Investigating the quality of mental models deployed by undergraduate engineering students in creating explanations: The case of thermally activated phenomena

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This paper describes a method aimed at pointing out the quality of the mental models undergraduate engineering students deploy when asked to create explanations for phenomena or processes and/or use a given model in the same context. Student responses to a specially designed written questionnaire are quantitatively analyzed using researcher-generated categories of reasoning, based on the physics education research literature on student understanding of the relevant physics content. The use of statistical implicative analysis tools allows us to successfully identify clusters of students with respect to the similarity to the reasoning categories, defined as "practical or everyday," "descriptive," or "explicative." Through the use of similarity and implication indexes our method also enables us to study the consistency in students' deployment of mental models. A qualitative analysis of interviews conducted with students after they had completed the questionnaire is used to clarify some aspects which emerged from the quantitative analysis and validate the results obtained. Some implications of this joint use of quantitative and qualitative analysis for the design of a learning environment focused on the understanding of some aspects of the world at the level of causation and mechanisms of functioning are discussed.

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I. INTRODUCTION

Many research reports present a new vision of science education which integrates the learning of disciplinary contents and scientific processes [1,2], i.e., the knowledge of scientific explanations and the practices needed to engage in scientific inquiry and design. It has been pointed out that, although the practices used to develop and represent scientific theories differ from one science domain to another, all sciences share certain common features at the core of their inquiry-based and problem-solving approaches. The processes of developing and using models play a central role in these.

In the last few years, researchers and educators have been increasingly interested in the role of models in science teaching, from various points of view [3–13]. In particular, it has been shown [14] that building and using models can help students to consolidate and improve their reasoning skills, helping them to analyze and assess a given phenomenon.

Among many cognitive theories, those explaining student reasoning in terms of structured cognitive conceptions, or mental models [15], are of special interest for physics education. For this reason a lot of research has been devoted to analyzing the mental models of students at different school and university levels [16]. This paper aims at supplying a method to point out the quality of mental models that undergraduate students deploy when they are asked to create explanations for an everyday-life phenomenon, relating it to the physics and chemistry they have already studied in previous courses. The method is based on quantitative and qualitative data analysis: it involves the construction of a tool (a specially designed open-answer questionnaire) and a quantitative analysis of student responses, supported by specifically designed interviews.

The questionnaire requires students to clarify the physical meaning of the quantities involved in the given phenomenon (the evaporation of a water puddle at different temperatures), discuss the related explicative model(s), and propose other experimental situations that can be explained by using the same model(s). The focus is on systems for which a process is thermally activated by overcoming a well-defined potential barrier ΔE, and is therefore described by an equation containing the Boltzmann factor $e^{-\Delta E/kT}$, where T is the system temperature and k is the Boltzmann constant.

The questionnaire answers are analyzed by means of similarity and implicative trees [17] which can be built on the basis of a phenomenographic [18,19] categorization. A qualitative analysis of the interviews aims at examining some aspects which emerged from the quantitative analysis in more detail, and at validating its results.

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The results discussed here have been obtained with students of the three-year bachelor degree program in chemical engineering at the University of Palermo (UniPA), Italy.

In the next section we describe the theoretical basis of our study, and then we present the different steps of our research by explaining the research questions, methods, and data analysis. In the last two sections we discuss our results and their implications for teaching.

II. THEORETICAL UNDERPINNINGS

Pedagogical modeling theories [7,20,21] focus on different phases of modeling processes and justify them with the need to move students' learning from the mere description of a phenomenon to explicative models. As it is well known [10–12,22–24], in physics an explicative model is different from a descriptive one because it supposes a system has properties which are not directly observable, but play a role in the observed regularities. Indeed, the model construction and validation process requires the building of several hypothesis typologies: empirical law hypothesis, synthesis of regularities (arising from phenomenological observations and condensed into rules), and hypothesis for the construction of explicative models introducing theoretical representations and often containing nonobservable entities. In the construction process of explicative models, inductive reasoning is involved, but an important role is also played by analogical reasoning [25], i.e., the ability to see similarities and differences between a "source" (something perceived as similar to what we are going to analyze) and the "target" (the real phenomena we are studying).

Research in science education has shown the strong presence in common reasoning of causal explanations [26–28]. For this reason, the teaching of formal laws and functional relationships alone seems insufficient for learning at school age and unsatisfactory for the students' need to understand. Pupils require a causal explanation that supplies a mechanism, which can account for the dynamics of facts and effects that have led to a given situation.

Cognitive theories explain learning in terms of changes in mental processes and knowledge structures resulting from a learner's efforts to make sense of the physical world [29]. Cognitive scientists have described people's personal conceptions of the world, introducing the term "mental model" and defining its main characteristics [15,30]. We share the Greca and Moreira's [8] definition of a mental model as "*an internal representation, which acts out as a structural analogue of situations or processes. Its role is to account for the individuals' reasoning both when they try to understand discourse and when they try to explain and predict the physical world behavior.*" To a greater or lesser extent, this representation contains structural information about the properties of the system and functional knowledge about its behavior.

Gilbert and Boulter [31] highlight the private nature of mental models and suggest that the researchers must rely on some *expressed form* of the mental model to infer what it can be. This is mainly done by means of external representations of individual's reasoning, like speech, writing, or other actions. Modeling literature analyzes the mental model properties [32] and describes people's reasoning as the "running" of the procedures present in their mental models.

Students' reasoning can be referred to mental models describing their personal views of the world (the spontaneous models [33]), or scientifically accepted models. However, students' reasoning can also be related to a different kind of mental model, defined in literature as "hybrid models" [8] or "synthetic models" [34], which represent a composite mental model that unifies different features of initial spontaneous models and scientifically accepted models. Such models are inconsistent (in one or more features) with both models from which they are derived. Research reveals [35,36] that a student can use different mental models in response to a set of situations or problems considered equivalent by an expert. In particular, Bao and Redish [35] developed a way to deal with these composite mental models "*by considering the student as being able to simultaneously possess multiple models with a distribution of probabilities for the activation of the different models*." They define students' model states and analyze changes of such states with specific contextual features in different equivalent questions. Moreover, they point out that probing the context dependence of students' mental models, and the consistency in their deployment, is relevant for teaching, as well as for the construction of assessment tools.

Many research papers studied the mental models deployed by students in order to make sense of given phenomenology, and students' understanding of models in different contexts [35–39], often using qualitative or quantitative analysis methods. However, in the last years there has been a move in social science towards multimethod approaches, which tend to emphasize the breadth of information which the use of more than one analysis method may provide to the researcher [40,41]. Research shows that external representations of the mental models are often activated in multimodal ways [42]. In many cases it may be useful to carefully study the relationships within and between the representations, and their dynamics in time [43,44].

Research results on eliciting and characterizing student mental models, based on the joint use of quantitative and qualitative methods, can be found in the literature [45,46]. Our paper develops this research context and is mainly focused on the development of a new quantitative and qualitative method to analyze students' scientific explanations [21]. Our aim is to study the effectiveness of the method to infer mental models students deploy in building explanation, and to study consistency in their deployment. This is done by analyzing the external representations of the mental models students use when tackling a written questionnaire and interviews, i.e., their ''answering strategies.''

III. THE RESEARCH

A. Research questions

Following the general theoretical framework and the research aims discussed above, this paper directly addresses the following research questions:

- Is our multimethod approach to the analysis of students' answering strategies adequate to highlight the characteristics of the mental models students deploy when searching for explanations to phenomena or situations?
- Can our methodological analysis point out consistency in students' deployment of mental models?

B. Sample and context

Our research sample consists of 34 freshmen, enrolled in the chemical engineering degree program during the academic year 2010–2011 at UniPA. Many of them attended secondary schools where physics is usually taught by following a traditional, teacher-centered approach, and where physics teaching is mainly based on teacher transmission of general concepts to students. In some cases, the lessons are integrated with laboratory activities, but these are often performed by the teachers themselves in a demonstrative way. As a consequence, skilled students are only allowed, at best, to contextualize ideas which have already been established, going from general laws to real-life situations mainly with a ''law verification'' approach. More passive students, on the other hand, tend to develop rigid mnemonic abilities, focused on repetition and description of the concepts they have studied, often only from a mathematical point of view.

During the first semester of their degree program the students attended general mathematics, physics, and inorganic chemistry courses, and they had already passed the related exams. When requested to participate in our study, they were attending a second semester physics course dealing with the fundamentals of electromagnetism, and voluntarily chose to participate in the survey. The total number of students on the course was about 60.

C. Methodology

In order to clarify the general framework used for this research, we will summarize it in six ''steps,'' that are shown below. These steps are then described in more detail in Secs. III D, III E, and IV B:

Step 1: The physics tasks and questionnaire items are formulated on the basis of a review of educational research literature and a survey conducted with some UniPA university teachers.

