymbol{\epsilon}} \simeq \gamma\epsilon_0\left(n_{co}^2 - n_{cl}^2\right)\delta(r-a)\cos 2\phi \tag{9}$$

where γ is a form factor, a is the average core radius and n_{co} and n_{cl} are the refractive indices of core and cladding respectively. Being a scalar perturbation, coupling can occur between transverse components of modes with the same polarization, or among longitudinal components. By evaluating (2) with respect to ϕ and considering that the perturbation is proportional to $\cos 2\phi$, we can see that coupling among transverse components can occur only among modes of the $\mathrm{LP}_{1,\mathrm{p}}$ manifold [16]. For these, the coupling matrix reads

$$\boldsymbol{C}_{1,p} = \chi_{1,p}\begin{pmatrix} \boldsymbol{\sigma}_0 & \mathbf{0} \\ \mathbf{0} & -\boldsymbol{\sigma}_0 \end{pmatrix} \tag{10}$$

where $\chi_{1,p}$ is a mode-dependent coefficient proportional to γ, to the ratio $n_{\mathrm{av}}/n_{\mathrm{eff}}$, and to the square of the maximum field intensity at the core-cladding interface. As for birefringence, the core ellipticity causes a detuning of the $\mathrm{LP}_{1,\mathrm{p}}$ modes. Note however that the signs of the detuning are different, because in this case, being the perturbation scalar, what matters is not the field polarization but the field pattern.

For the other modes inter-manifold coupling can occur only through longitudinal components, therefore it is a much weaker effect. For $\mathrm{LP}_{0,\mathrm{p}}$ modes (2) yields $\boldsymbol{C}_{0,\mathrm{p}} = \eta_{0,\mathrm{p}}\boldsymbol{\sigma}_1$, showing that for these modes core ellipticity has the same effect as birefringence. Differently, for $\mathrm{LP}_{2,\mathrm{p}}$ modes and $\mathrm{LP}_{\mathrm{n,p}}$ (with $\mathrm{n} > 2$) we have, respectively

$$\boldsymbol{C}_{2,p} = \begin{pmatrix} \eta_{2,p}\boldsymbol{\sigma}_1 & -\xi_{2,p}\boldsymbol{\sigma}_2 \\ -\xi_{2,p}\boldsymbol{\sigma}_2 & \eta_{2,p}\boldsymbol{\sigma}_1 \end{pmatrix}, \qquad \boldsymbol{C}_{n,p} = \eta_{n,p}\begin{pmatrix} \boldsymbol{\sigma}_1 & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\sigma}_1 \end{pmatrix}, \qquad \text{where} \quad \boldsymbol{\sigma}_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{11}$$

and coefficients η and ξ have quite complex expressions. Notice that for modes with azimuth order higher than 2, core ellipticity causes only detuning, whereas for modes with azimuthal order 2 it induces also mode coupling (off-diagonal terms). This is basically due to the fact that these modes have the same azimuth order of the perturbation.

2.4. Bend

The stress field induced on a fiber when it is bent is quite complex; however, two main contributions can be highlighted [21]. The first one is a longitudinal stress that stretches the fiber layers on the outer side of the bent and compresses those on the inner side. Nevertheless, considerations about the symmetry of this perturbation allow to conclude that it cannot induce coupling between modes of the same manifold [16].

Differently, the second term is a compressive stress exerted from the outer layers on the inner ones in the direction of the bending radius; it is a second order effect and reads $\sigma_x = \kappa^2(E/2)(x^2 - a^2)$. Considering that modes are concentrated in the core area, and noticing the quadratic dependence on x, we may set with a good approximation $\sigma_x \simeq -\kappa^2 a^2 E/2$. Owing to the elasto-optical properties of fused silica [22], [23], we can then conclude that the dielectric perturbations induced by this effect reads

$$\tilde{\boldsymbol{\epsilon}}_x = \frac{1}{2}(\kappa a)^2 \epsilon_0 n_{\text{av}}^4 q \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{12}$$

with $q = (1+\nu)(p_{1,2} - p_{1,1})/2 \simeq 0.087$, where $\nu \simeq 0.164$ is the Poisson ratio of fused silica, and $p_{1,1} \simeq 0.121$ and $p_{1,2} \simeq 0.270$ are its elasto-optical coefficients.

