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Size effect on fracture toughness of snow

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Abstract

Depending on the scale of observation, many engineered and natural materials show different mechanical behaviour. Thus, size effect theories, based on a multiscale approach, analyse the intrinsic (due to microstructural constraints, e.g., grain size) and extrinsic effects (caused by dimensional constraints), in order to improve the knowledge in materials science and applied mechanics. Nevertheless, several problems regarding Solid Mechanics and Materials Science cannot be solved by conventional approaches, because of the complexity and uncertainty of materials proprieties, especially at different scales.
For this reason, a simple model, capable of predicting a fracture toughness at different scale, has been developed and presented in this paper. This model is based on the Golden Ratio, which was firstly defined by Euclide as: "A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the less". Intimately interconnected with the Fibonacci sequence (1, 2, 3, 5, 8, 13, …), this number controls growth in Nature and recurs in many disciplines, such as art, architecture, design, medicine, etc.., and for man-made and natural brittle materials, the Golden Ratio permits to define the relationship between the average crack spacing and the thickness of quasi-brittle materials. In these cases, the theoretical results provided by the Golden Ratio, used to calibrate a size-effect law of fracture toughness, are in accordance with the experimental measurements taken in several test campaigns carried on different materials (i.e., rocks, ice, and concrete).
This paper presents the case of fracture toughness of snow, in which the irrational number 1.61803 recurs when the geometrical dimensions vary. This aspect is confirmed by the results of experimental campaigns performed on snow samples. Thus, we reveals the existence of the size-effect law of fracture toughness of snow and we argue that the centrality of the Golden Ratio in the fracture properties of quasi-brittle materials. Consequently, by means of the proposed model, the K_{IC} of large samples can be simply and rapidly predicted, without knowing the material performances but by testing prototypes of the lower dimensions.

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Keywords: Snow Properties; Fracture Toughness; Size Effect; Golden Scaling Law.

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1. Introduction

For most man-made materials, like concrete and metals, it is well known that failure originates from microscale damage and propagates to larger scales (Chiaia and Frigo, 2009). This is true also for natural materials like snow, since the imperfections in the snowpack play a leading role in snow avalanche release. The avalanche formation was analyzed by several and different approaches, i.e.,: stability index (Föhn, 1987); failure criteria (stress (McClung, 1979) and coupled stress-energy ones (Chiaia et al., 2008)); strain-rate (Bader and Salm, 1990) and fracture mechanics approaches (McClung, 1981); collapse theories (Heieli and Zaiser, 2008), and damage mechanics ones (Chiaia and Frigo, 2009). It is the clear evidence that fracturing is the most fundamental mechanisms on the release processes of avalanche formation: from damage process in the snowpack to the failure at the interface of two different snow layers (MODE II), to the following crown fracture in tension (MODE I), leading to dry snow slab avalanche.

Accordingly, much of the focus in Snow Mechanics and Snow Engineering, it is now to improve the knowledge on the fracture properties of snow, in which the referring mechanical property has always individuated in the fracture toughness or the stress intensity factor (McClung, 1981). But, the measurement of fracture proprieties presents many complications due to complexity of both the behavior of the snow - material (difficulties in the repeatability of the material and to maintain constant its properties during the tests) and to perform tests in laboratory and full-scale (for logistical, instrumental and extreme weather conditions reasons). Due to this, only a few data on fracture toughness of snow are still available (Kirchner et al., 2000; Kirchner et al., 2002a,b; Faillettaz et al., 2002; and Schweizer et al., 2004). In all previous studies, cantilever beam tests were used to determine fracture toughness applying the linear elastic fracture mechanics (LEFM) theory, varying the snow type and density. In 2004, arguing that the standard size requirements for LEFM were not fulfilled for the used snow specimens, Schweizer introduced the discussion on size effect pointing out the application of the test results to snow slope models (Schweizer et al., 2004). Related to avalanche release process, always in the framework of LEFM, Bažant et al. (2003) formulated a size effect law of fracture toughness in shear. These authors highlight how this propriety is strongly variable with the scale, showing an increase of the stress intensity factor of snow with increasing sample size. However, this behavior is in contradiction with most of the scaling laws, which identify a decreasing trend of the observed characteristic with an increase of the observation scale. A well-known example is the behavior of the ultimate tensile strength of concrete. Its decrease that increases with the size of the structure has been confirmed by experimental measurements.

