Workspace Topologies of Industrial 3R Manipulators

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Abstract: *A mathematical analysis is used to characterize workspace topologies of industrial 3R manipulators. A level-set reconstruction of the workspace is formulated to identify characteristic points with fairly simple algebraic expressions. Thus, industrial 3R manipulators are classified as functions of workspace kinematic properties. Examples are illustrated to show practical usefulness of the proposed workspace characterization.*
Keywords: *Manipulators, Workspace Analysis, Classification*

1. Introduction

Workspace analysis of serial manipulators is of great interest since the workspace geometry can be considered a fundamental issue for manipulator design, robot placement in a working environment, and trajectory planning.
Nowadays the majority of manipulators for industrial applications are of serial type. They often have geometric design simplifications, such as intersecting joint axes, orthogonal or parallel joint axes. Moreover, most of the industrial manipulators are wrist-partitioned, that is they consist of a concatenation of a 3R (Revolute) arm, i.e., regional structure, and a spherical wrist that is attached to the terminal link of the arm. The workspace analysis of such manipulators can be performed by considering both the positioning and orienting tasks, and the singularities determination.
Early studies have been developed for 3R manipulators for position workspace only (Freudenstein, F. & Primrose, E.J.F., 1984), (Roth, B., 1975). An algebraic formulation for determining the workspace of 3R manipulators has been presented by using the geometry of ring generation as described in a transversal plane in (Freudenstein, F. & Primrose, E.J.F., 1984), or in a cross-section plane in (Ceccarelli, M., 1989). Only the last approach has been generalized for nR manipulators in (Ceccarelli, M., 1996). The determination of the workspace boundary in Cartesian Space has been proposed also by using identification of singularities in workspace boundary, like for example in (Smith, D.R. & Lipkin, H., 1993).
Other papers are related to the singularity of the Jacobian matrix that is usually expressed in the Joint Space. Regions that are free of singularities in the Joint Space have been named C-sheets, (Burdick, J.W., 1995). In C-sheets it is possible to determine how to change posture without passing through singularities (Parenti-Castelli, V. & Innocenti, C., 1988). Manipulators that can change posture without meeting a singularity have been named cuspidal manipulators in (Wenger, P.,2000). The analysis and characterization of geometric singularities of the cross-section boundary curve has been proposed in (Ottaviano, E., Ceccarelli, M. & Lanni, C., 1999), (Ottaviano, E., Husty, M. & Ceccarelli, M., 2004).
Several authors have grouped manipulators into classes, as reported in (Burdick, J.W., 1995), (Wenger, P.,2000), (Zein, M., Wenger, P. & Chablat D., 2005), by considering special architectures, such as cuspidal or orthogonal manipulators, which have simplification in the architecture.
In this paper we present a classification of industrial 3R manipulators as based on kinematic properties of the workspace, but not only on parameters simplifications. A level-set reconstruction is used to analyze workspace topology by using algebraic expressions, as outlined in (Ottaviano, E., Husty, M. & Ceccarelli, M., 2006a). The two-parameter set of curves is used to characterize the workspace cross-section and gives an interesting insight of the internal structure of the workspace boundary as obtained as an envelope of generating circles. Characteristic points are determined for a fairly simple classification of industrial 3R manipulators with a direct kinematic interpretation, as pointed out in (Ottaviano, E., Husty, M. & Ceccarelli, M., 2006b). In this paper we have focused specific attention to workspace analysis for industrial manipulators.

2. A Formulation for Workspace Determination

Figures have to be made in high quality, which is suitable
A general 3R manipulator is sketched in Fig.1, in which the kinematic parameters are denoted by the Hartenberg

and Denavit (H-D) notation. Without loss of generality the base frame is assumed to be coincident with $X_1Y_1Z_1$ frame when $\theta_1 = 0$, $a_0=0$ and $d_1=0$. The end-effector point H can be usually chosen as either the center of the end-effector, or the tip of a finger. Point H is placed on the X_3 axis at a distance a_3 from O_3, as shown in Fig.1. The general 3R manipulator is described by the H-D parameters a_1, a_2, d_2, d_3, α_1 and α_2, and θ_i, for (i = 1,…,3), as shown in Fig.1.

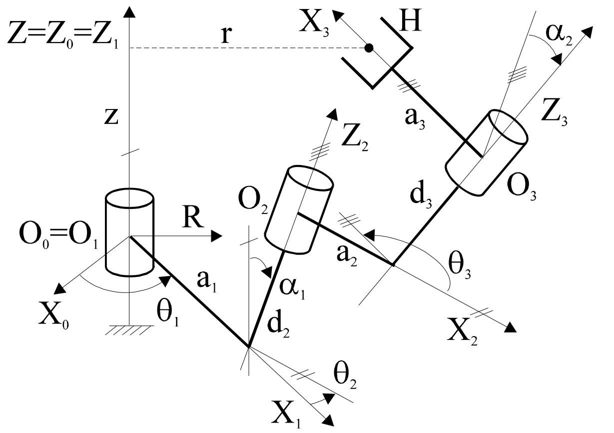


Fig. 1. A kinematic scheme for a general 3R manipulator

The position of point H with respect to reference frame $X_3Y_3Z_3$ can be represented by the vector $\mathbf{H}_3$. Using the transformation matrices T_i^{i+1}, the coordinates (x,y,z) of the operation point H with respect to the base frame $X_0Y_0Z_0$ are given by the position vector $\mathbf{H}_0$ in the form