Step 2: Validation of the questionnaire is performed: five physics freshmen, coming from the same secondary schools attended by our student sample, are asked to highlight problems in the questions, like unclear or ambiguous terminology. Then researchers make an independent analysis of the possible (*a priori*) student responses to the questionnaire items, which results in the singling out of a set of possible answering strategies for each item.

Step 3: After the submission of the questionnaire to the research sample, researchers independently analyze actual student responses to each item and place them in the appropriate *a priori* answering strategies, adding new ones as needed.

Step 4: It is assumed that each student has a latent cognitive structure underlying their answers to the questionnaire items, referred to as a mental model. Answering strategies are grouped into idealized sets. Each set is synthesized by typical reasoning procedures that allow us to infer an epistemic category of students' mental models, defined as ''practical or everyday,'' ''descriptive,'' or ''explicative.''

Step 5: The extent to which actual student answering strategies correspond to the idealized categories is studied by using statistical implicative analysis methods [17].

Step 6: An interview protocol is designed by the researchers and interviews are taken with a subset of the student sample in order to extend and validate the results obtained by means of the quantitative analysis.

The interviews were conducted immediately after the questionnaire submission, on a voluntary basis. All students were invited to participate in the interview, but only 15 accepted. The interview questions were aimed at supplying relevant information about the meaning of students' answers and at widening the analysis of their answering strategies, highlighting points of interest or unusual elements in the questionnaire answers. Checking the validity of the questionnaire items in actually revealing the students' reasoning when constructing explanations was another aim of the interviews. The interview protocol was predesigned by all three researchers, but the interviews were conducted by one of them, face to face with the students. In many cases, questions not included in the interview protocol were asked, in order to better clarify specific situations which emerged during the discussion. See Sec. IV B 2 for more details.

D. Questionnaire design

Before the questionnaire design phase, the researchers informally interviewed six lecturers and professors teaching physics or chemistry at the UniPA Engineering Faculty with the objective to collect information about the typical conceptual difficulties encountered by students when asked to describe and explain real-life, practical situations.

The main difficulties pointed out were the students' inability to identify the physical or chemical quantities relevant for the description or explication of a phenomenon and the relationships between them, their propensity to simply recall a mathematical law to explain a proposed situation, and, in general, their difficulty in generalizing the knowledge gained in theoretical contexts to solve a problem. These learning difficulties are well known in the science education research literature [47–50] and we took them into account in order to design the six-item questionnaire we used to collect data for our research.

In the first questionnaire item, students were requested to discuss a real-life situation (the evaporation of a water puddle at different environmental temperatures) dealing with a typical chemical kinetics subject, the Arrhenius law[1] [51]. Then, they were asked to try to explain the law, and clarify the physical meaning of the quantities involved in it. The remaining questionnaire items dealt with the explicative models behind the Arrhenius law and the possible phenomena showing temperature dependences similar to the one demonstrated by the chemical reaction rate, which can therefore be explained by a similar model.

All the questionnaire items were aimed at showing the different characteristics of the reasoning used by students to describe and explain the proposed situations, so as to infer the deployed mental models. They addressed the problem at different levels (verbal explanation, mathematical description, identification of relevant variables and relationships, identification of a "functioning mechanism," application of a model to other phenomena or situations). The questionnaire items can be found in Appendix A.

The questionnaire preparation was followed by a preliminary validation involving a group of five physics freshmen, coming from the same secondary schools attended by our student sample, in order to test the questionnaire face validity [52] and highlight other problems in the questions, like unclear or ambiguous terminology. Each student in the pilot group completed the questionnaire and then a focus group was conducted with the students, in order to clarify the meaning of their answers and get to the final version of the tool to be used with the research sample.

[1]The Arrhenius law describes the temperature dependence of the rate u of a chemical reaction: $u = Ae^{-E_A/kT}$, where A is a constant, T is temperature, k is the Boltzmann constant, and E_A is the so-called "activation energy." E_A can be described, to a first approximation, as the minimum energy the reactants must possess in order for the reaction to take place. The Arrhenius law contains the well-known Boltzmann factor, $e^{-\Delta E/kT}$, an expression which is useful to portray the behavior of natural systems that exchange energy with their environment. Arrhenius-like formulas are commonly used to describe the temperature dependence of many phenomena needing a minimum energy ΔE to be started, or activated. These phenomena are sometimes referred to as thermally activated or "threshold" phenomena.

Then an "*a priori*" analysis of the possible student responses to the questionnaire items was performed. According to Brousseau [53], this analysis allows the answering strategies expected from students facing a problematic situation to be highlighted, as well as the potential alternative responses that may appear. This can be very useful for the researcher, who can content validate the questionnaire [54,55]. In fact, the search for possible student answering strategies can highlight weak points in the questions, and allows the researcher to modify them before administering the questionnaire. The analysis is conducted independently of the observation (hence the term *a priori*), in order to provide a reference point for the subsequent study of the actual student answers to the questionnaire items.

In order to strengthen this questionnaire validation, the *a priori* analysis was independently performed by the three researchers and then a consensus procedure was negotiated to obtain a shared version that is optimized to the research aims.

The final version of the questionnaire was submitted to the research sample and the actual student answers were separately analyzed by the researchers, by comparing and contrasting them with the answering strategies found during the previous step (pilot validation and *a priori* analysis). From this comparison it emerged that some of the previously hypothesized strategies were not used by the students, but some more unforeseen ones were put into action. In line with previous research [56–58], these were "*a posteriori*" added to the *a priori* answering strategies, in order to obtain a comprehensive list of 61 strategies which can be used to classify students' mental models. This list is shown in Appendix A, where each question is followed by the related answering strategies.

E. Analysis of student answers

During the analysis of the student answering strategies, each researcher used the list to draw up a table summarizing them. Discordances between researchers' tables were found in some cases, when a student answer was classified under not just one of the *a priori* or *a posteriori* strategies, but two or more of them. In a few cases, discordances were due to different researchers' interpretations of students' statements. This happened 19 times when comparing the tables of researchers 1 and 2, 17 times for researchers 1 and 3, and 16 times for researchers 2 and 3. Hence, a good interrater reliability of the analysis is demonstrated, with accordance percentages of about 91%–92% between the analysis tables of each pair of researchers. The differences between the three tables were compared and discussed by the researchers to reach a consensus on a common table to use for the study.

The careful reading of the students' answers within a framework provided by domain-specific expertise and previous research in the field of the description of student

TABLE I. Categories of mental models deployed by students when tackling the questionnaire and the related reasoning procedures.

Practical or everyday	Descriptive	Explicative
Reflects the creation of situational meanings derived from practical, everyday contexts. The student uses other situations to try to explain the proposed ones.	The student describes and characterizes the analyzed process by finding or remembering the relevant variables and/or recalling from memory their relations, expressing them by means of different language (verbal, iconic, mathematical). They do not explain the causal relations of the physics parameters involved on the basis of a functioning model (microscopic or macroscopic).	The student proposes a model (qualitative and/or quantitative) based on a cause or effect relation or provides an explanatory hypothesis by introducing models which can be seen at a theoretical level.

TABLE II. Ideal student profiles and related answering strategies for the six-item questionnaire. Numbers refer to the item, lowercase letters to the mental model category [practical or everyday (pe), descriptive (de), or explicative (ex)], and uppercase letters to the specific answering strategy, as shown in Appendix A.

Practical or everyday	Descriptive	Explicative
1peA, 1peB	1deA, 1deB, 1deC, 1deD, 1deE, 1deF	1exA, 1exB, 1exC, 1exD
2peA, 2peB	2deA, 2deB, 2deC	2exA, 2exB
3peA, 3peB	3deA, 3deB, 3deC, 3deD, 3deE, 3deF, 3deG, 3deH, 3deI	3exA, 3exB
4peA	4deA, 4deB, 4deC, 4deD, 4deE, 4deF	4exA, 4exB
5peA, 5peB	5deA, 5deB, 5deC, 5deD	5exA, 5exB, 5exC
6peA	6deA, 6deB, 6deC, 6deD, 6deE, 6deF, 6deG, 6deH	6exA, 6exB

modeling competencies [59] allowed us to classify students' responses into three phenomenographic categories of mental models. They are *practical or everyday*, *descriptive*, and *explicative*, as described in Table I, where the reasoning procedures representative of each model category are also shown. In Appendix B there is an extended version of Table I, with examples of typical student answers classified in each category and the related answering strategy codes (as shown in Appendix A).

We then built a table which identifies three "idealized sets" containing the answering strategies that can be considered typical of each mental model category shown in Table I. Each set defines the ideal profile of a student answering all the questionnaire items always using strategies related to the same category of mental model. These profiles, shown in Table II using the answer codes from Appendix A, have been used for a similarity analysis of the research data, which is further explained in Sec. IV.

