Available online at www.sciencedirect.com

ScienceDirect

Nuclear Physics B 908 (2016) 279–301

NUCLEAR PHYSICS B

www.elsevier.com/locate/nuclphysb

Theory prospective on leptonic CP violation

S.T. Petcov [a,b,*]

[a] SISSA/INFN, Via Bonomea 265, 34136 Trieste, Italy

[b] Kavli IPMU (WPI), University of Tokyo, 5-1-5 Kashiwanoha, 277-8583 Kashiwa, Japan

Received 4 February 2016; received in revised form 24 February 2016; accepted 25 February 2016

Available online 2 March 2016

Editor: Tommy Ohlsson

Abstract

The phenomenology of 3-neutrino mixing, the current status of our knowledge about the 3-neutrino mixing parameters, including the absolute neutrino mass scale, and of the Dirac and Majorana CP violation in the lepton sector are reviewed. The problems of CP violation in neutrino oscillations and of determining the nature – Dirac or Majorana – of massive neutrinos are discussed. The seesaw mechanism of neutrino mass generation and the related leptogenesis scenario of generation of the baryon asymmetry of the Universe are considered. The results showing that the CP violation necessary for the generation of the baryon asymmetry of the Universe in leptogenesis can be due exclusively to the Dirac and/or Majorana CP-violating phase(s) in the neutrino mixing matrix U are briefly reviewed. The discrete symmetry approach to understanding the observed pattern of neutrino mixing and the related predictions for the leptonic Dirac CP violation are also reviewed.

1. Introduction: the three-neutrino mixing

There have been remarkable discoveries in the field of neutrino physics in the last 18 years or so. The experiments with solar, atmospheric, reactor and accelerator neutrinos have provided compelling evidences for the existence of neutrino oscillations [1,2] – transitions in flight between the different flavour neutrinos ν_e, ν_μ, ν_τ (antineutrinos $\bar{\nu}_e$, $\bar{\nu}_\mu$, $\bar{\nu}_\tau$) – caused by nonzero neutrino masses and neutrino mixing (see, e.g., Ref. [3] for review of the relevant data). The ex-

* Correspondence to: SISSA/INFN, Via Bonomea 265, 34136 Trieste, Italy.

http://dx.doi.org/10.1016/j.nuclphysb.2016.02.032

istence of flavour neutrino oscillations implies the presence of mixing in the weak charged lepton current:

$$\mathcal{L}_{\rm CC} = -\frac{g}{\sqrt{2}} \sum_{l=e,\mu,\tau} \overline{l_L}(x)\, \gamma_\alpha \nu_{lL}(x)\, W^{\alpha\dagger}(x) + \text{h.c.}\,,\;\; \nu_{l\rm L}(x) = \sum_{j=1}^{n} U_{lj}\, \nu_{j\rm L}(x)\,, \tag{1}$$

where $\nu_{lL}(x)$ are the flavour neutrino fields, $\nu_{j\rm L}(x)$ is the left-handed (LH) component of the field of the neutrino ν_j having a mass m_j, and U is a unitary matrix – the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) neutrino mixing matrix [1,2,4], $U \equiv U_{PMNS}$. All compelling neutrino oscillation data can be described assuming 3-neutrino mixing in vacuum, $n = 3$. The number of massive neutrinos n can, in general, be bigger than 3 if, e.g., there exist RH sterile neutrinos [4] and they mix with the LH flavour neutrinos. It follows from the current data that at least 3 of the neutrinos ν_j, say ν_1, ν_2, ν_3, must be light, $m_{1,2,3} \lesssim 1$ eV, and must have different masses, $m_1 \neq m_2 \neq m_3$.[1]

In the case of 3 light neutrinos, the 3×3 unitary neutrino mixing matrix U can be parametrised, as is well known, by 3 angles and, depending on whether the massive neutrinos ν_j are Dirac or Majorana particles, by one Dirac, or one Dirac and two Majorana, CP violation (CPV) phases [7]:

$$U = V\,P\,, \quad P = \text{diag}(1, e^{i\frac{\alpha_{21}}{2}}, e^{i\frac{\alpha_{31}}{2}})\,, \tag{2}$$

where $\alpha_{21,31}$ are the two Majorana CPV phases and V is a CKM-like matrix,

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}. \tag{3}$$

In eq. (3), $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$, the angles $\theta_{ij} = [0, \pi/2]$, and $\delta = [0, 2\pi)$ is the Dirac CPV phase. Thus, in the case of massive Dirac neutrinos, the neutrino mixing matrix U is similar, in what concerns the number of mixing angles and CPV phases, to the CKM quark mixing matrix. The PMNS matrix U contains two additional physical CPV phases if ν_j are Majorana particles due to the special properties of Majorana fermions (see, e.g., Refs. [7–9]). On the basis of the existing neutrino data it is impossible to determine whether the massive neutrinos are Dirac or Majorana fermions.

The probabilities of neutrino oscillation are functions of the neutrino energy, E, the source-detector distance L, of the elements of U and, for relativistic neutrinos used in all neutrino experiments performed so far, of the neutrino mass squared differences $\Delta m^2_{ij} \equiv (m^2_i - m^2_j)$, $i \neq j$ (see, e.g., Ref. [9]). In the case of 3-neutrino mixing there are only two independent Δm^2_{ij}, say $\Delta m^2_{21} \neq 0$ and $\Delta m^2_{31} \neq 0$. The numbering of neutrinos ν_j is arbitrary. We will employ the widely used convention which allows to associate θ_{13} with the smallest mixing angle in the PMNS matrix, and θ_{12}, $\Delta m^2_{21} > 0$, and θ_{23}, Δm^2_{31}, with the parameters which drive the solar (ν_e) and the dominant atmospheric ν_μ and $\bar{\nu}_\mu$ oscillations, respectively. In this convention $m_1 < m_2$, $0 < \Delta m^2_{21} < |\Delta m^2_{31}|$, and, depending on sgn($\Delta m^2_{31}$), we have either $m_3 < m_1$ or $m_3 > m_2$. The

[1] At present there are several experimental inconclusive hints for existence of one or two light sterile neutrinos at the eV scale, which mix with the flavour neutrinos, implying the presence in the neutrino mixing of additional one or two neutrinos, ν_4 or $\nu_{4,5}$, with masses m_4 ($m_{4,5}$) ~ 1 eV (see, e.g., Ref. [5]). For a discussion of these hints and of the related implications see, e.g., Ref. [6].

existing data allow us to determine Δm^2_{21}, θ_{12}, and $|\Delta m^2_{31(32)}|$, θ_{23} and θ_{13}, with a relatively good precision [10,11]. The best fit values (b.f.v.) and the 3σ allowed ranges of Δm^2_{21}, s^2_{12}, $|\Delta m^2_{31(32)}|$, s^2_{23} and s^2_{13} read [10]:

$$(\Delta m^2_{21})_{\rm BF} = 7.54 \times 10^{-5}\ {\rm eV}^2, \quad \Delta m^2_{21} = (6.99 - 8.18) \times 10^{-5}\ {\rm eV}^2\,, \tag{4}$$

$$(\sin^2\theta_{12})_{\rm BF} = 0.308, \quad 0.259 \leq \sin^2\theta_{12} \leq 0.359\,, \tag{5}$$

$$(|\Delta m^2_{31(32)}|)_{\rm BF} = 2.48\ (2.44) \times 10^{-3}\ {\rm eV}^2\,, \tag{6}$$

$$|\Delta m^2_{31(32)}| = (2.26\ (2.21) - 2.70\ (2.65)) \times 10^{-3}\ {\rm eV}^2\,, \tag{7}$$

$$(\sin^2\theta_{23})_{\rm BF} = 0.437\ (0.455)\,, \ \ 0.374(0.380) \leq \sin^2\theta_{23} \leq 0.628(0.641)\,, \tag{8}$$

$$(\sin^2\theta_{13})_{\rm BF} = 0.0234\ (0.0239)\,, \ \ 0.0176(0.0178) \leq \sin^2\theta_{23} \leq 0.0295(0.0298)\,, \tag{9}$$

where the value (the value in brackets) corresponds to $\Delta m^2_{31(32)} > 0$ ($\Delta m^2_{31(32)} < 0$). Note, in particular, that although $(\sin^2\theta_{23})_{\rm BF} = 0.437\ (0.455) < 0.5$, $\sin^2\theta_{23} = 0.5$ is within the 2σ interval of allowed values, while at 3σ we can have $\sin^2\theta_{23} \cong 0.6$ or 0.4. Thus, $\sin^2\theta_{23}$ can deviate significantly from 0.5.

It follows from the results quoted above that $\Delta m^2_{21}/|\Delta m^2_{31(32)}| \cong 0.03$. We have $|\Delta m^2_{31}| = |\Delta m^2_{32} - \Delta m^2_{21}| \cong |\Delta m^2_{32}|$. The value of $\theta_{12} = \pi/4$, i.e., maximal solar neutrino mixing, is ruled out at more than 6σ by the data. One has $\cos 2\theta_{12} \geq 0.28$ (at 99.73% C.L.). The quoted results imply also that $\theta_{23} \cong \pi/4$, $\theta_{12} \cong \pi/5.4$ and that $\theta_{13} \cong \pi/20$. Thus, the pattern of neutrino mixing differs significantly from the pattern of quark mixing.

There are also hints from data about the value of the Dirac phase[2] δ. In both analyses [10, 11] the authors find that the best fit value of $\delta \cong 3\pi/2$. The CP conserving values $\delta = 0$ and π ($\delta = 0$) are disfavoured at 1.6σ to 2.0σ (at 2.0σ) for $\Delta m^2_{31(32)} > 0$ ($\Delta m^2_{31(32)} < 0$). In the case of $\Delta m^2_{31(32)} < 0$, the value $\delta = \pi$ is statistically 1σ away from the best fit value $\delta \cong 3\pi/2$ (see, e.g., Fig. 3 in Ref. [10]). The hint that $\delta \cong 3\pi/2$ is strengthened somewhat by the first results of the *NO*νA neutrino oscillation experiment [14,15].

The relatively large value of $\sin\theta_{13} \cong 0.15$, measured in the Daya Bay [16], RENO [17] and Double Chooz [18] experiments, combined with the value of $\delta = 3\pi/2$ has far-reaching implications for the searches for CP violation in neutrino oscillations (see further). It has also important implications for the "flavoured" leptogenesis scenario of generation of baryon asymmetry of the Universe (BAU). As we will discuss in Section 3, if all CP violation necessary for the generation of BAU is due to the Dirac phase δ, a necessary condition for reproducing the observed BAU is [19] $|\sin\theta_{13}\sin\delta| \gtrsim 0.09$, which is comfortably compatible with the measured value of $\sin\theta_{13}$ and with the best fit value of $\delta \cong 3\pi/2$.