$$H_0 = T_0^1 \, T_1^2 \, T_2^3 \, H_3 \qquad (1)$$

The workspace of a general 3R manipulator can be expressed in the form of radial and axial reaches, r and z respectively, with respect to the base frame. r is the radial distance of point H from the Z_1-axis and z is the axial reach; both can be expressed as function of H-D parameters.

Reaches r and z can be evaluated as function of reach coordinates in the form

$$\begin{aligned} r^2 &= \left(H_0^x\right)^2 + \left(H_0^y\right)^2 \\ &= \left(H_1^x c\theta_1 - H_1^y s\theta_1\right)^2 + \left(H_1^x s\theta_1 + H_1^y c\theta_1\right)^2 \qquad (2) \\ z &= H_0^z \end{aligned}$$

which can be equivalently expressed in the form

$$\begin{aligned} r^2 &= \left(H_1^x\right)^2 + \left(H_1^y\right)^2; \\ z &= H_1^z \qquad (3) \end{aligned}$$

Equation (3) represents a 2-parameter family of curves, which gives the cross-section workspace in a cross-section plane (Freudenstein, F. & Primrose, E.J.F., 1984), (Ceccarelli, M., 1989), as function of the H-D parameters through H_1^x, H_1^y and H_1^z coefficients.

3. A Level-set Analysis for 3R Manipulators

In the following the above-mentioned two-parameter set is interpreted as a level-set. The level-set of a differentiable function $f : \Re^n \to \Re$ corresponding to a real value c is the set of points, (Sethian, J.A., 1996)

$$\left\{ (x_1, \ldots x_n) \in \Re^n : (x_1, \ldots x_n) = c \right\} \qquad (4)$$

The potentiality of the level-set method is now applied to the workspace analysis of 3R manipulators, as outlined in (Ottaviano, E., Husty, M. & Ceccarelli, M., 2006 b and c). In particular, the level-set reconstruction for a serial manipulator can be obtained by using the 2 parameter-family of curves that are expressed in Eq.(3).

The level-sets belonging to constant values of θ_3 are curves in the RZ-plane. Therefore, this one parameter set of curves can be viewed as the contour map of a surface S, which conveniently can be used to analyze the workspace of the manipulator. The surface S can be defined though the functions

$$\begin{aligned} X^2 &= r^2 \\ Y &= z \qquad (5) \\ Z &= \tan\left(\frac{\theta_3}{2}\right) \end{aligned}$$

By performing the half-tangent substitution v = tan (θ_2/2) in Eq.(5) and eliminating the v parameter one can obtain an implicit equation of the surface S in the form

$$S: F(X,Y,Z) = 0 \qquad (6)$$

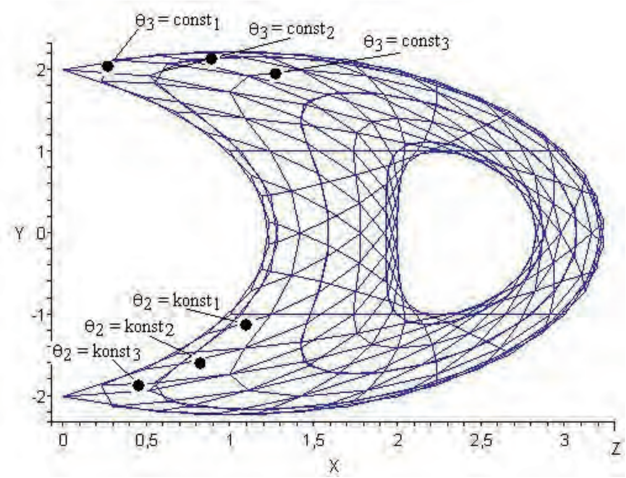
Equation (6) describes an algebraic surface which is of degree 20, as indicated in (Ottaviano, E., Husty, M. & Ceccarelli, M., 2006 b). It can be splitted into two parts