IV. DATA ANALYSIS

A concise overview of the quantitative method used

In this study we use two statistical implicative analysis (SIA) [60–62] functions, the *similarity* and the *implication indexes*. They are aimed at getting fine-grain detail about the properties of our sample system (a group of 34 students and 61 answering strategies), on the basis of our students' categorization. Here, we briefly define similarity and implication indexes and give some details about the use we make of them in this research. They are well described in [62], where a full theoretical discussion of their derivation and meaning is provided. An application of the use of these index in a piece of research focused on the epistemological approaches to knowledge of preservice elementary school teachers is thoroughly discussed in [58].

Let us consider two generic students i and j. Lerman's similarity index [63,64] classifies students according to hierarchical clustering [65,66] and allows similarities in their behavior (i.e., similar answering strategies) to be recognized. It is defined as follows:

$$s(i,j) = \begin{cases} \frac{n_{i\wedge j} - \frac{n_i n_j}{n}}{\sqrt{\frac{n_i n_j}{n}}} & \text{for } n_i \neq n_j; \quad n_i, n_j \neq 0; \quad n_i, n_j \neq n \\ 1 & \text{for } n_i = n_j, \end{cases}$$

where n_i and n_j are the number of answering strategies put into action by i and j, respectively; n is the total number of answering strategies (61 in our case); and $n_{i\wedge j}$ is the number of common answering strategies used by i and j.

For fixed values of n_i and n_j, the greater $n_{i\wedge j}$ (i.e., the more i and j are "similar") the more positive $s(i,j)$. When i and j put into action completely different strategies ($n_{i\wedge j} = 0$), the similarity index assumes negative values.

If we take into account two generic answering strategies a and b, we can define the implication index $q(a,\bar{b})$:

$$q(a,\bar{b}) = \begin{cases} \frac{n_{a\wedge\bar{b}} - \frac{n_a n_{\bar{b}}}{n}}{\sqrt{\frac{n_a n_{\bar{b}}}{n}}} & \text{for } n_a \neq n \ \wedge n_b \neq n \ \ n_a \neq 0; \ \ n_{\bar{b}} \neq 0 \\ 1 & \text{for } n_a = n \vee n_b = n, \end{cases}$$

where n_a is the number of students who put into action strategy a, $n_{\bar{b}}$ is the number of students not putting into action strategy b (i.e., using all possible strategies except b), n is the total number of students (34 in our case), and $n_{a\wedge\bar{b}}$ is the number of students both using strategy a and not using strategy b.

In order to clarify the use we made of them, we should point out that $s(i, j)$ here is mainly used to reveal if there is a grouping of student behaviors and if it is possible to identify clusters of behavior with respect to the similarity to the ideal profiles that we discussed in Sec. III E and reported in Table II. This can also help us to analyze the consistency of student mental models when tackling the questionnaire items.

The implication index, $q(a, \bar{b})$, allows us to find relations (i.e., implications) between strategies activated in each questionnaire answer and to study in more detail their consistency. The possibility offered by $q(a, \bar{b})$ to get fine-grain detail about implications between the strategies used allows us to better specify the similarity results [obtained by the use of $s(i, j)$], and to steer the qualitative analysis of students' paths of reasoning by means of semistructured interviews.

B. Data analysis

In order to analyze the data, we used CHIC (Classification Hiérarchique Implicative et Cohésitive) software [67–69]. It allows associations (implicative and similarity) from a set of data to be calculated and dendrograms to be constructed in the form of "implicative graphs" and "similarity trees," for an easy comparison of the results. The software also provides a level of significance for each index value. In fact, an implication between two strategies is identified on the basis of the percentage of students who make use of both the first answering strategy and the second. The similarity between two students is also expressed by a percentage indicating the similarity level, i.e., the reliability assigned by CHIC to the similarity relation between them.[2]

The 37×61 matrix we built in order to use CHIC is modeled on the one in Table III. In it, the rows represent the 34 real students plus the three ideal student profiles, and the columns report the 61 answering strategies.

For example, let us say that student s1 used strategies 1peB, 2exA, 3deB, 4deA, 5exC, and 6exB in his answers to the six questions. Therefore, the s1 row in Table III will contain the binary digit 1 in the six cells corresponding to these strategies, while all the other cells will be filled with 0. The last three rows represent the ideal student profiles described in Table II and are, therefore, filled with 1 and 0 according to these profiles.[3]

[2]For each answer to a question, CHIC identifies a student with one of the three "ideal profiles" if the student used at least one of the question-related answering strategies in Table II for that question, i.e., if a student used strategy 6exA and/or 6exB, he is classified as 100% similar (in question (6) to the explicative profile.

TABLE III. Data matrix for CHIC analysis. The 34 students are indicated as s1, s2, ..., s34. The three ideal student profiles are described as practical or everyday, descriptive, and explicative, respectively, and the 61 answering strategies are represented by 1peA, 1peB, ..., 6exB (see Appendix A for more details).

Student	Strategy						
	1peA	1peB	...	1exD	...	6exA	6exB
s1							
s2							
s3							
...	...	...	...	...	...	...	...
s34							
Practical or everyday							
Descriptive							
Explicative							

1. Quantitative analysis of the questionnaire

Figures 1–3 show the similarity trees of students obtained with the questionnaire data in relation to each of the three ideal student profiles, practical or everyday, descriptive, and explicative, respectively. In each graph, students are represented by s_i (where i goes from 1 to 34) in the upper line of the graph. The ideal profile is considered as a "student" and is also placed in the upper line. The similarity trees allow us to study the likeness between each student and the ideal student profiles (at the similarity level reported by the percentages shown in the figures), and also to show relations and similarities between the general answering strategies used by students.

The similarity levels between students are reported on the vertical axis.[4] For example, in Fig. 1 the similarity between s11 and s17 is weaker than the similarity between s4 and s8, as the link between the first two students is lower than the one between s4 and s8.

Figure 1 shows that 13 out of 34 students answered the questionnaire items mainly using strategies related to practical or everyday–type reasoning. Going into more detail, one student, s1, shows a 99% similarity level with this ideal student profile, i.e., he always answers the questionnaire by using practical or everyday–type strategies. Two students,

[3]More information about the software and its use in the framework of SIA can be found on the A.R.D.M. website (Association de Recherche en Didactique des Mathématiques) [70].

[4]Similarity trees are represented by CHIC without respecting a common scale factor in the similarity values reported on the vertical axis.

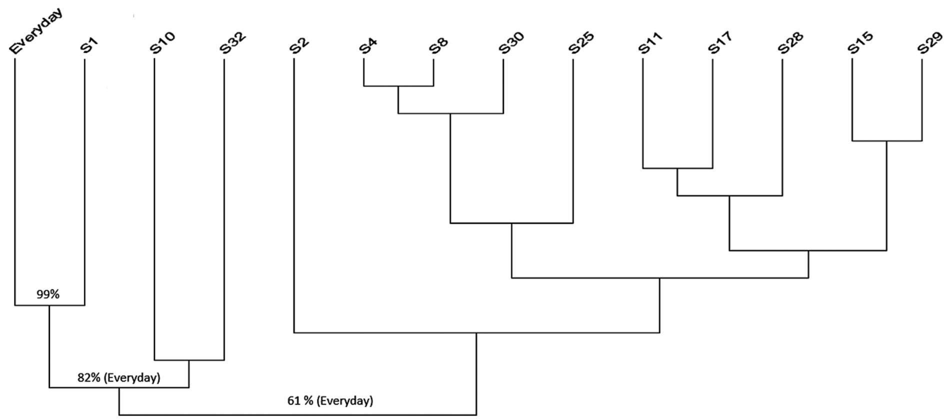

FIG. 1. Similarity tree of real students (s_i) in relation to the practical or everyday ideal student profile. Numbers represent the CHIC generated similarity levels between student clusters and the ideal profile.

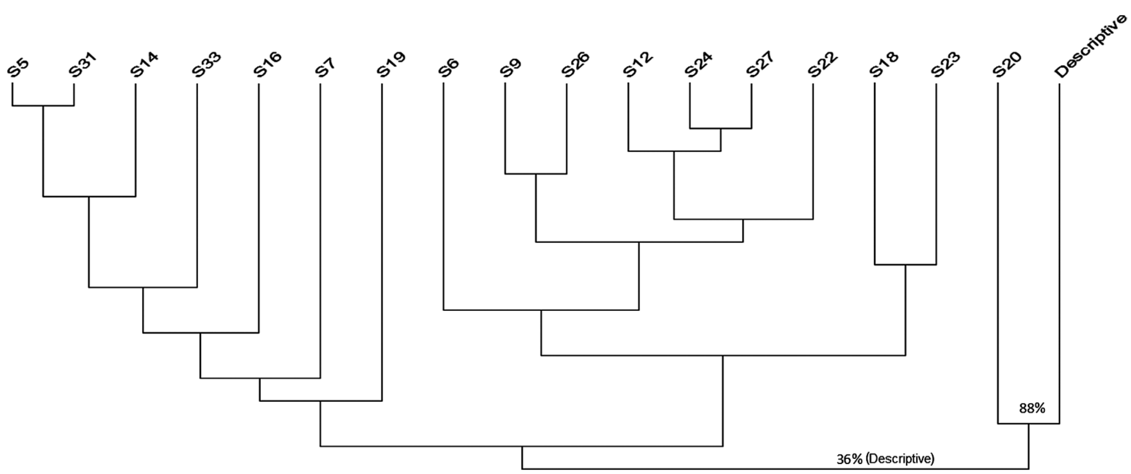

FIG. 2. Similarity tree of real students (s_i) in relation to the descriptive ideal student profile. Numbers represent the CHIC generated similarity levels between student clusters and the ideal profile.

s10 and s32, show an 82% similarity level with the practical or everyday profile and the remaining 10 students approach the questionnaire items with strategies at least 61% similar to the practical or everyday profile. It is worth noting that this means they in some cases adopt different answering strategies (descriptive and explicative).