The sign of $\Delta m^2_{31(32)}$ cannot be determined from the existing data. In the case of 3-neutrino mixing, the two possible signs of $\Delta m^2_{31(32)}$ correspond to two types of neutrino mass spectrum. In the convention of numbering of neutrinos ν_j employed by us the two spectra read:

[2] Using the most recent T2K data on $\nu_\mu \to \nu_e$ oscillations, the T2K collaboration finds for $\delta = 0$, $\sin^2\theta_{23} = 0.5$ and $|\Delta m^2_{31(32)}| = 2.4 \times 10^{-3}$ eV^2, in the case of $\Delta m^2_{31(32)} > 0$ ($\Delta m^2_{31(32)} < 0$) [12]: $\sin^2 2\theta_{13} = 0.140^{+0.038}_{-0.032}$ ($0.170^{+0.045}_{-0.037}$). Thus, the best fit value of $\sin^2 2\theta_{13}$ thus found in the T2K experiment is approximately by a factor of 1.6 (1.9) bigger than that measured in the Daya Bay experiment [13]: $\sin^2 2\theta_{13} = 0.090^{+0.008}_{-0.009}$. The compatibility of the results of the two experiments on $\sin^2 2\theta_{13}$ requires, in particular, that $\delta \neq 0$ (and/or $\sin^2\theta_{23} \neq 0.5$), which leads to the hints under discussion about the possible value of δ in the global analyses of the neutrino oscillation data.

i) spectrum with normal ordering (NO): $m_1 < m_2 < m_3$, $\Delta m^2_{31(32)} > 0$, $\Delta m^2_{21} > 0$, $m_{2(3)} = (m_1^2 + \Delta m^2_{21(31)})^{\frac{1}{2}}$;

ii) spectrum with inverted ordering (IO): $m_3 < m_1 < m_2$, $\Delta m^2_{32(31)} < 0$, $\Delta m^2_{21} > 0$, $m_2 = (m_3^2 + \Delta m^2_{23})^{\frac{1}{2}}$, $m_1 = (m_3^2 + \Delta m^2_{23} - \Delta m^2_{21})^{\frac{1}{2}}$.

Depending on the values of the lightest neutrino mass, $\min(m_j)$, the neutrino mass spectrum can also be:

a) Normal Hierarchical (NH): $m_1 \ll m_2 < m_3$, $m_2 \cong (\Delta m^2_{21})^{\frac{1}{2}} \cong 8.7 \times 10^{-3}$ eV, $m_3 \cong (\Delta m^2_{31})^{\frac{1}{2}} \cong 0.050$ eV; or

b) Inverted Hierarchical (IH): $m_3 \ll m_1 < m_2$, $m_{1,2} \cong |\Delta m^2_{32}|^{\frac{1}{2}} \cong 0.049$ eV; or

c) Quasi-Degenerate (QD): $m_1 \cong m_2 \cong m_3 \cong m_0$, $m_j^2 \gg |\Delta m^2_{31(32)}|$, $m_0 \gtrsim 0.10$ eV.

All three types of spectrum are compatible with the constraints on the absolute scale of neutrino masses. Determining the type of neutrino mass spectrum is one of the main goals of the future experiments in the field of neutrino physics[3] (see, e.g., Refs. [3,5,20]).

Data on the absolute neutrino mass scale (or on $\min(m_j)$) can be obtained, e.g., from measurements of the spectrum of electrons near the end point in ^{3}H β-decay experiments [22–24] and from cosmological and astrophysical observations. The most stringent upper bound on the $\bar{\nu}_e$ mass was reported by the Troitzk [25] experiment:

$$m_{\bar{\nu}_e} < 2.05 \text{ eV} \quad \text{at } 95\% \text{ C.L.}$$

Similar result was obtained in the Mainz experiment [23]: $m_{\bar{\nu}_e} < 2.3$ eV at 95% CL. We have $m_{\bar{\nu}_e} \cong m_{1,2,3}$ in the case of QD spectrum. The upcoming KATRIN experiment [26] is designed to reach sensitivity of $m_{\bar{\nu}_e} \sim 0.20$ eV, i.e., to probe the region of the QD spectrum.

Constraints on the sum of the neutrino masses can be obtained from cosmological and astrophysical data (see and, e.g.,Ref. [27]). Depending on the model complexity and the input data used one typically obtains [27]: $\sum_j m_j \lesssim (0.3\text{–}1.3)$ eV, 95% CL. Assuming the existence of three light massive neutrinos and the validity of the Λ CDM (Cold Dark Matter) model, and using their data on the CMB temperature power spectrum anisotropies, polarisation, on gravitational lensing effects and the low l CMB polarization spectrum data (the "low P" data), the Planck Collaboration reported the following updated upper limit [28]: $\sum_j m_j < 0.57$ eV, 95% C.L. Adding supernovae (light-curve) data and data on the Baryon Acoustic Oscillations (BAO) lowers the limit to [28]:

$$\sum_j m_j < 0.23 \text{ eV}, \quad 95\% \text{ C.L.} \tag{10}$$

Understanding the origin of the observed pattern of neutrino mixing, establishing the status of the CP symmetry in the lepton sector, determining the type of spectrum the neutrino masses obey and determining the nature – Dirac or Majorana – of massive neutrinos are among the highest priority goals of the programme of future research in neutrino physics (see, e.g., [3,5]). The principal goal is the understanding at a fundamental level the mechanism giving rise to

[3] For a brief discussion of experiments which can provide data on the type of neutrino mass spectrum see, e.g., Ref. [20]; for some specific proposals see, e.g., Ref. [21].

neutrino masses and mixing and to L_l-non-conservation. Are the observed patterns of ν-mixing and of $\Delta m^2_{21,31}$ related to the existence of a new fundamental symmetry of particle interactions? Is there any relation between quark mixing and neutrino mixing? What is the physical origin of CPV phases in the neutrino mixing matrix U? Is there any relation (correlation) between the (values of) CPV phases and mixing angles in U? Progress in the theory of neutrino mixing might also lead to a better understanding of the mechanism of generation of baryon asymmetry of the Universe.

2. Observables related to leptonic CPV phases

Apart from the hint that the Dirac phase $\delta \cong 3\pi/2$, no other experimental information on the Dirac and Majorana CPV phases in the neutrino mixing matrix is available at present. Thus, the status of CP symmetry in the lepton sector is essentially unknown. Our interest in the leptonic CPV phases is stimulated, in particular, by the fact that the values of these CPV phases, together with values of the neutrino mixing angles, might provide information about the existence of new fundamental symmetry in the lepton (and possibly – the quark) sector(s) (see, e.g., [29–31]), and by the intriguing possibility that the Dirac and/or the Majorana phases in $U_{\rm PMNS}$ can provide the CP violation necessary for the generation of the observed baryon asymmetry of the Universe [19].

2.1. Dirac CP violation

With $\theta_{13} \cong 0.16 \neq 0$, a CP nonconserving value of the Dirac phase δ can generate CP violating effects in neutrino oscillations [7,32,33], i.e., a difference between the probabilities of the $\nu_l \rightarrow \nu_{l'}$ and $\bar{\nu}_l \rightarrow \bar{\nu}_{l'}$ oscillations, $l \neq l' = e, \mu, \tau$. A measure of CP violation is provided, e.g., by the asymmetries:

$$A^{(l,l')}_{\rm CP} = P(\nu_l \rightarrow \nu_{l'}) - P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'})\,, \quad l \neq l' = e, \mu, \tau\,. \tag{11}$$

The magnitude of CPV effects in neutrino oscillations in the case of 3-neutrino mixing is controlled, as is well known [34], by the rephasing invariant $J_{\rm CP}$ associated with the Dirac phase δ: $A^{(e,\mu)}_{\rm CP} = A^{(\mu,\tau)}_{\rm CP} = -A^{(e,\tau)}_{\rm CP} = J_{\rm CP}\, F^{vac}_{osc}$,

$$J_{\rm CP} = {\rm Im}\left\{U_{e2}U^*_{\mu 2}U^*_{e3}U_{\mu 3}\right\}, \quad F^{vac}_{osc} = \sin(\frac{\Delta m^2_{21}}{2E}L) + \sin(\frac{\Delta m^2_{32}}{2E}L) + \sin(\frac{\Delta m^2_{13}}{2E}L)\,. \tag{12}$$

The $J_{\rm CP}$ factor in the expressions for the asymmetries $A^{(l,l')}_{\rm CP}$, $l \neq l'$ is analogous to the rephasing invariant associated with the Dirac phase in the Cabibbo–Kobayashi–Maskawa quark mixing matrix, introduced in Ref. [35]. In the “standard” parametrisation of the neutrino mixing matrix, eq. (2), $J_{\rm CP}$ has the form:

$$J_{\rm CP} \equiv {\rm Im}\,(U_{\mu 3}\, U^*_{e3}\, U_{e2}\, U^*_{\mu 2}) = \frac{1}{8}\, \cos\theta_{13} \sin 2\theta_{12}\, \sin 2\theta_{23}\, \sin 2\theta_{13}\, \sin\delta\,. \tag{13}$$

As we have discussed, the existing neutrino oscillation data allowed to determine the PMNS angles θ_{12}, θ_{23} and θ_{13} with a relatively high precision. The size of CPV effects in neutrino oscillations is still unknown because the value of the Dirac phase δ is not determined. Obviously, the values of $\delta = 0, \pi$ are CP-conserving. The current data imply $|J_{\rm CP}| \lesssim 0.039|\sin\delta|$, where we have used the 3σ ranges of $\sin^2\theta_{12}$, $\sin^2\theta_{23}$ and $\sin^2\theta_{13}$ given in eqs. (5), (8) and (9). For the b.f.v. of $\sin^2\theta_{12}$, $\sin^2\theta_{23}$, $\sin^2\theta_{13}$ and δ found in [10] we find for $\Delta m^2_{31(2)} > 0$ ($\Delta m^2_{31(2)} < 0$):

$J_{CP} \cong 0.034 \sin\delta \cong -0.032$ ($J_{CP} \cong 0.035 \sin\delta \cong -0.029$). Thus, if the indication that $\delta \cong 3\pi/2$ is confirmed by future more precise data, the CPV effects in neutrino oscillations would be relatively large provided the factor F_{osc}^{vac} is not suppressing the CPV asymmetries. Such a suppression would not occur if under the conditions of a given experiment both neutrino mass squared differences Δm^2_{21} and $\Delta m^2_{31(32)}$ are "operative", i.e., if neither $\sin(\Delta m^2_{21} L/(2E)) \cong 0$ nor $\sin(\Delta m^2_{31(32)} L/(2E)) \cong 0$. If, e.g., $\sin(\Delta m^2_{21} L/(2E)) \cong 0$, we would have $F_{osc}^{vac} \cong 0$ and consequently $A_{CP}^{(l,l')} \cong 0$.