This perturbation couples mode of the same azimuth order through both the transverse and the longitudinal components; since the former are much stronger, we neglect the latter. The resulting coupling matrices read

$$\boldsymbol{B}_{0,p} = -\frac{\omega^2 \mu_0 \epsilon_0}{4\beta_{0,p}} (\kappa a)^2 n_{\text{av}}^4 q \boldsymbol{\sigma}_1, \qquad \boldsymbol{B}_{n,p} = -\frac{\omega^2 \mu_0 \epsilon_0}{4\beta_{n,p}} (\kappa a)^2 n_{\text{av}}^4 q \begin{pmatrix} \boldsymbol{\sigma}_1 & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\sigma}_1 \end{pmatrix} \tag{13}$$

which means that the effect of bending on inter-manifold mode coupling is the same as that of birefringence.

2.5. Twist

When a fiber is twisted the shear stresses exerted on the silica and the corresponding elasto-optical effect induce a perturbation of the permittivity tensor that reads [20]

$$\tilde{\boldsymbol{\epsilon}} = g\tau\epsilon_0 n_{\text{av}}^4 r \begin{pmatrix} 0 & 0 & -\sin\phi \\ 0 & 0 & \cos\phi \\ -\sin\phi & \cos\phi & 0 \end{pmatrix} \tag{14}$$

where τ is the twist applied per unit length, $g \simeq 0.15$ is an elasto-optic coefficient, and n_{av} is the mean refractive index. This perturbation is the only one that couples transverse components of one mode with the longitudinal one of the other. Explicit evaluation of (2) yields the following coupling matrices:

$$\boldsymbol{T}_{0,p} = \frac{g\tau}{2}\boldsymbol{\sigma}_3, \qquad \boldsymbol{T}_{n,p} = \frac{g\tau}{2}\begin{pmatrix} \boldsymbol{\sigma}_3 & -jn\boldsymbol{\sigma}_1 \\ jn\boldsymbol{\sigma}_1 & \boldsymbol{\sigma}_3 \end{pmatrix}, \qquad \text{where} \quad \boldsymbol{\sigma}_3 = \begin{pmatrix} 0 & -j \\ j & 0 \end{pmatrix} \tag{15}$$

where we have used the approximation $\omega^2 \mu_0 \epsilon_0 n_{\text{av}}^2 \simeq \beta_{n,p}^2$. The result for $n = 0$ generalizes to the higher order $\text{LP}_{0,\text{p}}$ modes the effect that twist is known to have on the fundamental mode

$LP_{0,1}$ [20]. Namely, on these modes the twist induces a rotation of polarization that is phenomenologically equivalent to optical activity. The meaning of the result for $n > 0$ is less evident, but it can be shown that the matrix $\boldsymbol{T}_{n,p}$ induces a rotation of both the polarization and the field pattern of the mode. This can be proven by showing that the solution of the matrix equation $d\boldsymbol{F}/dz = -j\boldsymbol{T}_{n,p}(z)\boldsymbol{F}(z)$, with $\boldsymbol{F}(0) = \boldsymbol{I}$, is $\boldsymbol{F}(z) = \boldsymbol{R}_n[(g/2)\int_0^z \tau(s)ds]$, where $\boldsymbol{R}_n$ is just the rotation matrix defined in (5).

Therefore in general, the twist causes a rigid rotation of the LP modes by an angle that is almost equal for all modes and proportional only on the twist rate through the factor g.