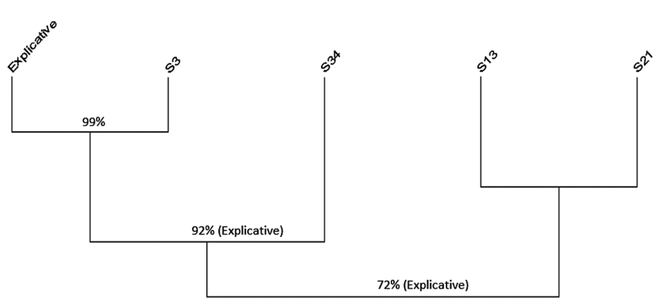

FIG. 3. Similarity tree of real students (s_i) in relation to the explicative ideal student profile. Numbers represent the CHIC generated similarity levels between student clusters and the ideal profile.

Figure 2 shows that one student, coded as s20, answers the test questions mainly with a descriptive approach (at 88% similarity level). For 16 other students, however, the similarity analysis does not give such clear results. They are grouped in a cluster which highlights an overall similarity level with the descriptive profile of only 36%. This means that these students answer the questionnaire items with a variety of approaches, typical of people without a clear line of reasoning. They answer questions by adopting mixed strategies, based on recalling real-life experience and/or memory of subjects they have studied and also use explicative strategies, in some cases. This will be discussed in more detail in Sec. IV B 2 and in the implicative analysis in Appendix C.

The remaining four students in our sample mainly used explicative strategies, as is shown in Fig. 3. s3 and s34 show 99% and 92% similarity levels with the explicative profile, respectively, and s13 and s21 can be considered 72% similar to this profile.

As a concluding remark, it should be pointed out that the first five questionnaire items were answered more or less by all 34 students, but only 25 of them answered item 6. This item requires the students to explain why the different phenomena they cited in the answer to item 5 can be considered similar, and asks them to explicitly state if a common physical quantity characterizing the phenomena can be identified. This question probably represented a greater challenge for students than the previous ones.

By means of CHIC we also studied the implications between some of the relevant answering strategies used by the students. These implications were mainly used in order to inform the strategy of interviews, and the related qualitative analysis. The implicative tree obtained can be seen in Appendix C, where the implications that can be considered most relevant, mainly in terms of their significance level (i.e., their implication index value) and of the number of students that highlighted these implications, are discussed.

2. Qualitative analysis of interviews

As stated in Sec. III C, the interviews were conducted by one of the researchers, face to face with the students, using a predesigned protocol. It was based on the questionnaire items and on data collected during the focus group conducted during the questionnaire validation phase. In many cases questions not included in the interview protocol were asked in order to better clarify students' answers to the questionnaire and specific situations that emerged during the discussion. The interviews were aimed at performing a deeper analysis of the cognitive styles and strategies of students, extending and validating the results obtained by means of the quantitative analysis [71–73].

According to many research papers [74–76] a detailed analysis of the language used by each student during an interview, or when carrying out an activity involving human interaction, can provide evidence of the cognitive style(s) used when tackling a given issue or problem. Therefore, the interviews were audio recorded and then analyzed by the three researchers, partly on the basis of a search for ''indicator words or utterances'' and specific aspects of students' answers which could help to answer the research questions. The analysis of the semantic properties of the student's language was based on the distinction made by the French psychologist Pauhlan between the sense and the meaning of a word and considering ''*the preponderance of the sense of a word over its meaning*'' ([77], p. 244): ''*the sense is ... the sum of all the psychological events aroused in our consciousness by the word. It is a dynamic, fluid, complex whole, which has several zones of unequal stability. Meaning is only one of the zones of sense, the most stable and precise zone. A word acquires its sense from the context in which it appears; in different contexts, it changes its sense*'' ([77], pp. 244–245).

Several methods of analyzing interview excerpts are described in previous research on this subject. One such method involves the use of coding schemes to associate the number of indicator word or phrases that occur with specific forms of reasoning [78–80]. However, we acknowledge that ''*the nature of language—in which any one grammatical form can be used to fulfill a range of pragmatic functions—renders any coding scheme of dubious value if used separately from a more contextually sensitive ... type of analysis*'' ([81], p. 372).

For this reason when analyzing the interview excerpts we tried to make sense of the students' use of indicator words or utterances in the specific context of the question itself [76,82], in order to highlight points of interest or controversial behavior in the related questionnaire answers. Furthermore, we also allowed the interviews, and the related qualitative analysis, to be driven by particularly relevant strategies used by students when answering the questionnaire items, and by their implications, as reported in the introductory remarks of each interview.

Below, we quote and discuss some excerpts from students' interviews,[5] taken immediately after they answered the questionnaire. We will start by quoting parts of a discussion between the interviewer and a student who was able to find significant analogies between different phenomena in his answer to question 6. He also answered question 4 by explaining the Arrhenius law and outlining the physical quantities involved, but had difficulties in clearly referencing them to an explicative model (see the discussion about implication between strategies 6deE and 4deF in Appendix C).

> Interviewer: *In your answer to question 5 you wrote that two phenomena which can be explained by a law similar to the Arrhenius law are the combustion of a substance and the ionization of an atom. In your answer to question 6 you identified temperature as the physical quantity which best characterizes combustion, but you cited energy as the quantity that influences ionization. Could you better explain your answers?*
>
> Aldo: *Yes. Combustion is a phenomenon which does not start immediately after the initial ''trigger.'' In fact, a minimum temperature must be reached in order to set light to a real-life object, like ... a match, if you strike it. On the other hand, if you want to ionize an atom with, let's say, a neutron, it needs a given energy to effectively collide with the atom and ionize it.*
>
> Interviewer: *In your answer to question 4 you identified energy, temperature and also time as relevant quantities in the Arrhenius law. Do you think energy could also be relevant for the combustion*

[5]Interview excerpts are not always literally translated from Italian into English. We tried to convey the sense of the originals, rather than reporting the exact terms and expressions used by students. Only the key words and typical expressions we identified as relevant for the analysis are directly translated.

process? And what about time? It does not explicitly appear in the Arrhenius law.

Aldo: *Well, energy could certainly influence the lighting of the match, i.e., I know that if I strike the match too weakly it will probably not light ... or maybe it will, but it will need more time to reach the ignition temperature .... Yes, I think that energy could also be relevant! On the other hand, energy and temperature are related .... With respect to the Arrhenius law, I know that time does not explicitly appear in the mathematical formula .... On the other hand, ... the law refers to the rate of a chemical reaction, so time should also be considered a relevant physical quantity in it.*

Interviewer: *Can you identify a clear microscopic model to explain the two phenomena you mentioned before?*

Aldo: *In the case of combustion, I think it is the content of flammable substances in the match head that affects the lighting speed. If the concentration of these substances is high, the match is lit in less time. With respect to the ionization of an atom, well, here I see that the collision between the neutron and the atom is the "mechanism" which causes the ionization. If the collision is weak, there can't be ionization, as an electron inside an atom needs some energy to be pushed out of it ....*

From these excerpts it is clear that Aldo does not properly use the threshold idea for the energy required to ionize an atom. Even when Aldo cites energy as relevant (in the case of ionization), the idea of a minimum energy threshold is never clearly stated, although the concept of "activation" is implicitly contained in statements like: "... *a neutron needs some energy to effectively collide with the atom and ionize it*" and "...*an electron inside an atom needs some energy to be pushed out of it*." We can also see that Aldo finds it easier to cite a microscopic model for the phenomenon of ionization than for that of combustion. This may be the result of previous instruction, where ionization was presented as the product of collisions between particles, while combustion was seen as a chemical reaction, i.e., something which is usually described at a macroscopic level.