The searches for Dirac CPV effects in neutrino oscillations is one of the principal goals of the future experimental studies in neutrino physics (see, e.g., Refs. [5,37,36]). As we have already noticed, in order for the CPV effects in neutrino oscillations to be observable, both $\sin(\Delta m^2_{31} L/(2E))$ and $\sin(\Delta m^2_{21} L/(2E))$ should be sufficiently large. In the case of $\sin(\Delta m^2_{31} L/(2E))$, for instance, this requires that, say, $\Delta m^2_{31} L/(2E) \sim 1$. The future experiments on CP violation in neutrino oscillations are planned to be performed with accelerator ν_μ and $\bar{\nu}_\mu$ beams with energies of ~ 0.7 GeV to a few GeV. Taking as an instructive example $E = 1$ GeV and using the best fit value of $\Delta m^2_{31} = 2.48 \times 10^{-3}$ eV^2, it is easy to check that $\Delta m^2_{31} L/(2E) \sim 1$ for $L \sim 10^3$ km. Thus, the chance to observe CP violation in neutrino oscillations requires experiments to have relatively long baselines. The MINOS, T2K and NOνA experiments, for example, which provide data on ν_μ oscillations (see, e.g., Ref. [3] and references therein), have baselines of approximately 735 km, 295 km and 810 km, respectively. The planned DUNE experiment [36], which is designed to search for CP violation effects in neutrino oscillations, will have a baseline of 1300 km.

Thus, in the MINOS, T2K, NOνA and in the future planned experiments DUNE [36] and T2HK [37] the baselines are such that the neutrinos travel relatively long distances in the matter of the Earth mantle. As is well known, the pattern of neutrino oscillations can be changed significantly by the presence of matter [38] due to the coherent (forward) scattering of neutrinos on the "background" of electrons (e^-), protons (p) and neutrons (n) present in matter. The scattering generates an effective potential V_{eff} in the neutrino Hamiltonian: $H = H_{vac} + V_{eff}$. This modifies the vacuum neutrino mixing since the eigenstates and eigenvalues of H_{vac} and of $H = H_{vac} + V_{eff}$ differ, leading to different oscillation probabilities with respect to those of oscillations in vacuum. The matter of the Earth (and the Sun), is not charge conjugation (C-) symmetric: it contains only e^-, p and n but does not contain their antiparticles. As a consequence, the oscillations taking place in the Earth, are neither CP- nor CPT- invariant [39]. This complicates the studies of CP violation due to the Dirac phase δ in long baseline neutrino oscillation experiments.

The expression for the $\nu_\mu \to \nu_e$ oscillation probability in the case of 3-neutrino mixing and for neutrinos crossing the Earth mantle, when both Δm^2_{21} and Δm^2_{31} contribute and the CPV effects due to the Dirac phase in $U_{\rm PMNS}$ are taken into account, has the following form in the constant density approximation and keeping terms up to second order in the two small parameters $|\alpha| \equiv |\Delta m^2_{21}|/|\Delta m^2_{31}| \ll 1$ and $\sin^2\theta_{13} \ll 1$ [40]:

$$P_m^{3\nu\ man}(\nu_\mu \to \nu_e) \cong P_0 + P_{\sin\delta} + P_{\cos\delta} + P_3 \,. \tag{14}$$

Here

$$P_0 = \sin^2\theta_{23}\,\frac{\sin^2 2\theta_{13}}{(A-1)^2}\,\sin^2[(A-1)\Delta]\,,\; P_3 = \alpha^2\,\cos^2\theta_{23}\,\frac{\sin^2 2\theta_{12}}{A^2}\,\sin^2(A\Delta)\,, \tag{15}$$

$$P_{\sin\delta} = -\,\alpha\,\frac{8\,J_{CP}}{A(1-A)}\,(\sin\Delta)\,(\sin A\Delta)\,(\sin[(1-A)\Delta])\,, \tag{16}$$

$$P_{\cos\delta} = \alpha\,\frac{8\,J_{\rm CP}\,\cot\delta}{A(1-A)}\,(\cos\Delta)\,(\sin A\Delta)\,(\sin[(1-A)\Delta])\,, \tag{17}$$

where

$$\alpha = \frac{\Delta m^2_{21}}{\Delta m^2_{31}}\,,\;\; \Delta = \frac{\Delta m^2_{31}\,L}{4E}\,,\;\; A = \sqrt{2}G_{\rm F}N_e^{man}\,\frac{2E}{\Delta m^2_{31}}\,, \tag{18}$$

N_e^{man} being the electron number density of the Earth mantle. The Earth matter effects in the oscillations are accounted for by the quantity A. The mean electron number density in the Earth mantle relevant for the experiments of interest is [41] $\bar{N}_e^{man} \cong 1.5\ {\rm cm}^{-3}\ {\rm N_A}$, ${\rm N_A}$ being Avogadro's number. N_e varies little around the indicated mean value along the trajectories of neutrinos in the Earth mantle corresponding to the experiments under discussion. Thus, in what concerns the calculation of neutrino oscillation probabilities, the constant density approximation $N_e^{man} = {\rm const.} = \bar{N}_e^{man}$, where $\bar{N}_e^{man}$ is the mean density along the given neutrino path in the Earth, was shown to be sufficiently accurate [34,42,43]. The[4] expression for the probability of $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillation can be obtained formally from that for $P_m^{3\nu\ man}(\nu_\mu \to \nu_e)$ by making the changes $A \to -A$ and $J_{\rm CP} \to -J_{\rm CP}$, with $J_{\rm CP}\cot\delta \equiv {\rm Re}(U_{\mu 3}U^*_{e3}U_{e2}U^*_{\mu 2})$ remaining unchanged. If the Dirac phase in the PMNS matrix U has a CP-conserving value, we would have $P_{\sin\delta} = 0$. However, we would still have $(P_m^{3\nu\ man}(\nu_\mu \to \nu_e) - P_m^{3\nu\ man}(\bar{\nu}_\mu \to \bar{\nu}_e)) \neq 0$ due to the Earth matter effects. It is possible, in principle, to experimentally disentangle the effects of the Earth matter and of $J_{\rm CP}$ in $A_{\rm CP}^{(e\mu)\ man}$ by studying the energy dependence of $P_m^{3\nu\ man}(\nu_\mu \to \nu_e)$ and $P_m^{3\nu\ man}(\bar{\nu}_\mu \to \bar{\nu}_e)$. This will allow to obtain direct information about Dirac CP violation in the lepton sector and to measure the Dirac phase δ. In the vacuum limit $N_e^{man} = 0$ $(A = 0)$ we have $A_{\rm CP}^{(e\mu)\ man} = A_{\rm CP}^{(e\mu)}$ (see eq. (12)) and only the term $P_{\sin\delta}$ contributes to the asymmetry $A_{\rm CP}^{(e\mu)}$.

The expressions for the probabilities $P_m^{3\nu\ man}(\nu_\mu \to \nu_e)$ and $P_m^{3\nu\ man}(\bar{\nu}_\mu \to \bar{\nu}_e)$ can be used in the interpretation of the results of MINOS, T2K, NOνA, and of the future planned T2HK and DUNE, experiments. For a discussion of the sensitivity of these experiments to δ see, e.g., Refs. [5,36,37]). If indeed $\delta \cong 3\pi/2$, the T2HK and DUNE experiments are foreseen to establish the existence of leptonic Dirac CP violation at the $\sim 5\sigma$ C.L.

2.2. Majorana CP violation phases and $(\beta\beta)_{0\nu}$-decay

The massive neutrinos ν_j are predicted to be Majorana particles by a large number of theories of neutrino mass generation (see, e.g., Refs. [31,44–46]). The 3-neutrino mixing matrix contains in the case of massive Majorana neutrinos two additional Majorana CPV phases [7], α_{21} and α_{31}. The phases α_{21} and α_{31} can play the role of the leptogenesis CPV parameter(s) at the origin of the baryon asymmetry of the Universe [19]. However, getting experimental information about the Majorana phases in $U_{\rm PMNS}$ is a remarkably difficult problem [47–52]. The flavour neutrino oscillation probabilities $P(\nu_l \to \nu_{l'})$ and $P(\bar{\nu}_l \to \bar{\nu}_{l'})$, $l, l' = e, \mu, \tau$, are insensitive to the phases $\alpha_{21,31}$ [7,39].

If the neutrinos with definite mass ν_j are Majorana fermions, their exchange can trigger processes in which the total lepton charge changes by two units, $|\Delta L| = 2$: $K^+ \to \pi^- + \mu^+ + \mu^+$, $e^- + (A, Z) \to e^+ + (A, Z-2)$, etc. The rates of these processes are typically proportional to

[4] For a detailed discussion of the conditions of validity of the analytic expression for $P_m^{3\nu\ man}(\nu_\mu \to \nu_e)$ quoted above see Ref. [40].

the factor $(m_j/M(|\Delta L|=2))^2$, $M(|\Delta L|=2)$ being the characteristic mass scale of the given process, and thus are extremely small. The experimental searches for neutrinoless double beta ($(\beta\beta)_{0\nu}$-) decay, $(A,Z) \to (A,Z+2) + e^- + e^-$, of even–even nuclei ^{48}Ca, ^{76}Ge, ^{82}Se, ^{100}Mo, ^{116}Cd, ^{130}Te, ^{136}Xe, ^{150}Nd, etc., are unique in reaching sensitivity that might allow to observe this process if it is triggered by the exchange of the light neutrinos ν_j (see, e.g., Refs. [8,53–55]). In $(\beta\beta)_{0\nu}$-decay, two neutrons of the initial nucleus (A,Z) transform by exchanging virtual $\nu_{1,2,3}$ into two protons of the final state nucleus $(A,Z+2)$ and two free electrons. The corresponding $(\beta\beta)_{0\nu}$-decay amplitude has the form (see, e.g., Refs. [9,54]): $A((\beta\beta)_{0\nu}) = G_F^2 \langle\, m\, \rangle M(A,Z)$, where G_F is the Fermi constant, $\langle\, m\, \rangle$ is the $(\beta\beta)_{0\nu}$-decay effective Majorana mass and $M(A,Z)$ is the nuclear matrix element (NME) of the process. The $(\beta\beta)_{0\nu}$-decay effective Majorana mass $\langle\, m\, \rangle$ contains all the dependence of $A((\beta\beta)_{0\nu})$ on the neutrino mixing parameters. We have (see, e.g., [9,54]):

$$|\langle\, m\, \rangle| = \left| m_1 |U_{e1}|^2 + m_2 |U_{e2}|^2 e^{i\alpha_{21}} + m_3 |U_{e3}|^2 e^{i(\alpha_{31}-2\delta)} \right| , \qquad (19)$$

$|U_{e1}| = c_{12}c_{13}$, $|U_{e2}| = s_{12}c_{13}$, $|U_{e3}| = s_{13}$. For the normal hierarchical (NH), inverted hierarchical (IH) and quasi-degenerate (QD) neutrino mass spectra, $|\langle\, m\, \rangle|$ is given by:

$$|\langle\, m\, \rangle| \cong |\sqrt{\Delta m_{21}^2}\, s_{12}^2 + \sqrt{\Delta m_{31}^2}\, s_{13}^2 e^{i(\alpha_{32}-2\delta)}|,\ \alpha_{32} = \alpha_{31} - \alpha_{21},\ \ \text{NH},$$

$$|\langle\, m\, \rangle| \cong \sqrt{|\Delta m_{32}^2|}\, \left| c_{12}^2 + s_{12}^2\, e^{i\alpha_{21}} \right|, \quad \text{IH},$$

$$|\langle\, m\, \rangle| \cong m_0\, \left| c_{12}^2 + s_{12}^2\, e^{i\alpha_{21}} \right|, \quad \text{QD}.$$

Obviously, $|\langle\, m\, \rangle|$ depends strongly on the Majorana phase(s): the CP-conserving values of $\alpha_{21} = 0, \pm\pi$ [56], for instance, determine the range of possible values of $|\langle\, m\, \rangle|$ in the cases of IH and QD spectrum. As is well-known, if CP-invariance holds, the phase factor $\eta_{jk} = e^{i\alpha_{jk}} = \pm 1$, $j > k$, $j,k = 1,2,3$, represents [56,9] the relative CP-parity of Majorana neutrinos ν_j and ν_k, $\eta_{jk} = \eta_j^{\nu CP} (\eta_k^{\nu CP})^*$, $\eta_{j(k)}^{\nu CP} = \pm i$ being the CP-parity of $\nu_{j(k)}$.