$$F(X,Y,Z) = S_1(X,Y,Z)\, S_2(X,Y,Z) \qquad (7)$$

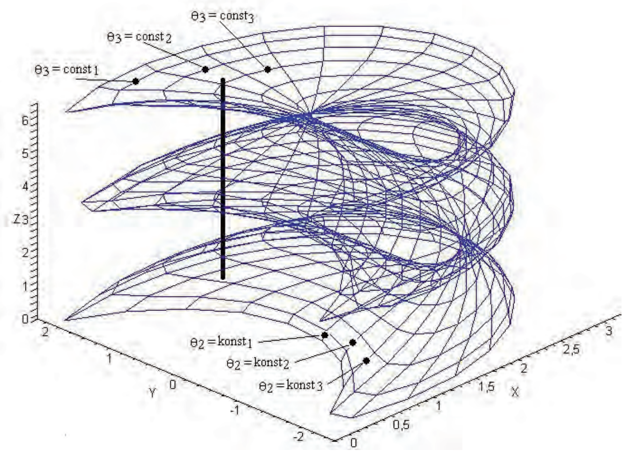
where S_1 represents four double planes parallel to XY plane, in which the height depends on the H-D parameters; S_2 is the graph of the level-set function in which the parameter lines belong to θ_2 = const or θ_3 = const. Geometrically S is generated by taking a cross-section of the workspace that is parameterized by θ_2 and θ_3 and explode the overlapping level-set curves in the direction of Z-axis, as illustrated in the example of Fig.2.

The major advantage of this procedure is that on S one can see clearly the number of solutions of the Inverse Kinematics (IK) belonging to one point of the workspace cross-section.

In Fig.2 this is shown for a general design case. In Fig.2a) the level-set curves are shown in the cross-section plane. It is to point out that in a workspace cross-section two different one-parameter sets of level-curves can be traced as function of θ_3 = const and θ_2 = const, respectively. On Fig.2b) the corresponding surface S is displayed. Geometrically the level-set curves of Fig.2a) are the orthogonal projections of the intersection curves with planes Z = const and the surface S onto the XY-plane. The level-set curves for θ_3 = const in Fig.2a) are therefore the contour curves for the surface S. Additionally we have displayed in Fig.2b) a gross line parallel to the Z-axis. This line shows clearly four intersection points with the surface S. Therefore, the corresponding point in the level-set plane in Fig.2a) corresponds to a four fold solution of the IK.



a)



b)

Fig.2. A numerical example for a general 3R manipulator: a) workspace cross-section; b) corresponding S surface for a level-set reconstruction

In Figs.2a) and 2b) the level-set curves are shown as function of different values of angles θ_2 and θ_3. In the displayed cross-section of the workspace the two different one-parameter sets of level-curves are displayed in order to show those lines that permit to built the level-set surface from the parametrization of the workspace cross-section.. The meridian ones, which are shown as horizontal in the 3D plot, belongs to θ_3 = const, and the other radial-like ones belongs to θ_2 = const.

In order to determine the algebraic degree of S_2 one has to homogenize and intersect with the plane at infinity. The resulting intersection is completely independent of the H-D parameters. It consists of an eight fold line Z = 0 and two complex double lines. Thus, the surface is of algebraic degree 12.

Manipulators having singularities on the surface S can be considered as an algebraically closed set. Singularities of the surface S can be found by considering the implicit equation of S, together with its partial derivatives with respect to X, Y, and Z, respectively, (Gibson, C.G., 1998). All these four functions have to vanish for a point on the surface being singular and giving singularity conditions that can be expressed as functions of H-D parameters.

An enumeration of all possible types of ring void has been presented in (Ottaviano, E., Ceccarelli, M. & Lanni, C., 1999), (Ottaviano, E., Husty, M. & Ceccarelli, M., 2006 a) by analyzing the internal branch of the cross-section boundary envelope curve. The internal branch of the boundary envelope curve in the cross section R-Z shows generally 3 loops. The middle loop delimits a ring void and it is a part of the boundary curve; the others are related to 4-solution regions for the IK problem. By considering a formulation for the cross-section workspace boundary of 3R manipulators as proposed in (Ottaviano, E., Husty, M. & Ceccarelli, M., 2004) it is possible to determine the singularities on the inner boundary curve, which is a part of the enveloping curve. These singularities can be either double points or acnodes or cusps of the cross-section boundary curve, (Ottaviano, E., Husty, M. & Ceccarelli, M., 2006 a).

The graph S of the level-set function reveals a very different nature of these highly interesting singular points that are related with the ring void boundary. Some of them arise just from the projection of S into the level-set plane and some of them come from singularities of the surface S.

A geometrical interpretation for the singularities of the graph of the level-set function is that there is a value of θ_3 for which the operation point H lies on Z_2 axis. Therefore, there is a free motion about Z_2 axis, which is completely independent by θ_2 angle. In this paper we have focused our analysis on this kind of singularity for the industrial 3R manipulators, which have been grouped in orthogonal and ortho-parallel manipulators.

3.1. A Formulation for Orthogonal Manipulators

Orthogonal manipulators are characterized by having three revolute joint axes, which are orthogonal to each other. Therefore, kinematic parameters can be identified as a_1, a_2, a_3, d_2, d_3, twist angles α_1 and α_2 are set equal to -90 and 90 deg. Joint variables are identified as θ_1, θ_2 and θ_3, respectively, and they will be assumed unlimited in this work. A kinematic scheme is displayed in Fig. 3.

The surface S of Eq.(6) can be studied by looking at factors S_1 and S_2 of S.