Another interesting point is Aldo's identification of time as a relevant quantity in the Arrhenius law, even though it is not present in the mathematical formula of the law. Aldo states that time should be considered an important quantity in a description of the law because "*it* [the law] *refers to the rate of a chemical reaction* ...." This is probably the result of the "common sense" identification of time as a relevant quantity in situations where "rate" is involved, even if it does not explicitly appear in the Arrhenius formula.

Aldo shows descriptive lines of reasoning, sometimes mixed with practical or everyday ideas, which are used "on the spot," when he needs to find a justification for statements which need to be clarified (as when Aldo cites a *real-life* object, i.e., a match, that needs a minimum temperature to light). In some cases rough explicative reasoning can be identified, mainly when referring to a model for atom ionization by means of "... *a neutron* (which) ... *collide*(s) *with the atom and ionize*[s] *it*." In Fig. 2 he is represented by code s31 and can be found in the big cluster of students using mixed strategies in answering the questionnaire items, and showing an overall similarity level to the descriptive profile of 36%. The excerpts of discussion reported above show a good consistency of its classification in this profile with the mixed lines of reasoning highlighted during the interview.

Some excerpts of a discussion between the interviewer and a student who persistently used a rigid mathematical approach in his answering strategies to the questionnaire items (see the discussion about implication between strategies 1deF and 4deC in Appendix C) are reported below:

Interviewer: *In your answer to question 1 you stated that when the temperature is higher, the phenomenon of evaporation is quicker, as "the environment contains more heat," and you wrote the formula $Q = mc\Delta T$ as a justification of your statement. However, you did not mention how this heat can be related to water evaporation in terms of a microscopic model. Can you explain this, and also link it to question 2?*

Francesco: *First of all, I know that a hot liquid evaporates quicker than a colder one. I also know that the temperature of a substance is linked to the environmental heat, so I suppose that this heat influences its evaporation. From questionnaire item 2 I understand that temperature is present in the formula expressing the speed of a chemical reaction, so I suppose that if evaporation can be represented as a reaction, the Arrhenius law is also valid for evaporation. But I am not sure how I can relate these considerations to a microscopic model.*

Interviewer: *In your answer to question 4 you also only described the Arrhenius law with reference to the mathematical role of the variables (u, T) and parameters (E) in the functional dependence of the reaction speed u on temperature T. Can you now discuss the physical meaning of these variables and a microscopic model related to the law?*

Francesco: *Well, E should be the energy of the chemical compound, i.e., the heat it receives from the environment. T is the compound temperature .... Speed depends exponentially on T, i.e., the reaction speed increases when the compound temperature increases, too .... But I'm not sure what the correct microscopic model would be to make sense of the mathematical form of the Arrhenius law.*

Interviewer: *As you clearly wrote in your answer to question 4, the variables in the Arrhenius law are the reaction speed and temperature, while energy is only a parameter that does not depend on u or T. How do you make this consistent with your previous statements: "E is the heat the compound receives from the environment" and "the temperature of a substance is linked to the environmental heat"?*

Francesco: *I am not sure … when I studied the Arrhenius law in chemistry, I understood that the meaning of E is a sort of … minimum energy the compound must have, i.e., a typical energy of the compound. But I know from physics, and also from my experience, that heat is energy, and energy is proportional to temperature, so … I am not able to make sense of these inconsistencies.*

Francesco's reasoning highlights many problems involving his knowledge of physics. Besides the confusion between heat and energy (unfortunately common in Italian university freshmen, particularly those coming from high schools not specializing in scientific studies), it is clear that Francesco's ideas are based on fragments of concepts he studied, which are not always consistent with each other. This is typical of common sense knowledge, and here it is combined with an approach to the explanation of a proposed situation based mainly on a rigid reference to mathematical formulas, i.e., a descriptive-type approach. The formulas are first taken into account by Francesco, and the subsequent explanation is based on what the formula "says." Although the symbols in a formula are recognized as physical quantities, no attempt is made by Francesco to find a model which is able to provide a microscopic interpretation of the quantities expressed by the formula and their relations. Francesco is student s25, and in Fig. 1 he is in the students' cluster collectively classified as 61% similar to the practical or everyday profile.

We continue by quoting some parts of the discussion between the interviewer and two more students. They used strategies based only on a formal mathematical approach in the analysis of a situation and appear not to be able to describe in physical terms the quantities which are relevant for the description of a phenomenon, again limiting themselves to a mathematical description (see the discussion about implication between strategies 5deB and 2deB in Appendix C).

Interviewer: *You answered question 5 by citing some phenomena which follow laws that are mathematically similar to the Arrhenius law. Do you think energy and temperature are also important to describe and explain those phenomena?*

Fabiana: *In my answer I mentioned the solubility of a gas and its formula, Henry's law. The mathematical formula of Henry's coefficient is similar to the formal expression of Arrhenius' law. I see temperature and enthalpy in it …. As the mathematical formulas are similar, I think that temperature and energy or enthalpy are likely to play the same roles.*

Matteo: *Liquid evaporation could be a phenomenon that depends on a law similar to the Arrhenius law. In fact, the Clausius-Clapeyron law states that equilibrium vapor pressure has a mathematical formula which is very similar to the Arrhenius law. With respect to energy and temperature …. Yes, temperature must be used to describe and explain liquid evaporation, as this phenomenon is influenced by the temperature of the liquid, and also by the environmental temperature …. I am not sure about energy … but I think it is relevant, too.*

Interviewer: *Do you think energy and temperature play separate roles in those phenomena, or is it also necessary to give meaning to the ratio between E and kT, as shown in the formulas?*

Fabiana: *Well, the ratio between E and kT in Arrhenius and Henry's laws only needs to be discussed if it is necessary to make sense of the mathematical dependence of the reaction speed and Henry's coefficient, respectively, on temperature. In fact, the correct match between those laws and the experimental data is obtained only if temperature is at the denominator of the exponential term.*

Matteo: *Well, thinking about this …, I now remember that when studying the equilibrium vapor pressure in liquids, it is necessary to take into account the interplay between the liquid's energy and the environmental temperature …. Maybe the ratio between E* and *kT is a correct way to express this interplay and reason ….*

In their answers to question 5, both Fabiana and Matteo mentioned phenomena which come straight from their experience as chemistry students. Moreover, they base their reasoning above all on a careful inspection of the mathematical formulas describing the phenomena, as do most of the other students. This is clear in Fabiana's statement: "*As the mathematical formulas are similar, I think that temperature and energy or enthalpy are likely to play the same roles*," and also in Matteo's: "*the equilibrium vapor pressure has a mathematical formula which is very similar to the Arrhenius law*." This is confirmed by the difficulty of the two students in understanding the physical meaning of the ratio between E and kT, and the fact they only discuss it in mathematical terms (although Matteo seems to "*remember*" something about the "*interplay between liquid energy and environmental temperature*," and tries to interpret the role of the ratio between E and kT in this "interplay"). In addition, they probably do not clearly understand that E is a characteristic parameter of the substance, a minimum energy the substance needs, to allow the activation of a process at a given temperature, and not generically the "*liquid's energy*," as stated by Matteo in his answer to the interviewer's second question. Matteo is student s20 in Fig. 2. He is classified as 88% similar to the descriptive profile. Fabiana is student s23, and in Fig. 2 she is in a student cluster collectively classified as descriptive at the much lower level of 36%. In these excerpts, however, they both show behavior clearly based on descriptive lines of reasoning: they display a clear tendency to recall previously studied subjects from memory in order to make analogies with the proposed situations, and a difficulty in finding the physical meaning of mathematical relations. However, Matteo seems better able to isolate relevant variables in mathematical formulas, and discuss relations between them, than Fabiana.

We now quote some parts of a discussion between the interviewer and a student that was able to find analogies between different phenomena and also made sense of the Arrhenius law, correctly explaining it in terms of the physical quantities involved (see the discussion about implication between strategies 6deE and 2exA in Appendix C):

> Interviewer: *In your answers to questions 5 and 6 you stated that the evaporation of liquids could be explained by a model similar to the one used to make sense of the Arrhenius law. You also recognized that temperature is a physical quantity influencing both a chemical reaction and the evaporation process. You wrote nothing about energy. What do you think about its role in the law and what about the ratio between energy and temperature in describing these phenomena?*
>
> Eleonora: *I know that energy is related to temperature, in the sense that kT can be considered like ... a sort of environmental energy. But E in Arrhenius-like formulas should be interpreted as an activation energy, or enthalpy .... So, yes, E must also be considered relevant. Come to think of it, I know that a ratio between two quantities is a sort of ... comparison between them, so maybe it is a comparison between activation energy and environmental energy.*
>
> Interviewer: *And can you relate the activation energy idea to a microscopic model?*
>
> Eleonora: *We studied the collision theory in chemistry. It states that the activation of a reaction is due to the transfer of energy caused by collisions between reactant molecules. A reaction is activated only if the colliding molecules exchange sufficient energy .... So, I think this could be the correct model.*

Although Eleonora did not consider energy as a common parameter in chemical kinetics or in evaporation processes, she was able to reconsider her idea after the interviewer's question. She also made use of a correct understanding of the concept of the mathematical ratio between two quantities to make sense of the physical meaning of the ratio E/kT. Eleonora clearly shows she is able to correctly link previous knowledge ("*we studied the collision theory in chemistry ...*") to a situation she is analyzing, in order to recognize a physical model which fits in with the Arrhenius law. She showed this in her answer to question 4, where she used strategy 4I, which is an explicative one (see Table II). She is s13 in the dendrogram shown in Fig. 3 and is classified explicative at a 72% confidence level.