Using the 3σ ranges of the allowed values of the neutrino oscillation parameters quoted in eqs. (4)–(9) one finds that:

i) 0.58×10^{-3} eV $\lesssim |\langle\, m\, \rangle| \lesssim 4.22 \times 10^{-3}$ eV in the case of NH spectrum;
ii) 1.3×10^{-2} eV $\lesssim |\langle\, m\, \rangle| \lesssim 5.0 \times 10^{-2}$ eV in the case of IH spectrum;
iii) 2.8×10^{-2} eV $\lesssim |\langle\, m\, \rangle| \lesssim m_0$ eV, $m_0 \gtrsim 0.10$ eV, in the case of QD spectrum.

The difference in the ranges of $|\langle\, m\, \rangle|$ in the cases of NH, IH and QD spectrum opens up the possibility to get information about the type of neutrino mass spectrum from a measurement of $|\langle\, m\, \rangle|$ [57]. The main features of the predictions for $|\langle\, m\, \rangle|$ are illustrated in Fig. 1, where $|\langle\, m\, \rangle|$ is shown as a function of the lightest neutrino mass $m_{\min} \equiv \min(m_j)$.

The experimental searches for $(\beta\beta)_{0\nu}$-decay have a long history (see, e.g., Ref. [58]). A positive $(\beta\beta)_{0\nu}$-decay signal at $> 3\sigma$, corresponding to $T_{1/2}^{0\nu} = (0.69\text{–}4.18) \times 10^{25}$ yr (99.73% C.L.) and implying $|\langle\, m\, \rangle| = (0.1\text{–}0.9)$ eV, is claimed to have been observed in [59], while a later analysis [60] reports evidence for $(\beta\beta)_{0\nu}$-decay at 6σ with $T_{1/2}^{0\nu}(^{76}\text{Ge}) = 2.23^{+0.44}_{-0.31} \times 10^{25}$ yr, corresponding to $|\langle\, m\, \rangle| = 0.32 \pm 0.03$ eV. The best lower limit on the half-life of ^{76}Ge, $T_{1/2}^{0\nu}(^{76}\text{Ge}) > 2.1 \times 10^{25}$ yr (90% C.L.), was found in the GERDA ^{76}Ge experiment [61]. By

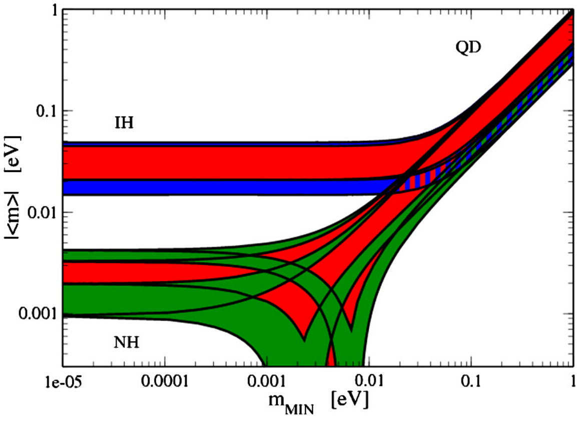

Fig. 1. The effective Majorana mass $|\langle m \rangle|$ (including a 2σ uncertainty), as a function of $m_{\min} = \min(m_j)$. The figure is obtained using the best fit values and the 2σ ranges of allowed values of Δm^2_{21}, $\sin^2\theta_{12}$, and $|\Delta m^2_{31}| \cong |\Delta m^2_{32}|$ from Ref. [10]. The phases $\alpha_{21,31}$ are varied in the interval $[0, \pi]$, δ is set to 0. The predictions for the NH, IH and QD spectra are indicated. The red regions correspond to at least one of the phases $\alpha_{21,31}$ and $(\alpha_{31} - \alpha_{21})$ having a CP violating value, while the blue and green areas correspond to $\alpha_{21,31}$ possessing CP conserving values. (From Ref. [3].) (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

combining the limits obtained in the Heidelberg–Moscow [62], IGEX [63] and GERDA experiments one gets [61] $T^{0\nu}_{1/2}(^{76}\text{Ge}) > 3.0 \times 10^{25}$ yr (90% C.L.).

Two experiments, NEMO3 [64] with ^{100}Mo and CUORICINO [65] with ^{130}Te, obtained the limits: $|\langle m \rangle| <$ (0.61–1.26) eV [64] and $|\langle m \rangle| <$ (0.16–0.68) eV [65] (90% C.L.), where estimated uncertainties in the NME are accounted for. The best lower limits on the $(\beta\beta)_{0\nu}$-decay half-life of ^{136}Xe were reported by the EXO and KamLAND-Zen collaborations: $T^{0\nu}_{1/2}(^{136}\text{Xe}) > 1.6 \times 10^{25}$ yr [66] and $T^{0\nu}_{1/2}(^{136}\text{Xe}) > 1.9 \times 10^{25}$ yr [67] (90% C.L.).

Most importantly, a large number of experiments of a new generation aim at sensitivity to $|\langle m \rangle| \sim$ (0.01–0.05) eV (see, e.g., Ref. [53,55]): CUORE (^{130}Te), GERDA (^{76}Ge), SuperNEMO, EXO (^{136}Xe), MAJORANA (^{76}Ge), AMoRE (^{100}Mo), MOON (^{100}Mo), COBRA (^{116}Cd), CANDLES (^{48}Ca), KamLAND-Zen (^{136}Xe), SNO+ (^{130}Te), etc. GERDA, EXO and KamLAND-Zen have provided already the best lower limits on the $(\beta\beta)_{0\nu}$-decay half-lives of ^{76}Ge and ^{136}Xe. The experiments listed above are aiming to probe the QD and IH ranges of $|\langle m \rangle|$; they will test the positive result claimed in Ref. [60]. If the $(\beta\beta)_{0\nu}$-decay will be observed in these experiments, the measurement of the $(\beta\beta)_{0\nu}$-decay half-life might allow to obtain constraints on the Majorana phase α_{21} [47,48,68] (see also Ref. [69]).

Proving that the CP symmetry is violated in the lepton sector due to Majorana CPV phases $\alpha_{21,31}$ is remarkably challenging [48–51]: it requires quite accurate measurements of $|\langle m \rangle|$ (and of m_0 for QD spectrum), and holds only for a limited range of values of the relevant parameters. For $\sin^2\theta_{12} = 0.31$ in the case of QD spectrum, for example, establishing at 2σ C.L. that the relevant phase α_{21} possesses a CP violating value requires [50,51] a relative error on the measured value of $|\langle m \rangle|$ and m_0 smaller than 15%, a "theoretical uncertainty" $F \lesssim 1.5$ in the value of $|\langle m \rangle|$ due to an imprecise knowledge of the corresponding NME, and value of α_{21} typically within the ranges of $\sim (\pi/4\text{–}3\pi/4)$ and $\sim (5\pi/4\text{–}7\pi/4)$.

Obtaining quantitative information on the neutrino mixing parameters from a measurement of $(\beta\beta)_{0\nu}$-decay half-life would be impossible without sufficiently precise knowledge of the corresponding NME of the process.[5] At present the variation of the values of the different $(\beta\beta)_{0\nu}$-decay NMEs calculated using the various currently employed methods is typically by factors $\sim$ (2–3) (see, e.g., [71,55]). The observation of $(\beta\beta)_{0\nu}$-decay of one nucleus is likely to lead to the searches and observation of the decay of other nuclei. The data on the $(\beta\beta)_{0\nu}$-decay of several nuclei might help to solve the problem of sufficiently precise calculation of the $(\beta\beta)_{0\nu}$-decay NMEs [48].

If the future $(\beta\beta)_{0\nu}$-decay experiments show that $|\langle m \rangle| < 0.01$ eV, both the IH and the QD spectrum will be ruled out for massive Majorana neutrinos. If in addition it is established in neutrino oscillation experiments that $\Delta m^2_{31(32)} < 0$ (IO spectrum), one would be led to conclude that either the massive neutrinos ν_j are Dirac fermions, or that ν_j are Majorana particles but there are additional contributions to the $(\beta\beta)_{0\nu}$-decay amplitude which interfere destructively with that due to the exchange of ν_j. The case of more than one mechanism generating the $(\beta\beta)_{0\nu}$-decay was discussed in, e.g., Refs. [72], where the possibility to identify the mechanisms inducing the decay was also analysed. If, however, $\Delta m^2_{31(32)}$ is determined to be positive, the upper limit $|\langle m \rangle| < 0.01$ eV would be perfectly compatible with massive Majorana neutrinos possessing NH mass spectrum, or NO spectrum with partial hierarchy, and the quest for $|\langle m \rangle|$ would still be open [73].

Let us emphasise that determining the nature of massive neutrinos is one of the fundamental, most challenging and pressing problems in today's neutrino physics (see, e.g., [5]).

3. The seesaw mechanism and leptogenesis

The existing data show that neutrino masses are significantly smaller than the masses of charged leptons and quarks. Taking as an indicative upper limit $m_j \lesssim 0.5$ eV, we have $m_j/m_{l,q} \lesssim 10^{-6}$, where m_l and m_q are the charged lepton and quark masses, $l = e, \mu, \tau$, $q = d, s, b, u, c, t$. It is natural to suppose that the remarkable smallness of neutrino masses is related to the existence of a new fundamental mass scale in particle physics, and thus to new physics beyond that predicted by the Standard Theory.