Anyhow, it is known that the variation of the nominal values of some quantities does not follow the decreasing performance rules of a solid with respect to the considered scale. An intuitive example is the size effect of the embrittlement increasing of the structural response with increasing sample size, already highlighted by Galileo Galilei in the "Discorsi e dimostrazioni matematiche a due nuove scienze attenenti alla mecanica & i movimenti locali" (1638).

The contrast is present just on Mechanics of Materials, considering the fracturing phenomenon where a variation of nominal values of some quantities follows the growth rule of performance with the considered scale.

Restricting the analysis to the materials with a brittle and quasi-brittle behavior (e.g., the snow), a significant examples are the fracture toughness and the fracture energy of ice (Frigo et al, submitted), rocks and concrete (Chiaia et al., 2013; Fantilli et al., 2014) that show this "contradictory" behavior.

Following the evidence in concrete (Fantilli and Chiaia, 2013), Fantilli et al. (2015 and 2016) analyze the crack pattern of brittle and quasi-brittle man-made composites (e.g., basic, reinforced and fiber-reinforced concrete elements), compared to natural ones (e.g., rocks). In both cases, the cracking phenomenon of brittle layers depends on the scale of observation and it is driven by a unique size-effect relationship ruled by the Golden Section number. Herein, a Golden Scaling Law is introduced and used to predict the crack pattern (i.e. the crack spacing) of concrete and rock structures at different scales referred to a reference scale.

The observation that crack pattern phenomena are basically driving by two mechanical proprieties, the fracture energy and the friction, leads to authors to investigate and demonstrate the relevance of the Golden Scaling Law also for the fracture properties of concrete, rock and ice subjected to a scaling effects (Chiaia et al., 2013; Fantilli et al., 2014, Frigo et al., submitted).

This paper reports the evidence of the Golden Scaling Law also in the framework of snow mechanics in accordance with the experimental measurements taken in several laboratory campaigns (Sigrist et al., 2005) carried on different scale.

2. The Golden Scaling Law

Usually denoted by the Greek letter Phi (ϕ), The Golden ratio (or the Golden section, Divine Proportion, etc.) is a special irrational number with infinitely digits.

Euclid (300 b.C.) proposed a first definition of the Golden Ratio. In particular, in the VI book of the Elements, the third definition states: “A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the less” (Fig. 1).

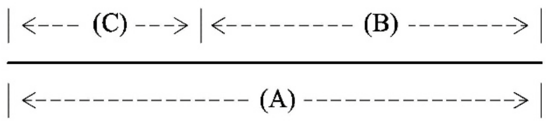

Fig. 1. Definition of the Golden Ratio for a straight segment (Fantilli et al., 2014)

For the geometric point of view, in the line segment depicted in Fig.1, it is possible to localize a point where the ratio of the whole line (A) to the large segment (B) is the same as the ratio of the large segment (B) to the small one (C). Only when this ratio is equal to the Golden Ratio (i.e., to the irrational number ϕ), can the proportion be satisfied:

$$A/B = B/C = \phi = 1.6180339887\ 498948420\ ... \tag{1}$$

Usually, the value of ϕ is rounded off to 1.618 (Hom, 2013).

The Golden Section seems to represent the standard of perfection, harmony and aesthetic beauty in architecture, design and arts (for music, to sculpture, painting, etc …). Typical Italian examples, which ϕ is applied, are: the Pantheon in Rome, the cathedral of Milano, the portal of Castel del Monte in Andria, the Last Supper and the Mona Lisa painted by Leonardo da Vinci; The Creation of Adam and the Sistine Chapel by Michelangelo; and The Birth of Venus by Botticelli. But the evidence of Phi can also be found in body proportions, geometrically represented by Leonardo da Vinci’s Vitruvian man, and by Le Courbusier’s “Anthropometric scale MODULOR”.

In addition, ϕ appears in several forms of nature, from the number of the flower petals (that follows the Fibonacci sequence), to the DNA molecules and spiral galaxies (Hom, 2013).