Last, we quote some excerpts of the interviews taken with two students classified as 61% similar to the practical or everyday profile (Valentina, student s15, and Filadelfio, s29 in Fig. 1). Each answered one of the six questionnaire items by using strategies that are considered explicative by our analysis, although in some cases "borderline" (here 1exA and 3exA, respectively). We start with Valentina:

> Interviewer: *In your answer to question 1 you wrote that at* 40 °C *a puddle dries faster than at* 20 °C *because at higher temperature collisions between molecules are more "energetic" than at a lower one. Can you explain this concept and try to be more explicit about the mechanism that makes evaporation faster at a higher temperature?*
>
> Valentina: *Yes. At* 40 °C *the thermal energy of molecules is greater than at* 20° *and so their ability to move, and I'll say to evaporate, too, is higher. It is something like the speed of a chemical reaction, which increases when temperature rises due to more energetic collisions between molecules ...*
>
> Interviewer: *OK, but how is the temperature related to collision energy?*
>
> Valentina (thinks about it for a few seconds): *... well, we know that the temperature is a ... level of energy ... it is a physical quantity that tells us when the system is capable of exchanging more energy ... like when we have two thermal sources at different temperatures and from the hotter one we can drain more energy .... We studied this in thermodynamics ...*
>
> Interviewer: *And besides temperature, is there any other physical quantity that you consider useful for the explication of the evaporation phenomenon?*
>
> Valentina (almost talking to herself): *uhm ... I would say energy ... but I said before that it is related to temperature ... and maybe the nature of the substance, in this case water ...*
>
> Interviewer: *Please explain what you mean when you say "the nature of the substance."*
>
> Valentina: *... I am not sure ... maybe like in enzyme action in an organic reaction, where different enzymes*

give different reaction rates depending on their nature, i.e., composition ....

Here, Valentina clearly demonstrates that her answer to questionnaire item 1, coded as explicative with code 1exA (see Appendix A for more details), was probably given on the basis of a close analogy with her experience as a chemistry student. She appears not to be able to properly describe a functioning mechanism for the evaporation of water at a given temperature and limits herself to an almost continuous recalling of concepts that she has studied. In one case she cites "*the nature of the substance*" as a useful quantity to explain the evaporation phenomenon, but when the interviewer tries to understand if this means that she considers the activation energy relevant, she again simply recalls an analogy with the familiar phenomenology of organic chemical reactions. We now continue with Filadelfio:

Interviewer: *You answered question 3 by considering a catalyst as a substance that lowers the energy needed for molecular collisions and makes the reaction development more effective. Please explain what you mean by "more effective."*

Filadelfio: *In my opinion, more effective means that the catalyst increases the number of collisions, i.e., energy exchange, and lowers the energy requirement for the reaction. For example, in a catalytic muffler, the metal coating catalyzes the oxidation of the unburnt hydrocarbons in the exhaust gas, lowering the energy needed for this effect. The same can be said when we add enzymes to facilitate the fermentation of wine. In all these cases the energy needed by the reactions is less than that required when the catalyst is not present.*

Interviewer: *Yes, but is there a relationship between this "energy needed by the reaction" and molecular energy?*

Filadelfio: *Well, the more energy is exchanged during collisions, the faster the reaction develops. Maybe this means that the presence of the catalyst makes molecular collisions easier ...*

Interviewer: *Is there any relationship between the catalyst action and the temperature of the reaction?*

Filadelfio: *I know that the reaction is quicker at higher temperatures, so I think that the catalyst effect is simply an empowering of the temperature action.*

Filadelfio seems to have some idea of a functioning mechanism relating molecular collisions to energy exchange, probably connected with his study experience. However, his subsequent reasoning is simply based on analogies with real-life situations (the action of catalytic mufflers in cars or of enzymes in wine fermentation), although mediated by expert-type understanding of the underlying chemistry. In his answer to the second question, Filadelfio continues to simply cite molecular collisions as related to the catalyst effect and never tries to connect it to the lowering of an energy gap. His whole line of reasoning appears to be experience or memory driven, without a clear search for mechanisms of functioning that explain what is observed.

Valentina's and Filadelfio's use of an explicative answering strategy can therefore only be considered "declarative," as it is limited to the repetition of words and concepts that they studied before, or to the recalling of real-life situations, without any evidence of a proper use or understanding of the underlying physical or chemical mechanisms.

V. DISCUSSION

The quantitative and qualitative data analyses reported above allow us to answer the research questions, which regard the adequacy of our multimethod approach (1) to highlight the characteristics of the mental models deployed by a sample of university freshmen when asked to create explanations for phenomena or situations, and (2) to point out consistency in students' deployment of mental models.

The similarity analysis allowed us to identify clusters of students whose answering strategies can be completely included into categories related to three different mental models. These categories highlight the reasoning procedures "ran" by students when searching for explanations about phenomena and/or proposed situations.

Going into more detail, the similarity trees reported in Figs. 1 and 3 show that our method identifies two clusters of students answering the questionnaire items using fairly definite lines of reasoning, i.e., highlighting the deployment of well-defined mental models. In fact, Fig. 1 shows that 13 students made use of answering strategies 61%, or higher, similar to the ones included in the practical or everyday ideal student profile. Moreover, 4 students used answering strategies at least 72% similar to the explicative profile, as shown in Fig. 3. These similarity levels identify these students mainly as holders of practical or everyday– or explanatory-type mental models, respectively, highlighting consistency in the deployment of mental models.

On the other hand, Fig. 2 shows that 16 out of 17 students use descriptive-type answering strategies at a 36% similarity level. This means that in many of their answers they also apply strategies that are included in the practical and explicative ideal profiles. Only one student can be considered 88% similar to the pure descriptive profile. As a consequence, the use of similarity analysis identifies that about half of our student sample seems to reveal mixed-type reasoning when they are first asked to make sense of a situation belonging to a real-life field and are then requested to find analogies and differences with a well-formalized law they have already studied in their general chemistry university courses. Therefore, our methodological approach allows us to underline a considerable lack of consistency in the deployment of mental models in about 50% of our research sample.

The analysis of implication indexes allow us to find relations (i.e., implications) between strategies activated by students in answering different questionnaire items. These implications are mainly used by us in order to inform the strategy of interviews, and the related qualitative analysis. However, the analysis of the most relevant implications,[6] also gives us indications about the consistency of student behaviors in the framework of epistemological approaches to modeling described above. In fact, high implication index values are obtained (as expected) among answering strategies related to the same mental model (practical or everyday, descriptive or explicative). However, in some cases significant implication indexes are obtained for answering strategies related to different mental models (see Appendix C, where one of these implications is discussed in some detail).

The analysis of the interviews allows us to go further and better characterize the ‘‘mixed-type’’ students. They clearly show to have more than one view about the nature and use of explications in science. They also often implement strategies which are inefficient at correctly connecting mathematical modeling to real situations, in order to build explanations. Very often, reference to a well-known mathematical model seems to stimulate a recalling procedure, i.e., a search in memory for examples that fit in with the formula, without a clear understanding of its physical meaning. Moreover, the analysis of interviews and the implicative graph discussed in Appendix C also highlight a significant use of approaches based on common-type knowledge, even in students who generally adopt descriptive strategies.

Our results are consistent with data from the literature [35–39,45,46] showing that the mental models students deploy in creating explanations can be eclectic, and sometimes contradictory. In fact, many students of our sample use different kinds of reasoning, with particular reference to ones which are inefficient for correctly associating explanations to real situations. A significant presence of everyday or descriptive ideas in student answers is highlighted, in some cases even in students who generally use explicative strategies.

When we compare qualitative and quantitative findings we notice that our multimethod approach also reveals its efficacy in clarifying controversial points emerging from the data analysis. In fact, in some cases the quantitative analysis highlights the use of explicative answering strategies by students who mainly make use of everyday-life or descriptive strategies. In particular, some of the ten students collectively classified as 61% similar to the practical or everyday ideal profile (see Fig. 1) also used explicative-type reasoning. An analysis of their answers to interview questions, however, clearly shows that this use is merely declarative (see Valentina's and Filadelfio's interview excerpts). In these cases, in fact, the explicative-type reasoning highlighted in the answer to a questionnaire item is not supported by its application to explain it or to make sense of similar situations proposed by the interviewer, but it is simply based on analogies with other, familiar circumstances. This is in accordance with the low similarity coefficient of Valentina and Filadelfio (19%) with the explicative ideal student profile.