The smallness of neutrino masses finds a natural explanations within the seesaw mechanism of neutrino mass generation [44]. The simplest version of this mechanism – the so-called "type I see-saw" – contains as an integral part $SU(2)_L$ singlet RH neutrinos ν_{lR}, $l = e, \mu, \tau$. The RH neutrinos ν_{lR} are assumed to have $SU(2)_L \times U_{\rm Yw}$ invariant Yukawa type coupling with the Standard Theory lepton and Higgs doublets $\psi_{lL}(x)$ and $\Phi(x)$, $(\psi_{lL}(x))^T = (\nu^T_{lL}(x)\ \ l^T_L(x))$, $l = e, \mu, \tau$, $(\Phi(x))^T = (\Phi^{(0)}\ \Phi^{(-)})$, as well as a Majorana mass term, $-0.5\overline{\nu}_{lR} M_{ll'} C (\overline{\nu}_{l'R})^T$, C being the charge conjugation matrix ($C^{-1}\gamma_\mu C = -\gamma^T_\mu$). The latter is an $SU(2)_L \times U_{\rm Yw}$ invariant dimension 3 operator. In the basis in which the Majorana mass matrix of RH neutrinos is diagonal we have:

$$\mathcal{L}_{\rm Y,M}(x) = -(\lambda_{kl}\, \overline{N_{kR}}(x)\, \Phi^\dagger(x)\, \psi_{lL}(x) + {\rm h.c.}) - \frac{1}{2}\, M_k\, \overline{N_k}(x)\, N_k(x)\,,$$

where λ_{lk} is the matrix of neutrino Yukawa couplings and $N_k(x)$ is the heavy (RH) Majorana neutrino field possessing a mass $M_k > 0$, $M_1 < M_2 < M_3$. The fields $N_k(x)$ satisfy the Majorana condition $C\overline{N_k}^T(x) = \xi_k N_k(x)$, where ξ_k is a phase. When the neutral component of the Higgs

[5] For discussions of the current status of the calculations of the NMEs for the $(\beta\beta)_{0\nu}$-decay see, e.g., Refs. [53,71,55]. A possible test of the NME calculations is suggested in Ref. [48] and is discussed in greater detail in Ref. [70] (see also, e.g., Ref. [71]).

doublet field acquires non-zero vacuum expectation value $v = 174$ GeV breaking the electroweak symmetry spontaneously, the neutrino Yukawa coupling generates a neutrino Dirac mass term: $m^D_{kl}\,\overline{N_{kR}}(x)\,\nu_{lL}(x) + \text{h.c.}$, with $m^D = v\lambda$. In the case when the elements of m^D are much smaller than M_k, $|m^D_{jl}| \ll M_k$, $j, k = 1, 2, 3$, $l = e, \mu, \tau$, the interplay between the Dirac mass term and the Majorana mass term of the heavy singlets N_k generates an effective Majorana mass (term) for the LH flavour neutrino fields $\nu_{lL}(x)$ [44]:

$$(m^\nu)_{l'l} \cong v^2 (\lambda^T\, M^{-1}\, \lambda)_{l'l} = ((m^D)^T\, M^{-1}\, m^D)_{l'l} = (U^*\, m\, U^\dagger)_{l'l}\,, \tag{20}$$

where $M \equiv \text{Diag}(M_1, M_2, M_3)$ ($M_{1,2,3} > 0$), $m \equiv \text{Diag}(m_1, m_2, m_3)$, $m_j \geq 0$ being the mass of the light Majorana neutrino ν_j, and U is the PMNS matrix. The PMNS neutrino mixing matrix appears in the charged current weak interaction Lagrangian $\mathcal{L}_{\text{CC}}(x)$, eq. (1), as a result of the diagonalisation of the mass matrix m^ν.

The Dirac mass m^D is typically of the order of the charged fermion masses in grand unified theories (GUTs) [44,45]. Taking indicatively $m^\nu \sim 0.05$ eV, $m^D \sim 10$ GeV, one finds $M_k \sim 2 \times 10^{12}$ GeV, which is close to the scale of unification of electroweak and strong interactions, $M_{GUT} \cong 2 \times 10^{16}$ GeV. The heavy singlet neutrinos N_k present in GUTs containing ν_{lR} indeed acquire naturally masses which are by few to several orders of magnitude smaller than M_{GUT} (see, e.g., Ref. [45]).

One of the characteristic predictions of the seesaw mechanism is that both the light and heavy neutrinos ν_j and N_k are Majorana particles. As we have discussed, if ν_j are Majorana particles, the $(\beta\beta)_{0\nu}$-decay will be allowed.

An appealing feature of the seesaw mechanism is that it relates via leptogenesis [75,76] the generation and smallness of neutrinos masses to the generation of the baryon asymmetry of the Universe (BAU) [77] – the observed difference in the present epoch of the evolution of the Universe of the number densities of baryons and anti-baryons, n_{B} and $n_{\bar{\text{B}}}$:

$$Y_{\text{B}} = \frac{n_B - n_{\bar{B}}}{s_0} = (8.67 \pm 0.15) \times 10^{-11}\,, \tag{21}$$

where s_0 is the entropy density in the current epoch.[6] The type I see-saw model is the simplest scheme in which the leptogenesis can be implemented. In its minimal version it includes the Standard Theory plus two or three heavy (RH) Majorana neutrinos, N_k. Thermal leptogenesis (see, e.g., Ref. [78]) can take place, e.g., in the case of hierarchical spectrum of the heavy neutrino masses, $M_1 \ll M_2 \ll M_3$, which we consider in what follows. Out-of-equilibrium lepton number and CP nonconserving decays of the lightest heavy Majorana neutrino, N_1, mediated by the neutrino Yukawa couplings, λ, generate a lepton asymmetry in the Early Universe. $(B - L)$-conserving but $(B + L)$-violating sphaleron interactions [76] which exist within the Standard Theory and are efficient at temperatures $100\text{ GeV} \lesssim T \lesssim 10^{12}$ GeV, convert the lepton asymmetry into a baryon asymmetry. In GUTs the heavy neutrino masses fall typically in the range of $\sim (10^8$–$10^{14})$ GeV (see, e.g., Ref. [45]). This range coincides with the range of values of M_k, required for a successful thermal leptogenesis [78]. For hierarchical heavy neutrino masses we consider, successful leptogenesis takes place in the Early Universe typically at temperatures somewhat smaller than the mass of N_1, but not smaller than roughly 10^9 GeV, $10^9\text{ GeV} \lesssim T < M_1$.

[6] The entropy density s at temperature T is given by $s = g_*(2\pi^2/45)T^3$, where g_* is the number of (thermalised) degree of freedom at temperature T. In the present epoch of the evolution of the Universe we have $s_0 = 7.04\, n_{\gamma 0}$, $n_{\gamma 0}$ being the number density of photons.

In what follows we will discuss briefly the appealing possibility [19,74] that the CP violation necessary for the generation of the baryon asymmetry of the Universe, Y_B, in the leptogenesis scenario can be due exclusively to the Dirac and/or Majorana CPV phases in the PMNS matrix, and thus can be directly related to the low energy leptonic CP violation (e.g., in neutrino oscillations, etc.). It proves convenient to use in our further analysis the "orthogonal parametrisation" of the matrix of neutrino Yukawa couplings [79]:

$$\lambda = v^{-1}\sqrt{M}\,R\,\sqrt{m}\,U^{\dagger}, \quad R\,R^T = R^T\;R = \mathbf{1}, \tag{22}$$

where R is, in general, a complex matrix. It is parametrised, in general, by 6 real parameters (e.g., 3 complex angles), of which 3 parameters can have CP violating values.

In the setting we are considering the only source of CP violation in the lepton sector is the matrix of neutrino Yukawa couplings λ. It is clear from eq. (22) that the CP violating parameters in λ can have their origin from the CPV phases in the PMNS matrix U, or from the CPV parameters present in the matrix R, or else from both the CPV parameters in U and in R. The CP invariance will hold if [19] the elements of U and R are real and/or purely imaginary such that we have:

$$P_{jkml} \equiv R_{jk}\,R_{jm}\,U^*_{lk}\,U_{lm} = P^*_{jkml}\,,\ \mathrm{Im}(P_{jkml}) = 0\,,\ k \neq m\,. \tag{23}$$

The realization that the CP violation necessary for the generation of the baryon asymmetry of the Universe can be due exclusively to the CPV phases in the PMNS matrix, is related to the progress made in the understanding of the importance of lepton flavour effects in leptogenesis [80,81] (for earlier discussion see Ref. [82]). In the case of hierarchical heavy neutrinos N_k, $M_1 \ll M_2 \ll M_3$, the flavour effects in leptogenesis can be significant for [80,81] 10^8 GeV $\lesssim M_1 \lesssim (0.5\text{–}1.0) \times 10^{12}$ GeV. If the requisite lepton asymmetry is produced in this regime, the CP violation necessary for successful leptogenesis can be fe25 provided entirely by the CPV phases in the neutrino mixing matrix [19].

Indeed, suppose that the mass of N_1 lies in the interval of interest, 10^9 GeV $\lesssim M_1 \lesssim 10^{12}$ GeV. The CP violation necessary for the generation of the baryon asymmetry Y_B in "flavoured" leptogenesis can arise both from the "low energy" neutrino mixing matrix U and/or from the "high energy" part of the matrix of neutrino Yukawa couplings λ – the matrix R, which can mediate CP violating phenomena only at some high energy scale determined by the masses M_k of the heavy Majorana neutrinos N_k. The matrix R does not affect the "low" energy neutrino mixing phenomenology. Suppose further that the matrix R has real and/or purely imaginary CP-conserving elements: we are interested in the case when the CP violation necessary for leptogenesis is due exclusively to the CPV phases in U. Under these assumptions, Y_B generated via leptogenesis can be written as [80,81]

$$|Y_B| \cong 3 \times 10^{-3}\,|\epsilon_\tau\,\eta|\,, \tag{24}$$

where ϵ_τ is the CPV asymmetry in the τ flavour (lepton charge) produced in N_1-decays,[7]

$$\epsilon_\tau = -\frac{3M_1}{16\pi v^2}\,\frac{\mathrm{Im}(\sum_{jk} m_j^{1/2} m_k^{3/2} U^*_{\tau j} U_{\tau k} R_{1j} R_{1k})}{\sum_i m_i |R_{1i}|^2}\,, \tag{25}$$

η is the efficiency factor [80],

[7] We have given the expression for Y_B normalised to the entropy density, see, e.g., Ref. [19].