2.6. Magnetic Field

Another effect that may cause mode coupling is a magnetic field with a non-negligible component oriented parallel to the fiber axis. As well known, this causes Faraday rotation of polarization [10], [20]. While this case is rarely of interest for communication systems, it has great importance in the framework of fiber optic sensors [24], [25]. The magnetic field perturbs the dielectric permittivity with the term

$$\tilde{\boldsymbol{\epsilon}} = j\frac{\lambda n_{av}}{\pi} B_{\tan} V \epsilon_0 \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{16}$$

where λ is the wavelength, $B_{\tan}$ is the magnetic induction component along the fiber axis, and V is the Verdet constant (about 0.6 rad/T/m at 1550 nm for silica). This effect couples only the transverse components of the modes, and the resulting coupling matrices read

$$\boldsymbol{M}_{0,p} = B_{\tan} V \boldsymbol{\sigma}_3, \qquad \boldsymbol{M}_{n,p} = B_{\tan} V \begin{pmatrix} \boldsymbol{\sigma}_3 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\sigma}_3 \end{pmatrix} \tag{17}$$

confirming that, also for higher order modes, Faraday rotation causes only a rotation of the field polarization.

2.7. Combined Effect of Several Perturbations

To conclude the analysis, we consider the case in which several sources of perturbation act together. In the limit of validity of the coupled mode theory, we may apply the superposition principle to state that the total coupling matrix is given by the sum of the matrices corresponding to each contribution [20]. Care, however, should be put in the case of twist.

Actually, in Section 2.5 we have considered only the effects due to the twist-induced torsional stress, but indeed when a fiber is twisted it undergoes also a rotation. If the fiber is not affected by other perturbations, i.e., if it has perfect cylindrical symmetry, this rotation has no effect. But if the fiber was already affected by perturbations, they will rotate at the applied twist. As we have seen, a physical rotation with respect to the reference frame is described in the space of coupling coefficients by the matrices defined in (4) and (5). Therefore, if $\boldsymbol{U}_{n,p}(z)$ is the total coupling matrix for the manifold $LP_{n,p}$ of an untwisted fiber, the coupling matrix of the same manifold after some twist with local rate $\tau(z)$ is applied reads

$$\boldsymbol{K}_{n,p}(z) = \boldsymbol{R}_n[\alpha(z)]\boldsymbol{U}_{n,p}(z)\boldsymbol{R}_n^T[\alpha(z)] + \boldsymbol{T}_{n,p}(z), \qquad \text{with} \quad \alpha(z) = \int_0^z \tau(s)ds. \tag{18}$$

The coupling matrix of any subsequent perturbation (such as bending, for example) will be added to one given above.

3. Comments and Conclusion

In this paper we have analyzed the coupling induced on a manifold of degenerate modes by different kinds of perturbation. Some results are straightforward generalization of what was already known the fundamental $LP_{0,1}$ mode. Birefringence and bending induce on higher modes the same detuning. Similarly, the effect of a magnetic field with a non-negligible component parallel to the fiber axis is the same Faraday rotation of polarization for all modes. Differently, core ellipticity has more involved effects on higher order modes. The $LP_{n,p}$ modes for $n \neq 2$ undergo detuning, which is stronger for modes of azimuthal order 1 and weaker for the others; the $LP_{2,p}$ modes, on the contrary, undergo both detuning and coupling. These differences of behavior should be traced back to the interplay between the azimuthal order of the perturbation and that of the modes. Finally, we have seen that twist causes a rotation of both the mode pattern and its polarization by an angle per unit length equal to $g\tau/2$, where $g \simeq 0.15$ and τ is the twist rate applied to the fiber. This result generalizes to multi-mode fibers the optical activity known to occur in twisted single-mode fibers.

It is important to notice that the coupling matrix defined by (2) is Hermitian (under the hypothesis that there are no losses, as assumed here). However, we have seen that the coupling matrices are real for all sources of perturbation, except for twisted fibers and for fibers affected by Faraday rotation, in which case the matrix is imaginary. Faraday rotation occurs only when the fiber is exposed to very intense magnetic field, which rarely happens to telecommunication fibers. Similarly, some experimental results suggest that also accidental twist is a quite rare event, with rates typically much lower than 1 rad/m [26], [27]. A deeper analysis would reveal that this observation is valid also for coupling among non-degenerate modes, as it pertains to the most general expression of the coupling integrals (2). For this reason we believe that the most accurate choice when modeling propagation in randomly coupled multi-mode fibers is to build the coupling matrix $\boldsymbol{K}$ real and symmetric.

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