[6]The relevance of an implication is given in terms of the significance level, i.e., the implication index value, and of the number of students that highlighted the implication (see Appendix C).

VI. CONCLUSION AND IMPLICATION FOR TEACHING

In this paper, we describe a multimethod approach to the analysis of mental models deployed by students when creating explanations for phenomena or processes. We study the student answering strategies to a written questionnaire and interviews. The statistical implicative analysis tools we proposed in this paper allowed us to successfully identify clusters of students with respect to the similarity to idealized sets. These sets are synthesized by typical reasoning procedures related to epistemic categories of mental models, defined as practical or everyday, descriptive, or explicative. Through the calculation of similarity and implication indexes, our analysis gives quantitative meaning to the relations between significant typical student reasoning and specific strategies used by students in answering the questionnaire items. The interview analysis allow us to validate the results obtained with the quantitative analysis of the questionnaire answers and to examine in more detail relevant aspects which emerged from the quantitative analysis.

The results we obtained are limited by the context of the Italian school and university systems. Moreover, the size of our research sample was restricted and not randomly selected, because of the voluntary basis of the students' participation in the survey. However, our multimethod analysis can supply hints for the effective design and development of a learning environment aimed at improving students' abilities in using explicative models. In fact, our data highlight the need for our students to better identify differences and similarities between descriptive and explicative procedures, as well as the way these are related to understanding reality [59,83–86]. This could help students to get some aspects of the world at the level of causation and mechanisms of functioning [23], by orientating them towards broader generalizations involved in theory construction.

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APPENDIX A

Questionnaire items and related answering strategies for each item on the basis of an *a priori* or *a posteriori* analysis. The unforeseen strategies are in italics.

(1) A puddle dries more slowly at 20 °C than at 40 °C. Assuming all other conditions (except temperature) equal in the two cases, explain the phenomenon, pointing out what the fundamental quantities are for the description of the phenomenon and for the construction of an interpretative model of the phenomenon itself.

1peA The relevant quantities are not identified.
1peB The relevant quantities are not identified, but a description or explanation based on common sense is given.
1deA The relevant quantities are identified, but they are not used properly to give an explanation.
1deB Only temperature is identified as relevant, but the phenomenon is not correctly described.
1deC *Only temperature is identified as relevant. It is used to give a rough description of the phenomenon.*
1deD The phenomenon is described by means of the macroscopic variables pressure and volume, but a microscopic model is not identified.
1deE The phenomenon is described by means of the macroscopic variables temperature, energy and heat, but a microscopic model is not identified.
1deF The phenomenon is described by means of a mathematical formula, but a microscopic model is not identified.
1exA *The phenomenon is not adequately described (by means of a mathematical formula or verbally), but a microscopic functioning mechanism is roughly presented in terms of molecular collisions.*
1exB The phenomenon is not adequately described (by means of a mathematical formula or verbally), but a microscopic functioning mechanism is presented in terms of energy exchange between molecules.
1exC The phenomenon is verbally described and a microscopic functioning mechanism is roughly sketched.
1exD The phenomenon is described by means of mathematical relations between macroscopic quantities and a microscopic functioning mechanism is found.

(2) In chemical kinetics it is well known that the rate of a reaction u between two reactants follows the Arrhenius law:

$$u = Ae^{-E/kT}.$$

Describe each listed quantity, clarifying its physical meaning and the relations with the other quantities.

2peA The fundamental quantities are not described and/or only examples of its application to everyday-life phenomenology are given.
2peB Some quantities are mentioned, but no description of the process is given.
2deA The relevant quantities are found, but only a few are described in terms of their physical meaning.
2deB *The relevant quantities are found, but only described in terms of their mathematical meaning in the formula. No relation between them is identified.*
2deC The relevant quantities are found and correctly described in terms of their physical meaning. No relation between them is identified.
2exA The relevant quantities are found and correctly described in terms of their physical meaning. Some relations between them are identified.
2exB The relevant quantities are found and correctly described in terms of their physical meaning. The relations between them are correctly identified.

(3) What do you think the role of a catalyst is, in the development of a chemical reaction?

3peA A definition of catalyst is given, which does not conform to the scientifically correct one.
3peB A definition of catalyst based on an analogy with the concept of enzyme is given. The analogy is recalled without providing additional reasoning.

3deA The catalyst is described as a substance which speeds up a chemical reaction. No additional explanation is supplied.
3deB The catalyst is described as a substance which shifts the chemical equilibrium towards the products. No additional explanation is supplied.
3deC The catalyst is described as a substance which speeds up a chemical reaction. An explanation is given using common language.
3deD The catalyst is presented as a substance which shifts the chemical equilibrium towards the products. An explanation is given using common language.
3deE The catalyst is presented as a substance which speeds up a chemical reaction. The concept is generically described in terms of energy.
3deF The catalyst is presented as a substance which shifts the chemical equilibrium towards the products. The concept is generically described in terms of energy.
3deG *The catalyst is presented as a substance which speeds up a chemical reaction. The concept is described by simply citing the energy gap concept, without any explanation.*
3deH *The catalyst is presented as a substance which shifts the chemical equilibrium towards the products. The concept is described by simply citing the energy gap concept, without any explanation.*
3deI *The role of a catalyst in a chemical reaction is discussed referring to the energy gap concept, but only in macroscopic terms.*
3exA The role of a catalyst in a chemical reaction is discussed taking into account the energy gap concept. The concept is explained considering a microscopic model regarding collisions between molecules.
3exB The role of a catalyst in a chemical reaction is discussed taking into account the energy gap concept. The concept is explained considering a microscopic model which links the energy gap concept with the molecular energy.

(4) Can you give your own microscopic interpretation (model) of the Arrhenius law?

4peA Everyday-life concepts are mentioned, without any correct relation to the Arrhenius law.
4deA Scientific concepts, such as energy, temperature, or molecular thermal agitation, are mentioned, but they are not correctly related to the Arrhenius law.
4deB Arrhenius law is described as a mathematical function of T or E. No explanation of the meaning of these quantities is given.
4deC Arrhenius law is described as a mathematical function of both T and E. No explanation of the meaning of these quantities is given.
4deD Arrhenius law is described as a function of both T and E and the meaning of these two quantities is outlined mainly in mathematical terms.
4deE Arrhenius law is described as a function of both T and E. The physical meaning of these two quantities and/or of their ratio in the Arrhenius law is outlined.
4deF *Arrhenius law is described outlining the physical quantities involved. Collision theory is sometimes mentioned, but a clear reference to a microscopic model is not always present.*
4exA A generic explanation based on a microscopic model of collisions between molecules is given. The activation energy concept is outlined but its relation with kT is not clearly presented.
4exB A quantitative explanation in terms of the "collision theory" is given. A correct microscopic model is presented and the role of the activation energy and of kT is clearly expressed.

(5) Can you think of other natural phenomena which can be explained by a similar model?

5peA A few phenomena not related to the model are mentioned. No explanation is given.
5peB A few phenomena not related to the model are mentioned. An explanation is given using common language.
5deA A few phenomena not related to the model are mentioned. An explanation is given using mathematical formulas.
5deB Some phenomena related to the model are mentioned, but these are limited to the context of the attended graduation program (chemical engineering). An explanation is given using mathematical formulas.
5deC *Some phenomena related to the model are mentioned, and nonchemical phenomena are also taken into account, but a clear explanation is not given.*
5deD Some phenomena related to the model are mentioned, and nonchemical phenomena are also taken into account. An explanation is given using mathematical formulas.
5exA Some phenomena related to the model are mentioned, but these are limited to the context of the attended graduation program (chemical engineering). An explanation is given outlining a common microscopic model.
5exB *Some phenomena related to the model are mentioned, and nonchemical phenomena are also taken into account. An explanation is given outlining a common microscopic model, but energy and temperature are not clearly interrelated.*

5exC Some phenomena related to the model are mentioned, and nonchemical phenomena are also taken into account. An explanation is given outlining a common microscopic model. The role of energy and temperature in the model is clearly discussed.

(6) Which similarities can be identified in the previous phenomena? Is it possible to find a common physical quantity which characterizes all the systems you discussed in the previous questions?