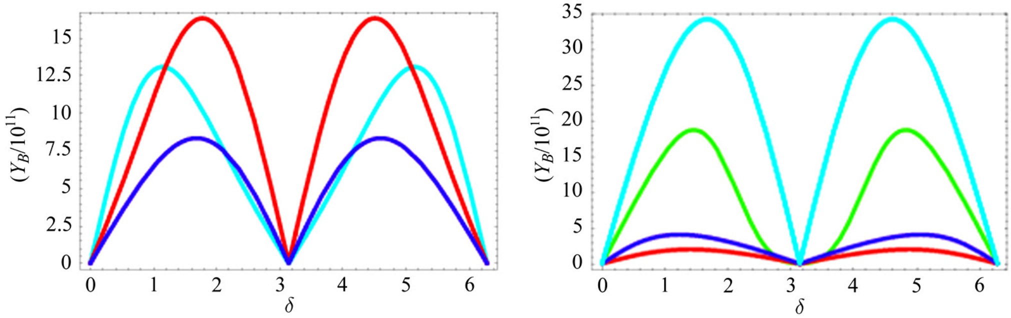

Fig. 2. The baryon asymmetry $|Y_B|$ as a function of the Dirac phase δ varying in the interval $\delta = [0, 2\pi]$ in the case of Dirac CP violation and hierarchical heavy neutrinos. Left panel: NH light neutrino mass spectrum, $\alpha_{32} = 0;\ 2\pi$, $M_1 = 5 \times 10^{11}$ GeV, real R_{12} and R_{13} satisfying $|R_{12}|^2 + |R_{13}|^2 = 1$, $|R_{12}| = 0.86$, $|R_{13}| = 0.51$, sign $(R_{12}R_{13}) = +1$, and i) $\alpha_{32} = 0$, $s_{13} = 0.2$ (red line) and $s_{13} = 0.1$ (dark blue line), ii) $\alpha_{32} = 2\pi$, $s_{13} = 0.2$ (light blue line). Right panel: IH light neutrino mass spectrum, $\alpha_{21} = \pi$, $M_1 = 2 \times 10^{11}$ GeV, $R_{11}R_{12} = i\,\kappa\,|R_{11}R_{12}|$ $(|R_{11}|^2 - |R_{12}|^2 = 1)$, $\kappa = -1$ (red and dark blue lines), $\kappa = +1$ (light blue and green lines), $s_{13} = 0.1$ (red and green lines), $s_{13} = 0.2$ (dark blue and light blue lines); values of $|R_{11}|$, which maximise $|Y_B|$, have been used. (From Ref. [19].) (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

$$|\eta| \cong |\eta(0.71\widetilde{m_2}) - \eta(0.66\widetilde{m_\tau})|\,, \tag{26}$$

$\widetilde{m_{2,\tau}}$ being the wash-out mass parameters which determine the rate of the processes in the Early Universe that tend to “erase”, or “wash-out”, the asymmetry,

$$\widetilde{m_2} = \widetilde{m_e} + \widetilde{m_\mu}\,,\quad \widetilde{m_l} = |\sum_j m_j\ R_{1j}\ U^*_{lj}|^2\,,\quad l = e, \mu\,. \tag{27}$$

Approximate analytic expression for $\eta(\widetilde{m})$ is given in [80,81]. We shall consider next a few specific examples.

A. NH Spectrum, $m_1 \ll m_2 \ll m_3 \cong \sqrt{\Delta m^2_{31}}$. Assume for simplicity that $m_1 \cong 0$ and $R_{11} \cong 0$ (N_3 decoupling). If $R_{12}R_{13}$ is real and $\alpha_{32} = 0$, the only source of CPV is the Dirac phase δ in U, and $\epsilon_\tau \propto \sin\theta_{13}\sin\delta$. For $R_{12}R_{13} > 0$, $s_{13} = 0.15$, $\delta = 3\pi/2$, and $R_{12} \cong 0.86$ (which maximises $|Y_B|$), we have [19]: $|Y_B| \cong 2.7 \times 10^{-13}\ (\sqrt{\Delta m^2_{31}}/0.05\ \text{eV})\ (M_1/10^9\ \text{GeV})$, where we have used the best fit values of Δm^2_{21}, $\sin^2\theta_{12}$ and $\sin^2\theta_{23}$ (see Fig. 2, left panel). For the values of $M_1 \lesssim 5 \times 10^{11}$ GeV for which the flavour effects in leptogenesis can be significant, the observed value of the baryon asymmetry, taken conservatively to lie in the interval $|Y_B| \cong (8.1\text{–}9.3) \times 10^{-11}$, can be reproduced if

$$|\sin\theta_{13}\sin\delta| \gtrsim 0.09\,. \tag{28}$$

The ranges of values of $|\sin\theta_{13}\sin\delta|$ we find in the case being considered are comfortably compatible with the measured value of $\sin\theta_{13}$ and with the hints that $\delta \cong 3\pi/2$. Since both Y_B and J_{CP} depend on s_{13} and δ, for given values of the other relevant parameters there exists a correlation between the values of $|Y_B|$ and J_{CP}.

As was shown in [19], we can have successful leptogenesis also if the sole source of CP violation is the difference of the Majorana phases $\alpha_{32} = \alpha_{31} - \alpha_{21}$ of U_{PMNS}. In this case values of $M_1 \gtrsim 4 \times 10^{10}$ GeV are required.

B. IH Spectrum, $m_3 \ll m_{1,2} \cong \sqrt{|\Delta m^2_{32}|}$. Under the simplifying conditions of $m_3 \cong 0$ and $R_{13} \cong 0$ (N_3 decoupling), leptogenesis can be successful for $M_1 \lesssim 10^{12}$ GeV only if

$R_{11}R_{12}$ is not real [19,83], so we consider the case of purely imaginary $R_{11}R_{12} = i\kappa|R_{11}R_{12}|$, $\kappa = \pm 1$. The requisite CP violation can be due to the i) Dirac phase δ (Fig. 2, right panel), and/or ii) Majorana phase α_{21}, in the neutrino mixing matrix U. If, e.g., in the second case we set $\sin\delta = 0$ (say, $\delta = \pi$), the maximum of $|Y_B|$ for, e.g., $\kappa = -1$, is reached for [19] $|R_{11}|^2 \cong 1.4$ ($|R_{12}|^2 = |R_{11}|^2 - 1 = 0.4$), and $\alpha_{21} \cong 2\pi/3; 4\pi/3$, and at the maximum $|Y_B| \cong 1.5 \times 10^{-12}(\sqrt{|\Delta m^2_{32}|}/(0.05\ \text{eV})(M_1/10^9\ \text{GeV})$. The observed $|Y_B|$ can be reproduced for $M_1 \gtrsim 5.4 \times 10^{10}$ GeV.

Similar results can be obtained [19] in the case of quasi-degenerate in mass heavy Majorana neutrinos.

The interplay in "flavoured" leptogenesis between contributions in Y_B due to the "low energy" and "high energy" CP violation, originating from the PMNS matrix U and the R-matrix, respectively, was investigated in Ref. [84]. It was found, in particular, that under certain physically plausible conditions (IH spectrum, $(-\sin\theta_{13}\cos\delta) \gtrsim 0.1$, etc.), the "high energy" contribution in Y_B due to the R-matrix, can be so strongly suppressed that it would play practically no role in the generation of baryon asymmetry compatible with the observations. One would have successful leptogenesis in this case only if the requisite CP violation is provided by the Majorana phases in the PMNS matrix U.

4. Predicting the leptonic CP violation

As we have already emphasised, understanding the origin of the patterns of neutrino masses and mixing, emerging from the neutrino oscillation, ^{3}H β-decay, cosmological, etc. data is one of the most challenging problems in neutrino physics. It is part of the more general fundamental problem in particle physics of understanding the origins of flavour, i.e., of the patterns of quark, charged lepton and neutrino masses and of the quark and lepton mixing.

4.1. Origins of the pattern of neutrino mixing: the discrete symmetry approach

We believe, and we are not alone in holding this view, that with the observed pattern of neutrino mixing Nature is "sending" us a message. The message is encoded in the values of the neutrino mixing angles, leptonic CP violation phases and neutrino masses. We do not know at present what is the content of Nature's message. However, on the basis of the current ideas about the possible origins of the observed pattern of neutrino mixing, the Nature's message can have two completely different contents: ANARCHY or SYMMETRY. In the ANARCHY approach [85] to understanding the pattern of neutrino mixing it is assumed that Nature "threw dice" when Nature was "choosing" the values of the neutrino masses, mixing angles and CPV phases. The main prediction of the ANARCHY explanation of the pattern of neutrino mixing is the absence of whatever correlations between the values of the neutrino masses, between the values of the neutrino mixing angles, and between the values of the neutrino mixing angles and the CPV phases, all of them being random quantities. In contrast, the most distinctive feature of the SYMMETRY approach to understanding the pattern of neutrino mixing (and possibly the pattern of neutrino masses when it will be uniquely determined) is the prediction of existence of correlations between the values of at least some the neutrino mixing angles and/or between the values of the neutrino mixing angles and the Dirac and Majorana CPV phases, as well as other possible correlations.

In what follows we will review aspects of the SYMMETRY approach to the understanding the form of neutrino mixing, which is based on non-Abelian discrete flavour symmetries and is widely explored at present (see, e.g. [86,87] and references therein). It leads to specific correlations between the values of at least some of the mixing angles of the neutrino mixing matrix $U_{\rm PMNS}$ and, either to specific fixed values of CPV phases present in U, which are "trivial" (e.g., $\delta = 0$ or π, $\alpha_{21} = \alpha_{31} = 0$), (see, e.g., [88]), or to a correlation between the values of the neutrino mixing angles and of the Dirac CPV phase of U [29,30,89–92]. In the case of Majorana massive neutrinos one can obtain (under specific conditions) also correlations between the values of the two Majorana CPV phases present in $U_{\rm PMNS}$ and of the three neutrino mixing angles and of the Dirac CPV phase [29]. As a consequence of the correlation involving the Dirac CPV phase δ, $\cos\delta$ can be expressed in terms of the three neutrino mixing angles of U [29,30,89–91], i.e., one obtains a sum rule for $\cos\delta$. This sum rule depends on the underlying discrete symmetry used to derive the observed pattern of neutrino mixing and on the type of breaking of the symmetry, necessary to reproduce the measured values of the neutrino mixing angles. It depends also on the assumed status of the CP symmetry before the breaking of the underlying discrete symmetry.

The approach of interest is based on the assumption of existence at some energy scale of a (lepton) flavour symmetry corresponding to a non-Abelian discrete group G_f. Groups that have been considered in the literature include S_4, A_4, T', A_5, D_n (with $n = 10, 12$) and $\Delta(6n^2)$, to name several (see, e.g., Ref. [93] for definitions of these groups and discussion of their properties[8]). The choice of these groups is related to the fact that they lead to values of the neutrino mixing angles, which can differ from the measured values at most by subleading perturbative corrections. For instance, the groups A_4, S_4 and T' are commonly utilised to generate tri-bimaximal (TBM) mixing [94]; the group S_4 can also be used to generate bimaximal (BM) mixing[9] [95]; A_5 can be utilised to generate golden ratio type A (GRA) [96] mixing; and the groups D_{10} and D_{12} can lead to golden ratio type B (GRB) [97] and hexagonal (HG) [98] mixing.