For orthogonal manipulators the surface S_1 can be expressed in the form

$$S_1 = k_4 Z^4 + k_2 Z^2 + k_0 \tag{8}$$

where the coefficients k_i depend on a_2, a_3 and d_3 only, in the form

$$k_4 = (a_3 - a_2)^2 + d_3^2$$

$$k_0 = (a_3 + a_2)^2 + d_3^2 \qquad (9)$$

$$k_2 = 2(a_3 - a_2)(a_3 + a_2) + 2d_3^2 .$$

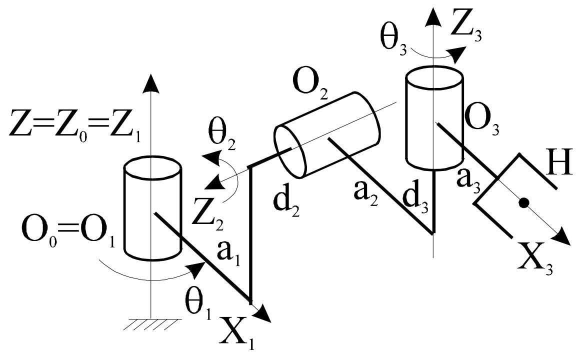


Fig. 3. A kinematic scheme for orthogonal 3R manipulators

In general the equation for S_1 can have real solutions. The necessary and sufficient condition for having real solutions is: $d_3 = 0$ and $a_3 > a_2$. Other singularities can be found by analyzing surface S_2. Zeros of the set of equations $S_2 = 0$; $S_{2X} = 0$; $S_{2Y} = 0$; and $S_{2Z} = 0$, yield the geometric singularities of the surface S_2.
Singularities of S_2 surface can be can expressed by the product of two polynomials in the form

$$P_1 = d_3^2 + (a_3 - a_2)^2 ;$$

$$P_2 = \left(a_2^2 + a_3^2 + d_3^2\right)^2 - 4a_2^2 a_3^2 \qquad (10)$$

The zeros of the set of equations: $S_2 = 0$; $S_{2X} = 0$; $S_{2Y} = 0$; and $S_{2Z} = 0$, yield the geometric singularities of S_2.
The herein proposed classification allows obtaining design information related to workspace properties. Geometrically, when α_2 is equal to 90 deg. then a member of the θ_3 parameter set of curves belonging to different values of a_3 can intersect the Z_2 axis only when d_3 is equal to zero. In particular, each possible intersection of a θ_3 curve represents a singularity of the level-set graph. Only three cases can arise: no intersection, two distinct intersections and two coincident intersections. The three cases represent three classes of industrial-type manipulators.

3.2 A Formulation for Ortho-Parallel Manipulators

Ortho-parallel manipulators are characterized by having the first two revolute joint axes orthogonal to each other, and the last revolute joint axis is parallel to the second one. Therefore, kinematic parameters can be identified as a_1, a_2, a_3, d_2, d_3, and twist angles α_1 and α_2 are set equal to -90 and 0 deg. Joint variables are identified as θ_1, θ_2 and θ_3, respectively, and they will be assumed unlimited in this work. A kinematic scheme is displayed in Fig. 4.

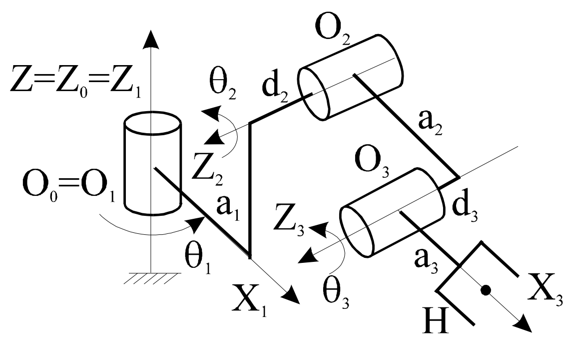


Fig.4. A kinematic scheme for ortho-parallel 3R manipulators

The factors S_1 and S_2 of S are analyzed separately. S_1 can be expressed in the form

$$S_1 = k_2 Z^2 + k_0 , \qquad (11)$$

in which k_i coefficients depend on a_2 and a_3 only, in the form

$$k_2 = (a_3 - a_2)^2$$

$$k_0 = (a_3 + a_2)^2 \qquad (12)$$

A necessary and sufficient condition for having real solutions can be determined through only when there are changes in the signs of coefficients k_i. In particular, the number of real roots is equal to the number of changes of sign in the k_i coefficients. Therefore, S_1 has no real solutions. Other singularities can be found by analyzing surface S_2. Zeros of the set of equations $S_2 = 0$; $S_{2X} = 0$; $S_{2Y} = 0$; and $S_{2Z} = 0$, yield the geometric singularities of the surface S_2. Singularities of S_2 surface are given in the form

$$P_1 = (a_3 - a_2)^2 \qquad (13)$$

Geometrically, when α_2 is equal to 0 deg then the θ_3 parameter set of curves can intersect the Z_2 axis only when a_2 is equal to a_3. For this case, a θ_3 curve is in a plane, which is orthogonal to Z_2 axis. Only two cases can arise: no intersection and two coincident intersections. These two cases yield two different classes of industrial-type manipulators.