6peA No similarities are detected and questions (1) and (2) are identified as being related to a different context on the basis of everyday-life reasoning.
6deA *No similarities are detected and questions (1) and (2) are identified as being related to a different context. An explanation is given, mentioning physical quantities which are not really relevant to the correct explanation of the questions.*
6deB *A few correct similarities are found, but physical quantities are given, which are not really relevant to the correct explanation of the questions.*
6deC Incorrect similarities are found on the basis of a mathematical formula.
6deD A few correct similarities are found on the basis of a mathematical formula.
6deE Correct similarities are found, but E and T are not always considered common to all phenomena.
6deF Some correct similarities are found. E or T is considered to be characteristic of the various phenomena, but a clear justification is not given.
6deG Some correct similarities are found. E or T is considered to be characteristic of the various phenomena, clearly explaining why.
6deH Some correct similarities are found. E or T is considered to be characteristic of the various phenomena, but the relevance of their ratio in explaining the energy threshold processes is not clearly presented.
6exA Some correct similarities are found. E or T is considered to be characteristic of the various phenomena. The activation energy role is correctly discussed in all the mentioned phenomena, but only in macroscopic terms.
6exB Some correct similarities are found. E or T is considered to be characteristic of the various phenomena. The activation energy role is correctly discussed in all the mentioned phenomena, on the basis of a microscopic model.

APPENDIX B

Categories of mental model, related reasoning procedures, typical students' answers, and related strategy codes as reported in Appendix A.

Mental model	Reasoning procedures	Examples of answers from the data	Answering strategy code
Practical or everyday	Reflects the creation of situational meanings derived from everyday contexts. The student uses other already known situations to try to explain the proposed ones	*The puddle dries faster at* $T = 40\,^\circ\mathrm{C}$ *than at* $20\,^\circ\mathrm{C}$ *because at this temperature there is more heat in the environment. It is like when in a warm day I wash my hands and they dry after a few minutes.*	1peB
		E is energy (generic concept, no description), *t is time, k is the Boltzmann constant.*	2peB
		A catalyst makes reactions go faster, but does not affect other reaction parameters. Enzymes in biological systems are actually catalysts. They make biological reactions easier, and so, faster.	3peB
		From experience we see that at low temperature all reactions are very slow, hence the reason for the Arrhenius law behavior ….	4peA
		I read that a thunderbolt strikes when a charge threshold is reached in the cloud. I think that this could depend on the temperature ….	5peB
		I don't see analogies between the puddle evaporation and Arrhenius-like phenomena. In fact, the former is due to the environmental energy (heat), while in the Arrhenius law temperature is a relevant quantity ….	6peA

Descriptive	The student describes and characterizes the analyzed process by finding or remembering the relevant variables and/or recalling from memory their relations, expressing them by means of different language (verbal, iconic, mathematical). They do not explain the causal relations among the physics parameters involved on the basis of a functioning model (microscopic or macroscopic).	*The speed of reaction depends on energy and temperature in the* $e^{-E/kT}$ *factor. So, any increase of E slows down the reaction speed, and any temperature increase makes the reaction speed higher.*	1deF
		E is the activation energy (some description of the concept is given), *t is time*, *k is the Boltzmann constant.*	2deA
		A catalyst speeds up reactions by lowering the activation energy or energy gap and making the creation of reaction products at equilibrium easier	3deH
		When temperature increases, the reaction rate increases, too. In fact, the quantity T is at denominator of the exponential term in the Arrhenius formula.	4deB
		A situation which can be explained with a law similar to the Arrhenius one is the charge or discharge of an RC circuit. In fact, in both situations we can find a negative exponential function of time	5deA
		I see that the common physical quantity is energy. It is present in all the mathematical expressions describing the phenomena	6deD
Explicative	The student proposes a (qualitative and/or quantitative) model based on a cause or effect relation or provides an explanatory hypothesis introducing models which can be seen at a theoretical level.	*If temperature increases, the molecular energy is higher. So, more water molecules can overcome the evaporation energy barrier at* 40 °C, *rather than at* 20 °C.	1exC
		E is the activation energy (a description of the concept and of its role in the formula is given), *t is time*, *k is the Boltzmann constant*.	2exA
		As explained by the collision theory, a catalyst lowers the energy barrier the molecules must overcome in order to allow the reaction development	3exB
		A microscopic interpretation of the Arrhenius law can be given if we think about a gas as composed by particles which collide with each other and gain enough energy to overcome the energy barrier	4exA
		In chemistry all reactions follow the Arrhenius law. The collision theory explains that when temperature is high, collisions between particles are more energetic	5exA
		The common physical quantities are temperature and activation energy. In all these phenomena there is a threshold energy which must be reached to activate them, and the phenomenon rate depends on temperature	6exA

APPENDIX C

We here report and briefly discuss the implicative graph between some of the strategies used by students when answering to the questionnaire items, by following the coding scheme used in Appendix A. The implicative analysis was performed by using the CHIC software. It builds implicative trees, like the one reported in Fig. (A1), where student strategies are connected to each other by means of arrows.

For simplicity, in Fig. (A1) we chose to represent only the answering strategies implying another strategy with a significance level greater than 99% (red, double lines), 95% (blue, solid lines), 90% (green, dashed lines), and 85% (grey, dash-dotted lines). We have to point out that in CHIC graphs, implications are only to be read between pairs of strategies. For example, the implication chain 4deB-1deB-3deE is to be read considering that 95% of all students using answering strategy 4deB (14 in our survey data) also use strategy 1deB, and 95% of all students using strategy 1deB (15) also use strategy 3deE.

We will now discuss some of the implications, considering the higher percentages of implications, but also taking into account the number of students involved.[7]

[7]Here we will not discuss implications involving less than six students.

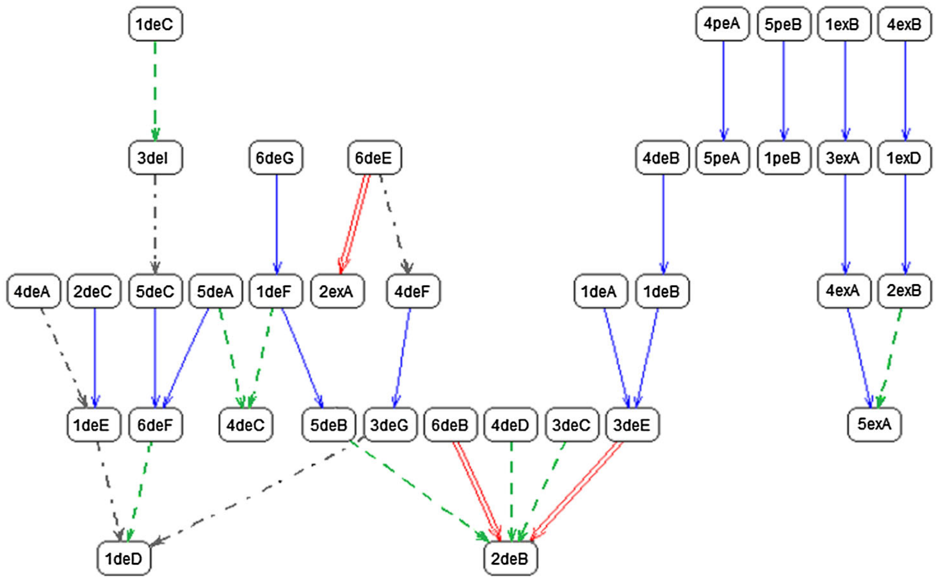

FIG. A1 (color online). Implicative graph of the answers to the test. Rectangles represent students' answering strategies as reported in the *a priori* or *a posteriori* analysis in Appendix A. Red, double line arrows indicate an incidence of implication greater than 99% between two connected strategies; blue, solid arrows a 95% incidence; green, dashed arrows indicate a 90% incidence, and gray, dash-dotted lines an incidence of implication greater than 85%. Implications are to be read only between neighboring pairs of strategies.

An overall look to the implicative test graph highlights a significant persistence of answering strategies with respect to the three categories. The 95% implication between the practical or everyday–type strategies 5peB and 1peB, the 99% one between the descriptive-type strategies 6deB-2deB and 3deE-2deB, the 95% between 1deB and 3deE, and the 95% implicative chains revealed between the explicative-type strategies 1exB-3exA-4exA-5exA and 4exB-1exD-2exB are examples of this persistence.

The implication between strategies 6deE and 4deF shows that being able to find analogies between different phenomena can imply (at 85% confidence level) the ability to explain a law not only in mathematical terms, but also outlining the involved physical quantities. However, this ability to explain is not always supported by a clear reference to a microscopic model which can make sense of the previously found physical quantities. The 90% implication between strategies 1deF-4deC and the 95% one between 1deF-5deB also highlight a persistence in finding analogies between different situations only in terms of mathematical formulas. Moreover, the implications 4deD-2deB and 5deB-2deB confirm that at least 90% of students who use strategies based only on a formal mathematical approach in the analysis of a situation are not able to describe in physical terms the quantities which are relevant for the description of a phenomenon, limiting themselves to a mathematical description.

Only in the 99% implication between strategies 6deE and 2exA, the use of a descriptive strategy implies an explicative one. This implication shows that all the eight students who used strategy 6deE, proving, in the challenging question 6, to be able to find analogies between different phenomena, also made sense of the Arrhenius law, correctly explaining it in terms of the physical quantities involved.

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