The flavour symmetry group G_f can be broken, in general, to different symmetry subgroups, or "residual symmetries", G_e and G_ν of the charged lepton and neutrino mass terms, respectively. Given a discrete G_f, there are more than one (but still a finite number of) possible residual symmetries G_e and G_ν. The subgroup G_e, in particular, can be trivial.

Non-trivial residual symmetries constrain the forms of the 3×3 unitary matrices U_e and U_ν, which diagonalise the charged lepton and neutrino mass matrices, and the product of which represents the PMNS matrix:

$$U_{\rm PMNS} = U_e^\dagger\, U_\nu\,. \tag{29}$$

Thus, the residual symmetries constrain also the form of $U_{\rm PMNS}$.

In general, there are two cases of residual symmetry G_ν for the neutrino Majorana mass term when a part of G_f is left unbroken in the neutrino sector (see, e.g., [86]): G_ν can either be a $Z_2 \times Z_2$ symmetry or a Z_2 symmetry. In models where $G_\nu = Z_2$, the matrix U_ν contains two free parameters, i.e., one angle and one phase, as long as the neutrino Majorana mass term does not have additional "accidental" symmetries, e.g., the $\mu - \tau$ symmetry. In the latter case as well as in the case of $G_\nu = Z_2 \times Z_2$, the matrix U_ν is completely determined by symmetries up to re-phasing on the right and permutations of columns. The latter can be fixed by considering a specific model.

---

[8] S_4 is the group of permutations of 4 objects and the symmetry group of the cube. A_4 is the group of even permutations of 4 objects and the symmetry group of the regular tetrahedron. T' is the double covering group of A_4. A_5 is the icosahedron symmetry group of even permutations of five objects, etc.

[9] Bimaximal mixing can also be a consequence of the conservation of the lepton charge $L' = L_e - L_\mu - L_\tau$ (LC) [46], supplemented by $\mu - \tau$ symmetry.

In the general case of absence of constraints, the PMNS matrix can be parametrised in terms of the parameters of U_e and U_ν as follows [99]:

$$U = U_e^\dagger \, U_\nu = (\tilde{U}_e)^\dagger \, \Psi \tilde{U}_\nu \, Q_0 \, . \tag{30}$$

Here $\tilde{U}_e$ and $\tilde{U}_\nu$ are CKM-like 3×3 unitary matrices and Ψ and Q_0 are given by:

$$\Psi = \mathrm{diag}\left(1, \mathrm{e}^{-i\psi}, \mathrm{e}^{-i\omega}\right), \quad Q_0 = \mathrm{diag}\left(1, \mathrm{e}^{i\frac{\xi_{21}}{2}}, \mathrm{e}^{i\frac{\xi_{31}}{2}}\right), \tag{31}$$

where ψ, ω, ξ_{21} and ξ_{31} are phases which contribute to physical CPV phases. Thus, in general, each of the two phase matrices Ψ and Q_0 contain two physical CPV phases. The phases in Q_0 contribute to the Majorana phases in the PMNS matrix and can appear in eq. (30) as a result of the diagonalisation of the neutrino Majorana mass term, while the phases in Ψ can result from the charged lepton sector ($U_e^\dagger = (\tilde{U}_e)^\dagger \, \Psi$), from the neutrino sector ($U_\nu = \Psi \tilde{U}_\nu Q_0$), or can receive contributions from both sectors.

4.2. Predicting the Dirac CPV phase

In the present subsection we will discuss two rather general settings or models in which the value of the Dirac CPV phases δ is predicted, while the values of the Majorana phases $\alpha_{21,31}$ can be predicted provided the phases $\xi_{21,31}$ in the matrix Q_0 in eq. (31) are known [29]. We will consider only the predictions for the Dirac phase δ.

Following Ref. [29] we will consider the cases when, as a consequence of underlying and residual symmetries, the matrix U_ν, and more specifically, the matrix $\tilde{U}_\nu$ in eq. (30), has the i) TBM, ii) BM, iii) GRA, iv) GRB and v) HG forms. For all these forms we have $\tilde{U}_\nu = R_{23}(\theta^\nu_{23}) R_{12}(\theta^\nu_{12})$ with $\theta^\nu_{23} = -\pi/4$, R_{23} and R_{12} being 3×3 orthogonal matrices describing rotations in the 2–3 and 1–2 planes:

$$\tilde{U}_\nu = R_{23}\left(\theta^\nu_{23}\right) R_{12}\left(\theta^\nu_{12}\right) = \begin{pmatrix} \cos\theta^\nu_{12} & \sin\theta^\nu_{12} & 0 \\ -\dfrac{\sin\theta^\nu_{12}}{\sqrt{2}} & \dfrac{\cos\theta^\nu_{12}}{\sqrt{2}} & -\dfrac{1}{\sqrt{2}} \\ -\dfrac{\sin\theta^\nu_{12}}{\sqrt{2}} & \dfrac{\cos\theta^\nu_{12}}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} \end{pmatrix}. \tag{32}$$

The value of the angle θ^ν_{12}, and thus of $\sin^2\theta^\nu_{12}$, depends on the form of $\tilde{U}_\nu$. For the TBM, BM, GRA, GRB and HG forms we have: i) $\sin^2\theta^\nu_{12} = 1/3$ (TBM), ii) $\sin^2\theta^\nu_{12} = 1/2$ (BM), iii) $\sin^2\theta^\nu_{12} = (2+r)^{-1} \cong 0.276$ (GRA), r being the golden ratio, $r = (1+\sqrt{5})/2$, iv) $\sin^2\theta^\nu_{12} = (3-r)/4 \cong 0.345$ (GRB), and v) $\sin^2\theta^\nu_{12} = 1/4$ (HG).

The TBM form of $\tilde{U}_\nu$, for example, can be obtained from a $G_f = A_4$ symmetry, when the residual symmetry is $G_\nu = Z_2$. In this case there is an additional accidental $\mu - \tau$ symmetry, which together with the Z_2 symmetry leads to the TBM form of $\tilde{U}_\nu$ (see, e.g., [100,91]). The TBM form can also be derived from $G_f = T'$ with $G_\nu = Z_2$, provided the left-handed (LH) charged lepton and neutrino fields each transform as triplets of T'[10] (see, e.g., [91] for details). Finally, one can obtain the BM from, e.g., the $G_f = S_4$ symmetry, when $G_\nu = Z_2$. There is an accidental $\mu - \tau$ symmetry in this case as well [101].

[10] When working with 3-dimensional and 1-dimensional representations of T', there is no way to distinguish T' from A_4 [102].

For all the forms of $\tilde{U}_\nu$ considered in [29] and listed above we have i) $\theta^\nu_{13} = 0$, which should be corrected to the measured value of $\theta_{13} \cong 0.15$, and ii) $\sin^2\theta^\nu_{23} = 0.5$, which might also need to be corrected if it is firmly established that $\sin^2\theta_{23}$ deviates significantly from 0.5. In the case of the BM and HG forms, the values of $\sin^2\theta^\nu_{12}$ lie outside the current 3σ allowed ranges of $\sin^2\theta_{12}$ and have also to be corrected.

The requisite corrections are provided by the matrix U_e, or equivalently, by $\tilde{U}_e$. The approach followed in [29,30,89,90] corresponds to the case of G_f completely broken by the charged lepton mass term. In this case the matrix $\tilde{U}_e$ is unconstrained and was chosen in [29,30,89] on phenomenological grounds to have the following two forms:

$$\mathbf{A}: \ \tilde{U}_e = R^{-1}_{23}(\theta^e_{23})\, R^{-1}_{12}(\theta^e_{12})\,; \quad \mathbf{B}: \ \tilde{U}_e = R^{-1}_{12}(\theta^e_{12})\,. \tag{33}$$

These two forms appear in a large class of theoretical models of flavour and studies, in which the generation of charged lepton masses is an integral part (see, e.g., [31,103]).

In the setting we are considering with $\tilde{U}_\nu$ having one of the five symmetry forms, TBM, BM, GRA, GRB and HG, and $\tilde{U}_e$ given by the form **A** in eq. (33), the Dirac phase δ of the PMNS matrix satisfies the following sum rule [29]:

$$\cos\delta = \frac{\tan\theta_{23}}{\sin 2\theta_{12}\sin\theta_{13}}\left[\cos 2\theta^\nu_{12} + \left(\sin^2\theta_{12} - \cos^2\theta^\nu_{12}\right)\left(1 - \cot^2\theta_{23}\,\sin^2\theta_{13}\right)\right]. \tag{34}$$

Within the approach employed this sum rule is exact. It was shown to hold also for the form **B** in [90], where it was found to be valid for any value of the angle θ^ν_{23} as well. The difference between the cases **A** and **B** of forms of $\tilde{U}_e$ in eq. (33) is, in particular, in the relation between $\sin^2\theta^e_{12}$ and $\sin^2\theta_{13}$ and, most importantly, in the correlation between the values of $\sin^2\theta_{23}$ and $\sin^2\theta_{13}$. In case **B** we have [90]:

$$\sin^2\theta_{13} = |U_{e3}|^2 = \sin^2\theta^e_{12}\sin^2\theta^\nu_{23}\,, \quad \sin^2\theta_{23} = \frac{|U_{\mu 3}|^2}{1-|U_{e3}|^2} = \frac{\sin^2\theta^\nu_{23} - \sin^2\theta_{13}}{1-\sin^2\theta_{13}} \tag{35}$$

For $\theta^\nu_{23} = -\pi/4$ of interest, we get $\sin^2\theta_{23} = 0.5(1-2\sin^2\theta_{13})/(1-\sin^2\theta_{13})$. Thus, $\sin^2\theta_{23}$ can deviate from 0.5 only by $\sim \sin^2\theta_{13}$. In contrast, for the form **A** of $\tilde{U}_e$ the values of $\sin^2\theta_{23}$ and $\sin^2\theta_{13}$ are not correlated and $\sin^2\theta_{23}$ can differ significantly from 0.5 [29,89]. The equality $\sin^2\theta^e_{12} = \sin^2\theta_{13}/4$ does not have to hold either, although $\sin^2\theta^e_{12} \propto \sin^2\theta_{13}$.

Qualitatively, the result in eq. (34) for δ can be understood as follows. In the parametrisation defined in eq. (30) with $\tilde{U}_\nu$ and $\tilde{U}_e$ given in (32) and, e.g., by form **B** in (33),

$$U_{\rm PMNS} = R_{12}(\theta^e_{12})\,\Psi\, R_{23}(\theta^\nu_{23})\, R_{12}(\theta^\nu_{12})\, Q_0\,. \tag{36}$$

The phase ω in the phase matrix Ψ is unphysical, while the phase ψ serves as a source for the Dirac phase δ (and gives a contribution to the Majorana phases $\alpha_{21,31}$). It follows from eq. (36) that in the case under discussion, the three angles θ_{12}, θ_{23}, θ_{13} and the Dirac phase δ of the standard parametrisation of $U_{\rm PMNS}$ are expressed in terms of the three parameters θ^e_{12}, ψ and θ^ν_{12} ($\theta^\nu_{23} = -\pi/4$). This suggests that it will be possible to express one of the four parameters θ_{12}, θ_{23}, θ_{13} and δ, namely δ, in terms of the other three, hence eq. (34). Although the case of $\tilde{U}_e$ having the form **A** in eq. (33) is somewhat more complicated, in what concerns $\cos\delta$ one arrives to the same conclusion and result [29,89].