4. A Classification for Industrial 3R Manipulatros

The following groups contain all possible topologies of orthogonal and ortho-parallel manipulators, which can be characterized by the presence of singularities on the surface S. Furthermore, if the surface has real singularities then they also correspond to singularities in the cross-section of the boundary curve. A classification of industrial 3R manipulators can be proposed as shown in Table 1 by referring to the general scheme in Fig.5, in agreement with the following discussion. In particular, the shape of S surface function is characterized by the relative location of the characteristic points A and B that represent singularities for S function.

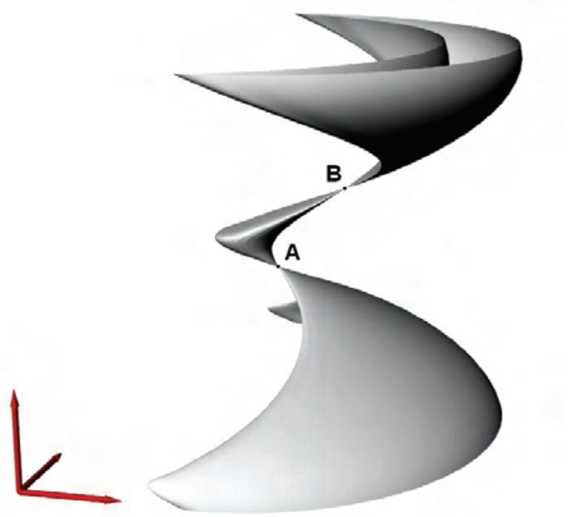
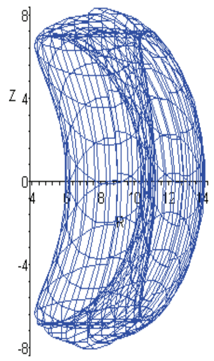


Fig. 5. A general scheme for the level-set reconstruction function S for the workspace of 3R industrial manipulators

Class	1	2	3a	3b	3c
Characteristic points	A→ ∞ B→ ∞	A = B	A ≠ B	A ≠ B A→ ∞	A ≠ B B→ ∞

Table 1. A classification of industrial 3R manipulators through relative location of points A and B in Fig.5

4.1 Class 1: A General Manipulator



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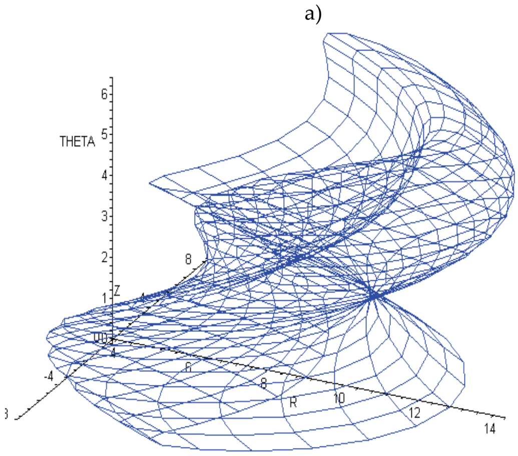


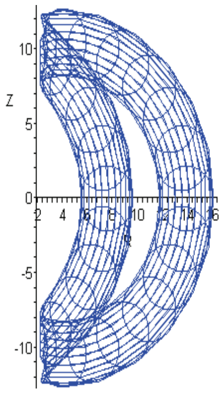
b)

Fig. 6. A numerical example of Class 1 orthogonal manipulators without void, with a1=2.61 u, a2=0.97 u, a3=3.12 u, d2=7.21 u, d3=6.92 u : a) workspace cross-section; b) surface S. (u is unit length and angles are in radians)

A manipulator that belongs to the Class 1 has no (real) singularities on the surface S since both the characteristics points A and B are at infinite and in opposite directions. It may have either a changing posture behavior or it can present a void within the workspace. A characteristic shape with corresponding cross-section figures is reported in the examples of Figs. 6 and 7 for orthogonal manipulators and ortho-parallel manipulators, respectively. The corresponding cross-section boundary curve can have only cusps and/or double points. Such general manipulators are characterized to have no singularities on the graph of the level-set. In addition, it can be observed that in general cuspidality behavior is not strictly related to special designs.

4.2 Class 2 Industrial Manipulators

A manipulator that belongs to the Class 2 has only one singularity on the surface S that is determined by the coincidence of the characteristic points A and B. Class 2 manipulators can be characterized by the presence of 4-solution regions for the IK as identified in the S area that is delimited by characteristic points A and B. The cross-section boundary curve for Class 2 manipulators contains one acnode as a singular point, (Ottaviano, E., Husty, M. & Ceccarelli, M., 2006 a).