Golden Section can be instituted in mathematical series and geometrical patterns (Akhtaruzzaman et al., 2011), from the construction of the Pentagon and the Pentagram (the first written testimony on the subject is in the XIII book of the Elements by Euclide), to the Fibonacci sequence (1, 2, 3, 5, 8, 13…). In physics, Phi, intimately interconnected with the Fibonacci sequence, controls growth in several natural patterns. In fact, the limit of the ratios of two successive terms of the series tends to the Golden Ratio.

As the Golden Section has been observed in the crack pattern of some brittle materials (Fantilli and Chiaia, 2013; Fantilli et al., 2015 and 2016), we investigate the possibility of extending the validity of the Golden Scaling Law (GSL) also to some snow material properties. Thus, according to (Chiaia et al., 2013; Fantilli et al., 2014), if the fracture propriety of natural brittle material increases with the size, a size effect law having the general form of a power function (Rilem TC QFS, 2004) can be introduced:

$$\left(\frac{s_r}{s_{r_0}}\right) = \left(\frac{D}{D_0}\right)^{\beta} \tag{2}$$

where D_0 is the reference dimension, D the dimension at the generic scale, s_r is the characteristic of the physical property, s_{r0} the characteristic of the physical property at the generic scale; the exponent β is calculated thanks to the equation:

$$\beta = \left(\frac{\log n}{\log \phi}\right) \tag{3}$$

in which n is the ratio between D and D_0 .

The reliability of the law has been verified through comparison with experimental data obtained in earlier studies on samples of brittle and quasi-brittle materials, e.g. concrete (Perdikaris and Romeo, 1992) and rock (Bažant et al., 1991 and 1993). Fitting experimental data, according to the experimental results reported by Fantilli et al. (2014), when $D/D_0 = 4$, the fracture properties of brittle and quasi-brittle materials (concrete, rock and ice) increase of a factor ϕ, and therefore the exponent $\beta \cong 0.35$.

3. Validation of the model in snow samples

To validate the model for the fracture toughness of snow, a set of samples are herein taken into consideration (Fig. 2). The three-point bending tests (3PB-tests) are those of the experimental campaign performed by Sigrist et al. (2005) on the decomposed and fragmented, small rounded, 0.5-1 mm, F-4F (according to ICSSG, Colbeck at al., 1990) snowpack in the surroundings of Davos, Switzerland.

As reported in the original paper (Sigrist et al., 2005), the snow specimens were extracted by a naturally deposited snow, with a density around 186 ± 12 (kg m^{-3}), and each one was cut out by snow cover thanks to a beam-shaped aluminum cases. The samples presented a uniform (10 cm) thickness B in order to avoid a possible thickness effect ("2D similarity", according to Bazant and Planas, 1998), and all are notched at central cross section with a thin metal saw blade for a length.

The experiments were conducted in the SLF (Institute for Snow and Avalanche Research) cold laboratories in Davos at temperatures between -7 and -15 ˚C in a standard material testing apparatus.

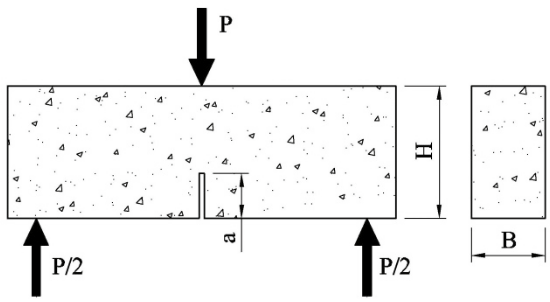

Fig. 2. Three-point bending tests of snow beams: geometrical dimensions of the specimens.

The load P (see Fig. 2) was applied in displacement control by means an aluminum cylinder with a 5 cm diameter (Sigrist et al, 2005), and other two aluminum cylinders with a 6 cm diameter supported the samples at the base in order to avoid high local snow deformations at the loading points.

To explore the size effect of fracture toughness of snow, Sigrist et al. (2005) tested a size range of 1:4 3PB samples, varying the size H (and the length, L) of the beam from 8 cm to 32 cm (Set "E" of Table 2 and Fig. 10 in original paper). Only four different sizes of cases were tested (Table 1).

