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# Discussion of models for LCF small crack growth

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#### Abstract

The life assessment for a component subjected to high strain concentration in critical regions can be treated as a crack growth estimation starting from the first cycle of the component life. This approach assumes that the presence of same small surface defects or slip bands causes a quickly crack nucleation in critical regions where Low Cycle Fatigue (LCF) design is adopted.

The work is focused on the analysis of LCF tests carried out by the aim of smooth cylindrical specimens and specimens with semi-circular notches made of a quenched and tempered steel. The crack growth has been monitored using a thin acetate-foil based replica technique. Two different approaches have been analyzed in order to define a model able to predict the crack growth rate for high constant strain amplitudes. Such a tool is useful to assess the residual life of a component in presence of a crack in critical regions where high stress concentrations cause cyclic yielding of the material. Finally a new analytical model able to predict the crack growth rates is presented.

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#### 1. Introduction

Murakami and Miller [1] suggested that the Low Cycle Fatigue (LCF) life assessment of a component can be seen as a crack growth estimation starting from the first cycle of the component life together with an appropriate crack growth model. This approach does not seem to be straightforward applicable to High Cycle Fatigue (HCF) where short and small cracks have a growth rate [2] very different from the traditional Linear Elastic Fracture Mechanics (LEFM) description [3]. On the other hand it is important to

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identify appropriate crack growth models for mechanical components whose prospective life is of the order of 10000 cycles

The scope of this paper is to discuss the application of existing models to a quenched and tempered structural steel. In particular, a series of experimental tests in the LCF regime have been performed in order to analyze the crack propagation in presence of relatively small plastic deformations. The results have been examined in terms of crack growth rates and a series of parameters have been used to correlate these data with the external applied load.

# 2. Experimental

The material used is a quenched and tempered steel. All low cycle fatigue tests have been performed using a MTS 810 servo-hydraulic testing machine. The strain have been controlled with a longitudinal extensometer with a gage length of 10 mm. All tests have been carried out at room temperature.

Two types of specimens have been used. Smooth cylindrical specimens of 8 mm diameter and 20 mm gauge length have been used to obtain the cyclic stress-strain curve and the Manson Coffin curve under constant strain amplitude loading. Moreover, in order to observe the initiation and growth of a fatigue crack, additional cylindrical specimen with an artificial defect of 100 or 400  $\mu$ m depth have been tested. In the right-bottom part of Fig 1a is shown a picture of the section of the artificial defect.



Fig. 1. (a) Specimen and initial artificial defect geometries; (b) crack length measurements; (c) experimental results.

For these specimens, during the interruption of fatigue tests, the crack advancement has been detected using the plastic replica technique with a thin foil of acetate. An example of the application of this technique is presented in Fig 1b in which can be observed the surface development of the crack emanating from the notch. All the results are reported in Fig 1c: full black points represent the results obtained with smooth specimens at strain ratio R=-1, the other points show the effect of a different initial depth (100 - 400  $\mu$ m) and the effect of the strain ratio R. The number of cycles to fracture,  $N_{f_i}$  is the number of cycles necessary to reach a load drop of 25%, corresponding to a final crack length  $a_f = 3$  mm. Due to confidential issues, experimental results are presented in normalized form.

### 3. Analysis and discussion

#### 3.1. Fracture Mechanics parameters

The approach based on the Fracture Mechanics has been proposed by several authors, Dowling and Iyyner [4-5], Skelton [6-7], Zezulka and Polàk [8-9]. In order to find a way to correlate the crack growth data experimentally obtained, an attempt has been made with two parameters widely adopted on the literature.

The first parameter (Starkey and Skelton [7]) is derived from the concept of the cyclic strain intensity factor  $\Delta K_{\varepsilon} = \Delta \varepsilon \sqrt{\pi} \sqrt{a}$ . In particular, assuming that the crack opens and closes at the same stress level it is possible to define an equivalent strain range  $(q\Delta\varepsilon_{el} + \Delta\varepsilon_p)$  where the parameter q denotes the stress level for the effect of the closure:

$$\Delta K_{eq} = EY \left( q \Delta \varepsilon_{el} + \Delta \varepsilon_{pl} \right) \sqrt{\pi a} \tag{1}$$

where *E* is the Young modulus and *Y* is a geometric factor.