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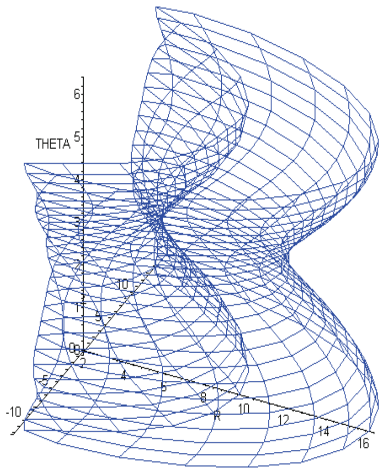


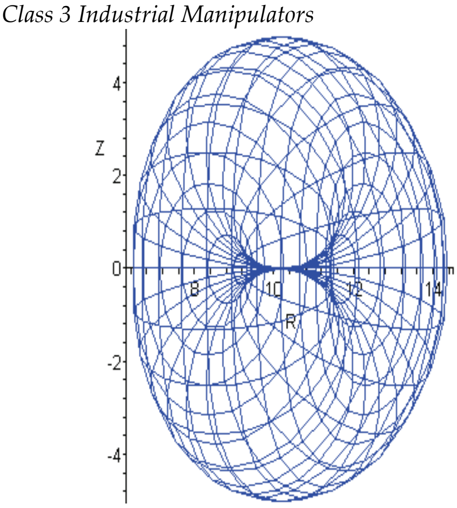
b)

Fig. 7. A numerical example of Class 1 ortho-parallel manipulators with void, with a1=3.28 u, a2=10.50 u, a3=2.20 u, d2=1.69 u, d3=0.492 u: a) workspace cross-section; b) surface S. (u is unit length and angles are in radians)

A Class 2 orthogonal manipulator is characterized by having $a_2 = a_3$; and $d_3 = 0$. If $a_2 \leq a_3$, the operation point H can meet the second joint axis whenever $\cos \theta_3 = \pm\ (a_2/a_3)$, which was found also in (Zein, M., Wenger, P. & Chablat D., 2005). Characteristic shape with corresponding cross-sections for Class 2 orthogonal manipulators is reported in the example of Fig. 8. For class 2 orthogonal manipulators, S_1 expression vanishes and singularities can be found by checking S_2 polynomial expression. Class 2 orthogonal manipulators are characterized to have two coincident singular configurations that depend on θ_3 parameter only.

A Class 2 ortho-parallel manipulator is characterized by having $a_2 = a_3$. Characteristic shapes of workspace cross-section and surface S are shown in the example of Fig.9.

4.3 Class 3 Industrial Manipulators



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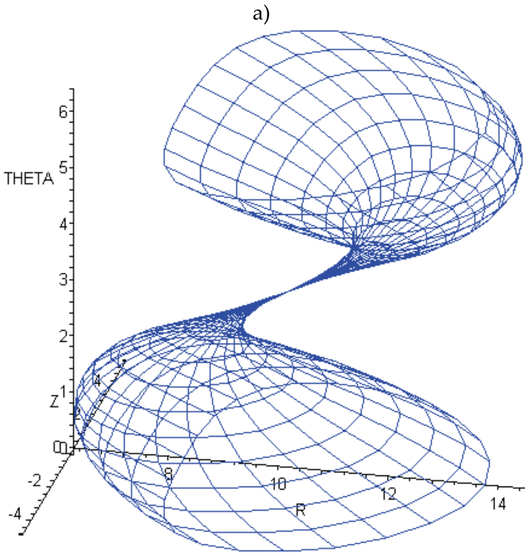
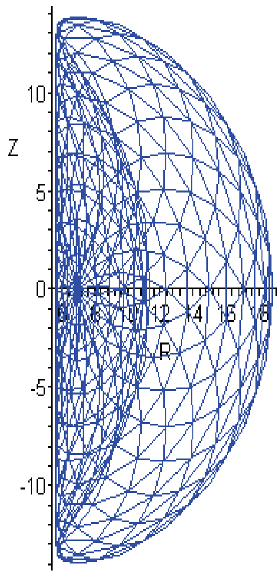


b)

Fig. 8. A numerical example of Class 2 orthogonal manipulators, with a1=6.00 u, a2=a3=2.51 u, d2=8.22 u: a) workspace cross-section; b) surface S. (u is unit length and angles are in radians)

This class of manipulators is characterized by having two distinct singularities on the surface S and in general 4-solution regions for the IK. Class 3 orthogonal manipulators have $a_3 > a_2$ and $d_3=0$. The meaning of a singularity is that for a θ_3 value exists a line passing through the operation point H and intersecting one of the manipulator axes. If d_3 is not equal to zero then the generating curve has no real solutions; if a_3 is less than a_2 then no singularities are on the S surface. A characteristic shape and corresponding cross-section of manipulator workspace is reported in the example of Fig. 10. Class 3 manipulators can have a void only when the projections of the singularities of the S surface belong to the workspace boundary too. At the two singularities, point H meets the second joint axis and the manipulator has infinite IK solutions [10]. It has been found that ortho-parallel manipulators cannot have two distinct singularities.