On 1989, to investigate the size effect on fracture of ice, underlying the centrality of fracture toughness in Ice Mechanics, Dempsey defined the basic rules of experiments on ice and related measurements of the MODE I criticalstress-intensity-factor (K_{Ic}). As the theory of LEFM is generally applied to freshwater ice, K_c in MODE I (opening mode) is symbolically substituted by K_Q (Dempsey, 1991), following the ASTM E399-83:

$$K_Q = K_{apparent}^{initiation} = K(a, r, P_Q) \tag{4}$$

where K_Q assembles all the hypothesis of a standard test and the knowledge deficiency of any different ice. The apparent fracture toughness depends on the length of the crack a, on the crack tip radius r, and on the maximum load P_Q.

Following the same philosophy, Sigrist et al. (2005) calculated the MODE I K_Q combining the 3PB-solution and the pure bending one given by Tada at al. (2000), defining:

$$K_Q = \sqrt{\pi a}\,\frac{6}{H^2}\left[0.95\frac{P_Q\,s}{4B}f_1\left(\frac{a}{H}\right) + M_m\,f_2\left(\frac{a}{H}\right)\right] \tag{5}$$

where s is a span of the specimen, the functions $f_1\,(a/H)$ and $f_2(a/H)$ are reported in Tada et al. (2000), and M_m is the moment per beam width due to the body weight (Sigrist et al., 2005) calculated as:

$$M_m = \frac{1}{8}\left(\frac{G_{3PB}}{s}\right)\left[s^2 - (L-s)^2\right] \tag{6}$$

in which $G_{3PB} = sH\rho g$ and ρ is the snow density, g the the gravity acceleration. For each 3PB test, the related values of K_Q are reported in Table 1.

Table 1. Three-point bending tests by Sigrist et al. (2005).

Snow sample	H [cm]	L [cm]	a/H	H/L	B [cm]	K_Q [kPa m^{0,5}]
3PB1	8	20	0.1	0.4	10	1.0717
3PB1	8	20	0.1	0.4	10	1.1575
3PB1	8	20	0.1	0.4	10	1.2
3PB2	13	31	0.1	0.4	10	0.9286
3PB2	13	31	0.1	0.4	10	0.9858
3PB2	13	31	0.1	0.4	10	1.1858
3PB2	13	31	0.1	0.4	10	1.6572
3PB3	20	50	0.1	0.4	10	1.2572
3PB3	20	50	0.1	0.4	10	1.4143
3PB3	20	50	0.1	0.4	10	1.4429
3PB4	32	80	0.1	0.4	10	1.3858
3PB4	32	80	0.1	0.4	10	1.4715
3PB4	32	80	0.1	0.4	10	1.6
3PB4	32	80	0.1	0.4	10	1.8143

As for the fracture properties of concrete, rock and ice specimens, the fracture toughness of snow becomes ϕ times higher when the geometrical dimensions of the specimen are scaled with a size factor 4 (i.e., due to constant thickness of the specimens). Similarly, the fracture toughness of the 3PB samples, tested by Sigrist et al. (2005) and made with a natural snow, with a density around 186 ± 12 (kg m^{-3}), varies according to the proposed size effect law. As Fig. 3

shows, Eq.(2) with β = 0.35 gives a reasonable approximation of MODE I K_Q also in the case of large specimens (i.e., size factor = 4).