The second parameter introduced is the J-integral in which the plastic component is evaluated following the definition given by Sih and Hutchinson [10]. The final equation for the selected geometry of the specimens is suggested by J. Polàk and P. Zezulka [9]:

$$\Delta J = \frac{4K_a^2}{E} + 1.72G(n')\sigma_a\varepsilon_a a \tag{2}$$

where  $K_a$  is the stress intensity factor amplitude, n' is the exponent of the cyclic stress-strain curve and function G(n') is:

$$G(n') = (1-n') \left( 3.85 \sqrt{\frac{1}{n'}} (1-n') - \pi n' \right)$$
(3)

Fig 2 shows the crack growth rates plotted against these two parameters. As can be observed, the equivalent SIF  $K_{eq}$  shows a better correlation then the J-integral.



Fig. 2. (a) Crack growth rates against range of equivalent SIF  $K_{eq}$  (Eq 1); (b) crack growth rates against range of J-integral (Eq 2). Strain ratio R = -1.

#### 3.2. Analytical model based on Manson-Coffin law

Polàk and Zezulka [8,9] reported an analytical model based on the observation that the crack growth curve for cracks embedded in fully plastic regions follows an exponential law:

$$a = a_i \exp\left(k_g N\right) \tag{4}$$

where *a* is the crack growth measured from the free surface to the deepest point of the crack, *N* is the number of cycles,  $k_g$  is a non-dimensional parameter that depends on the applied plastic strain amplitude  $\varepsilon_{a,pl}$  and  $a_i$  is the hypothetical initial crack length obtained by extrapolation of the crack growth curve for N=0.

Deriving Eq 4 whit respect to the number of cycles it is possible to obtain a linear relation between the crack growth rate and the crack length:

$$\frac{da}{dN} = k_g a \tag{5}$$

The parameter  $k_g$  depends strongly on the applied plastic strain amplitude and it can be expressed by a power law [9]:

$$k_g = k_{g0} \varepsilon_{ap}^d \tag{6}$$

where  $k_{g0}$  and d are parameters that determine the growth of short cracks in a material.

The crack growth law given by Eq 5 and Eq 6 is in agreement with the law proposed by Tomkins [11] where the parameter d should be equal to 1+2n', being n' the exponent of the cyclic stress-strain curve.

In the present paper  $k_{g0}$  and d have been related to the Coffin-Manson curve. In particular, since:

$$\varepsilon_{ap} = \varepsilon_f' \left( 2N_f \right)^c \tag{7}$$

introducing two new parameters  $a_i$  and  $a_f$  which represent the hypothetical initial and final crack lengths for the Coffin-Manson curve obtained from smooth specimens:

$$d = -1/c, \qquad k_{g0} = 2\left(\varepsilon_f\right)^{1/c} \ln\left(\frac{a_f}{a_i}\right)$$
(8)

Using three different values for the hypotethical initial crack length  $a_i = 5 - 20 - 50$  µm the predicted crack growth rates are shown in Fig 3. For each strain amplitude the quality of the prediction increases decreasing  $a_i$  (the range 5–50 µm also corresponds to inhomogenities detected at the origin of smooth specimens failures). As can be expected from the hypothesis of the model, the crack growth rates estimates are not satisfactory for low strain amplitudes. In fact, decreasing the total strain amplitude leads raising the elastic component with respect to the plastic one and the model loses accuracy. In order to overcome this shortcoming the indroduction of a term based on the elastic component of the total applied strain amplitude is proposed.

#### 3.3. Proposed model

In Fig 4 the experimental data are plotted by means of the parameter  $J_{el} = (1-v^2)K^2/E$ . Furthermore are introduced the experimental data obtained from long crack tests carried out at stress ratio R=0.7. It can be observed that these data define an upper-bound for the LCF crack growth rates. Taking into account the closure effect by defing an effective stress intensity factor  $\Delta K_{eff} = U\Delta K$  it is possible to have an estimation of the R=-1 long crack growth rate curve. In Fig 4 different crack growth rate curves are plotted as a function of the U value.



Fig. 3. Prediction of crack growth rates using the Tomkins model (Eq 4) and the new proposed model (Eq 9). Strain ratio R=-1.