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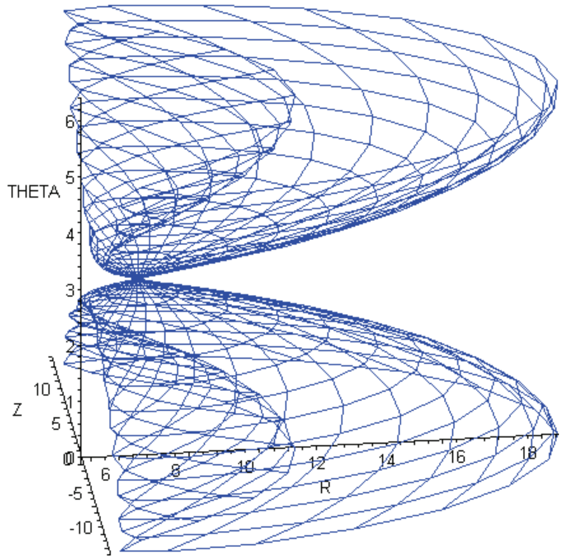
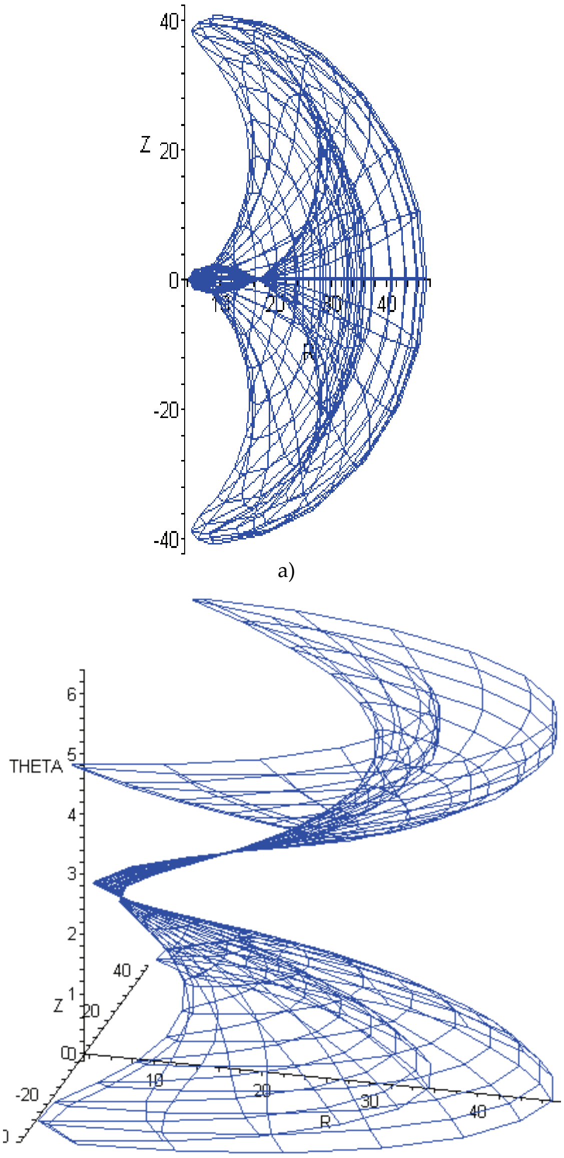


b)

Fig. 9. A numerical example of Class 2 ortho-parallel manipulators, with a1=4.04 u, a2=a3=6.98 u, d2=0.957 u, d3=4.64 u: a) workspace cross-section; b) surface S. (u is unit length and angles are in radians)

5. Numerical examples

Figure 11 refers to the manipulator arm of the Miller Hybrid Arc Welding Robot System, (Miller, 2001), that shows an orthogonal architecture. The results of the workspace analysis through the proposed level-set reconstruction are shown in Fig.11b) from which the manipulator can be recognized as a Class 3 type manipulator.



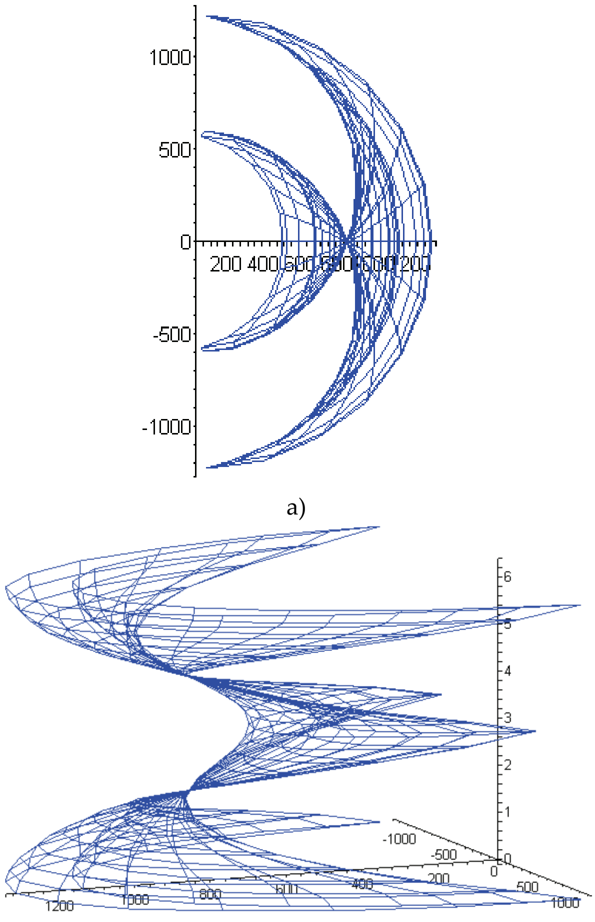
a)

b)