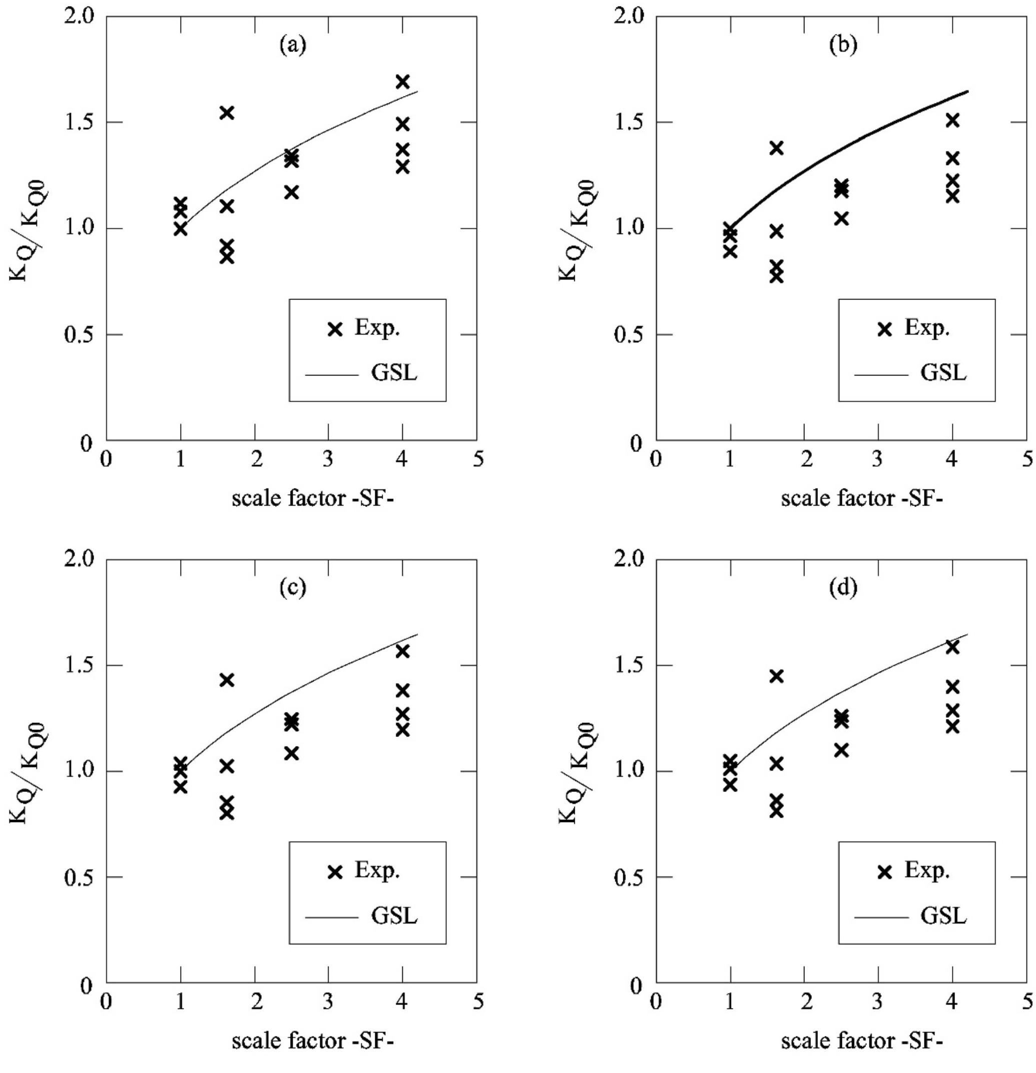

Fig. 3. Comparison between the measured fracture toughness of snow and the proposed size effect law (Eq. 2)

Fig. 3 reports the variability of the GSL depending on the choosing value of K_Q of D_0, equal to 8 (3PB1). The applicability of the GSL is relyed to the choose of the $K_{Q0.}$ Fig. 3 reports the comparison between the the measured fracture toughness of snow and the GSL defined by K_{Q0} equal to (a) minimum value of 3PB1 test with D_0 = 8; (b) the maximum and (c) the median one (Table 1), (d) the calculated average value (equal to 1.1431 kPa $m^{0,5}$).

We note that the GSL (with β =0.35) fits the fracture toughness measurements, also considering as a K_{Q0} the avegare value of the three data. The best fit is obtained with lower value of K_Q, (Fig. 3.a) equal to 1.0717 kPa $m^{0,5}$.

Conclusions

According the results shown in the previous sections, the following conclusions can be drawn:

- As concrete, ceramic, rocks and ice, snow shows grain size depending behavior and present a structurally similar brittle response.
- Similarly to the crack pattern of reinforced concrete ties and beams, the golden ratio also recurs in the fracture mechanism of snow. In particular, GSL can be used to calculate the fracture toughness of snow at different scale.
- With respect to the existing experimental data, the proposed GSL can predict fracture properties (i.e, K_{IQ}) for snow with good accuracy by using a single parameter (and a single test, as well).

Future works will report the comparison of the GSL with existing traditional size effect models (e.g.; Size Effect Law – SEL (Bažant, 1984) and the Multifractal Scaling Law – MFSL (Carpinteri, 1994; Carpinteri A., Chiaia, B. 1997)) for the given data of fracture toughness, testing the accuracy of the proposed scaling law.

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