Taking into account the results reported in Fig 4, the idea is to introduce in the previously analyzed model, Eq 5, an elastic term similar to the classical Paris law using the elastic definition of the J-integral and the material constant obtained from the R=-1 curve

$$\frac{da}{dN} = k_g a + C' \left(\Delta J_{el}\right)^{m'} \tag{9}$$

The crack growth rates estimated by this model are in good agreement with the experimental data (Fig 3, C' and m' determined for U=0.35). In particular, the additional term increases the prediction capabilities for the low strain amplitude data, where the contribution of plastic starin is negligible. It is also worth

remarking that the term  $C'(\Delta J_{el})^{m'}$  could be also substituted, without any loss in precision) by a term  $C(\Delta K)^m$ .



Fig. 4. Crack growth rates against range of Jel and comparison with the long crack propagation behavior.

# 3.4. Fatigue life prediction

The proposed analytical models can also be used to predict the fatigue life. In Fig 5 the prediction has been obtained with an initial crack length  $a_i$  equal to the artificial defect size and a final crack length  $a_f$  equal to 3 mm, as experimentally observed at the end of the fatigue tests.

Two models have been compared, the equivalent SIF model, see Eq 1, and the new proposed model, see Eq 9. The equivalent SIF model gives good results but, in the case of  $a_i = 400 \mu m$ , the prediction tends to be unsafe. This problem can be solved by the application of the new model. However, as can be observed, the new model is a function of the closure parameter U. A U value of 0.45 seems to give a good correlation with the experimental results.



Fig. 5. Fatigue life prediction. a) and b) predictions with the improved model respectively for ai = 100 mm and ai = 400 mm; c) predictions with equivalent SIF model.

#### 4. Concluding remarks

Different low cycle fatigue tests have been performed in order to obtain the Manson-Coffin curve on smooth specimens and the crack growth rate curve on specimens containing artificial defects. The crack advancement has been detected using the plastic replica technique with a thin foil of acetate.

Different analytical model have been implemented and check against the capabilities of well predict the experimental crack growth rate as well as the fatigue life. Two models, the J integral and the Tomkins model, have shown some limitation. The first one is characterized by a large scatter in the correlation between the crack growth rate and the range of J integral. The second one estimates a no satisfactory crack growth rate decreasing the strain amplitude.

The equivalent SIF model shows an unsafe prediction of fatigue life in the case of  $a_i = 400 \ \mu\text{m}$ . The predictive capabilities of the new method are good but it depends on the closure parameter U. To confirm

its applicability is necessary to experimentally measure the U value. Further activities will deal with the measure of the closure levels by the DIC (Digital Image Correlation) technique.

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#### References

[1] K. J. Miller, Y. Murakami. What is fatigue damage? A view point from the observation of low cycle fatigue process. *International Journal of Fatigue*. 2005; 27(8):991-1005.

[2] K. J. Miller. Material science perspective of metal fatigue resistance. Material Science and Technology. 1993; 9:453-462.

[3] Paris, P. C., M. P. Gomez, and W. E. Anderson. A rational analytic theory of fatigue. The Trend in Engineering at the University of Washington. 1961;13(1):9-14.

[4] N. E. Dowling and N. S. Iyyner. Fatigue Crake Growth and Closure at High Cyclic Strains. *Material Science Engineering*. 1987; 96:99 – 107.

[5] N. E. Dowling. J-integral estimates for cracks in infinite bodies. Engineering Fracture Mechanics. 1987; 26:333–348.

[6] R. P. Skelton. The prediction of crack growth rates from total endurances in high strain fatigue-thirty years on. *Fatigue and Fracture of Engineering Materials and Structures*. 2008; 32(2):81-83.

[7] M. S. Starkey, R. P. Skelton. A comparison of the strain intensity and cyclic J approaches to crack growth. *Fatigue and Fracture of Engineering Materials and Structures*. 1982; 5(4):329-341.

[8] J. Polàk. Cyclic deformation, crack initiation, and low cycle fatigue. *In: Comprehensive Structural Integrity* (Edited by I. Milne, R. O. Ritchie and B. Karihaloo). Vol. 4, Elsevier, Amsterdam, pp. 1-39.

[9] P. Zezulka J. Polàk. Short crack growth and fatigue life in austenitic-ferritic duplex stainless steel. *Fatigue and Fracture of Engineering Materials and Structures*. 2005; 28: 923-935.

[10] Sih, C. F. and Hutchinson, J.W. Fully plastic solutions and large scale yielding estimates for plane stress crack problems. *J. Engng Mater. Technol.* 1976; 98:289–295.

[11] B. Tomkins. Fatigue crack propagation-an analysis. Phil. Mag. 1968; 18: 1041-1066.