Fig. 10. A numerical example of Class 3 orthogonal manipulators, with a1=5.77 u, a2=19.20 u, a3=21.45 u, d2=6.31 u: a) workspace cross-section; b) surface S. (u is unit length and angles are in radians)

In Figs. 12 and 13 the results of the proposed analysis are shown for the KUKA KR30 robot, (KUKA 2006), by considering two structure configurations that are related to the cases of freezing a joint in the 4R manipulator chain with the aim to simulate operations with unused or damaged mobility of joints. An ortho-parallel configuration of the manipulator of industrial robot KUKA KR30 is obtained by considering the first three joints of the manipulator chain only when the last joint is frozen. The workspace characteristics are shown in Fig.12, indicating a Class 2 manipulator. An orthogonal architecture of the manipulator of industrial robot KUKA KR30 is obtained by considering by freezing the third joint of the robot arm and by considering the last joint as the extreme joint of the 3R manipulator chain. The workspace characteristics are shown in Fig.13, identifying it as a Class 1 manipulator.

Such an investigation of sub-structures of manipulator arms can be of interest for practical operation of the robot when for any reason not all the joints are used or when partial motions refer to those sub.-structures.



a)

b)

Fig. 11. Workspace analysis for Miller MRH industrial orthogonal manipulator, (Miller 2001) with a1=88.0 mm, a2=315.0 mm, a3=912.0 mm, d2=0; d3= 0: a) workspace cross-section; b) surface S

6. Conclusions

Workspace topologies of 3R industrial manipulators are characterized by using a level-set reconstruction. The proposed algebraic expressions have been useful to classify industrial 3R manipulators and to identify design conditions for avoiding singularities in the workspace.

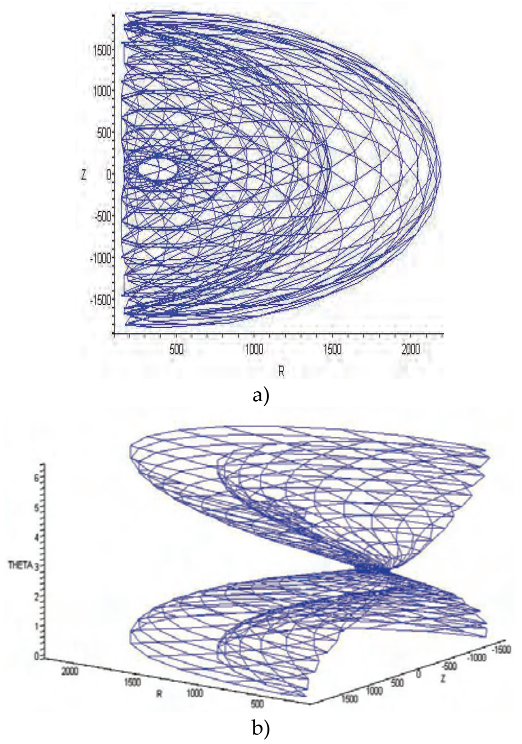


Fig.12. Workspace analysis of manipulator arm of industrial robots KUKA KR 30, (KUKA 2006), with a1= 350 u, a2 = 850 u, a3= 990 u, d2 = 0 u, d3 = 145 u, ◎1 = 90 deg; ◎2 = 0 deg: a) workspace cross-section; b) surface S. (u is unit length and angles are in radians)

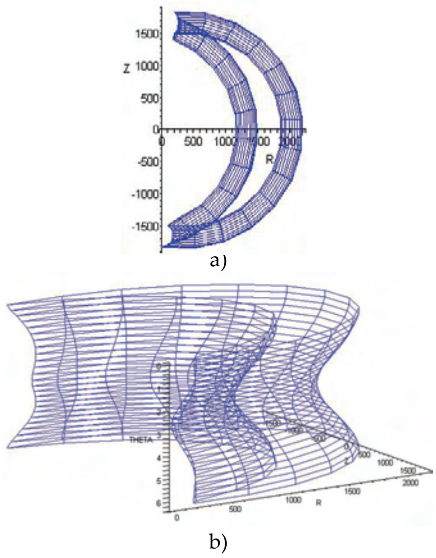


Fig. 13. Workspace analysis of manipulator arm of industrial robots KUKA KR 30, KUKA 2006), with a1= 350 u, a2 = 1,670u, a3= 170 u, d2 = 0 u, d3 = 145 u, ◎1 = 90 deg; ◎2 = 90 deg: a) workspace cross-section; b) surface S. (u is unit length and angles are in radians)

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