Given the values of $\sin\theta_{23}$, $\sin\theta_{23}$, $\sin\theta_{13}$ and θ^ν_{12}, $\cos\delta$ is determined uniquely by the sum rule (34). This allows us to determine also $|\sin\delta|$ uniquely. However, in the absence of additional information, $\text{sgn}(\sin\delta)$ remains undetermined, which leads to a two-fold ambiguity in the determination of the value of δ from the value of $\cos\delta$.

The fact that the value of the Dirac CPV phase δ is determined (up to an ambiguity of the sign of $\sin\delta$) by the values of the three mixing angles θ_{12}, θ_{23} and θ_{13} of the PMNS matrix and the value of θ^{ν}_{12} of the matrix $\tilde{U}_{\nu}$, eq. (32), is the most striking prediction of the models considered. This result implies that in the schemes under discussion, the rephasing invariant $J_{\rm CP}$ associated with the Dirac phase δ, eq. (13), is also a function of the three angles θ_{12}, θ_{23} and θ_{13} of the PMNS matrix and of θ^{ν}_{12}:

$$J_{\rm CP} = J_{\rm CP}(\theta_{12}, \theta_{23}, \theta_{13}, \delta(\theta_{12}, \theta_{23}, \theta_{13}, \theta^{\nu}_{12})) = J_{\rm CP}(\theta_{12}, \theta_{23}, \theta_{13}, \theta^{\nu}_{12})\,. \tag{37}$$

This allows to obtain predictions for the possible values of $J_{\rm CP}$ for the different symmetry forms of $\tilde{U}_{\nu}$ (specified by the value of θ^{ν}_{12}) using the current data on θ_{12}, θ_{23} and θ_{13}.

In [29], by using the sum rule in eq. (34), predictions for $\cos\delta$, δ and the $J_{\rm CP}$ factor were obtained in the TBM, BM, GRA, GRB and HG cases for the b.f.v. of $\sin^2\theta_{12}$, $\sin^2\theta_{23}$ and $\sin^2\theta_{13}$. It was found that the predictions of $\cos\delta$ vary significantly with the symmetry form of $\tilde{U}_{\nu}$. For the b.f.v. of $\sin^2\theta_{12} = 0.308$, $\sin^2\theta_{13} = 0.0234$ and $\sin^2\theta_{23} = 0.437$ found in [10], for instance, one gets [29] $\cos\delta = (-0.0906)$, (-1.16), 0.275, (-0.169) and 0.445, for the TBM, BM (LC), GRA, GRB and HG forms, respectively. For the TBM, GRA, GRB and HG forms these values correspond to $\delta = \pm 95.2°, \pm 74.0°, \pm 99.7°, \pm 63.6°$. The unphysical value of $\cos\delta$ in the BM (LC) case is a reflection of the fact that the scheme under discussion with BM (LC) form of the matrix $\tilde{U}_{\nu}$ does not provide a good description of the current data on θ_{12}, θ_{23} and θ_{13} [89]. Physical values of $\cos\delta$ can be obtained, e.g., for the b.f.v. of $\sin^2\theta_{13}$ and $\sin^2\theta_{23}$ if $\sin^2\theta_{12}$ has a larger value [30].[11] The results quoted above imply [29] that a measurement of $\cos\delta$ can allow to distinguish between at least some of the different symmetry forms of $\tilde{U}_{\nu}$, provided θ_{12}, θ_{13} and θ_{23} are known, and $\cos\delta$ is measured, with sufficiently good precision.[12] Even determining the sign of $\cos\delta$ will be sufficient to eliminate some of the possible symmetry forms of $\tilde{U}_{\nu}$. It was also concluded in [29] that distinguishing between the TBM, GRA, GRB and HG forms of $\tilde{U}_{\nu}$ be measuring $J_{\rm CP}$ would require extremely high precision measurement of the $J_{\rm CP}$ factor.

These conclusions were confirmed by the statistical analyses performed in Ref. [30] where predictions of the sum rule (34) for i) δ, $\cos\delta$ and the rephasing invariant $J_{\rm CP}$ using the "data" (best fit values and χ^2-distributions) on $\sin^2\theta_{12}$, $\sin^2\theta_{13}$, $\sin^2\theta_{23}$ and δ from [10], and ii) for $\cos\delta$, using prospective uncertainties on $\sin^2\theta_{12}$, $\sin^2\theta_{13}$ and $\sin^2\theta_{23}$, were derived for the TBM, BM (LC), GRA, GRB and HG symmetry forms of the matrix $\tilde{U}_{\nu}$.

The aim of the first analysis, the results of which are summarised in Table 1, was to derive the allowed ranges for $\cos\delta$ and $J_{\rm CP}$, predicted on the basis of the current data on the neutrino mixing parameters for each of the symmetry forms of $\tilde{U}_{\nu}$ considered. We have found [30], in particular, that the CP-conserving value of $J_{\rm CP} = 0$ is excluded in the cases of the TBM, GRA, GRB and HG neutrino mixing symmetry forms, respectively, at approximately 5σ, 4σ, 4σ and 3σ C.L. with respect to the C.L. of the corresponding best fit values which all lie in the interval $J_{\rm CP} = (-0.034) - (-0.031)$ (see Table 1). The best fit value for the BM (LC) form is much smaller and close to zero: $J_{\rm CP} = (-5 \times 10^{-3})$. For the TBM, GRA, GRB and HG forms at 3σ we have $0.020 \leq |J_{\rm CP}| \leq 0.039$. Thus, for these four forms the CP violating effects in neutrino oscillations are predicted to be relatively large and observable in the T2HK and DUNE experiments [36,37].

[11] For, e.g., $\sin^2\theta_{12} = 0.34$ allowed at 2σ by the current data, we have $\cos\delta = -0.943$. Similarly, for $\sin^2\theta_{12} = 0.32$, $\sin^2\theta_{23} = 0.41$ and $\sin\theta_{13} = 0.158$ we have [29]: $\cos\delta = -0.978$.

[12] Detailed results on the dependence of the predictions for $\cos\delta$ on $\sin^2\theta_{12}$, $\sin^2\theta_{23}$ and $\sin^2\theta_{13}$ when the latter are varied in their respective 3σ experimentally allowed ranges can be found in [30].

Table 1
Best fit values of J_{CP} and $\cos\delta$ and corresponding 3σ ranges (found fixing $\chi^2 - \chi^2_{min} = 9$) for the five symmetry forms, TBM, BM, GRA, GRB and HG, and $\tilde{U}_e$ given by the form **A** in eq. (33), obtained using the data from [10] for NO neutrino mass spectrum. (From Ref. [30], where results for IO spectrum are also given.)

Scheme	$J_{CP}/10^{-2}$ (b.f.v.)	$J_{CP}/10^{-2}$ (3σ range)	$\cos\delta$ (b.f.v.)	$\cos\delta$ (3σ range)
TBM	−3.4	[−3.8, −2.8] ∪ [3.1, 3.6]	−0.07	[−0.47, 0.21]
BM (LC)	−0.5	[−2.6, 2.1]	−0.99	[−1.00, −0.72]
GRA	−3.3	[−3.7, −2.7] ∪ [3.0, 3.5]	0.25	[−0.08, 0.69]
GRB	−3.4	[−3.9, −2.6] ∪ [3.1, 3.6]	−0.15	[−0.57, 0.13]
HG	−3.1	[−3.5, −2.0] ∪ [2.6, 3.4]	0.47	[0.16, 0.80]

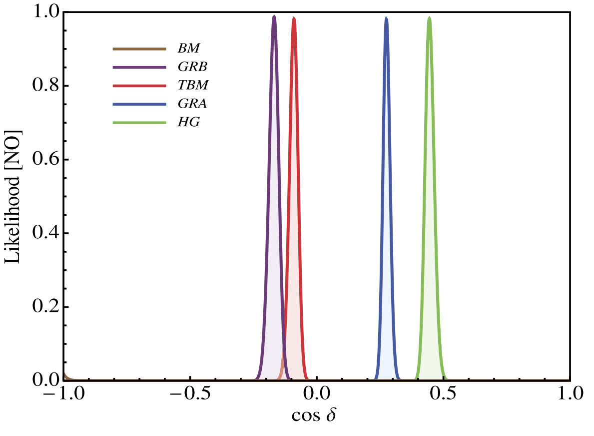

Fig. 3. The likelihood function versus $\cos\delta$ for NO neutrino mass spectrum after marginalising over $\sin^2\theta_{13}$ and $\sin^2\theta_{23}$, for the TBM, BM (LC), GRA, GRB and HG symmetry forms of the mixing matrix $\tilde{U}_\nu$. Th figure is obtained by using the prospective 1σ uncertainties in the determination of $\sin^2\theta_{12}$, $\sin^2\theta_{13}$ and $\sin^2\theta_{23}$ within the Gaussian approximation. The three neutrino mixing parameters are fixed to their current best fit values (i.e., $\sin^2\theta_{12} = 0.308$, etc.). See text for further details. (From Ref. [30].)

In Fig. 3 we present the results of the statistical analysis of the predictions for $\cos\delta$, namely the likelihood function versus $\cos\delta$ within the Gaussian approximation (see [30] for details) performed using the current b.f.v. of the mixing angles for NO neutrino mass spectrum given in Ref. [10] and the prospective 1σ uncertainties in the determination of $\sin^2\theta_{12}$ (0.7% from JUNO [104]), $\sin^2\theta_{13}$ (3% derived from an expected error on $\sin^2 2\theta_{13}$ of 3% from Daya Bay, see Refs. [5,105]) and $\sin^2\theta_{23}$ (5% derived from the potential sensitivity of NOvA and T2K on $\sin^2 2\theta_{23}$ of 2%, see Ref. [5], this sensitivity can also be achieved in planned neutrino experiments as T2HK [106]). The BM (LC) case is very sensitive to the b.f.v. of $\sin^2\theta_{12}$ and $\sin^2\theta_{23}$ and is disfavoured at more than 2σ for the current b.f.v. found in [10]. This case might turn out to be compatible with the data for larger (smaller) measured values of $\sin^2\theta_{12}$ ($\sin^2\theta_{23}$). The measurement of $\sin^2\theta_{12}$, $\sin^2\theta_{13}$ and $\sin^2\theta_{23}$ with the quoted precision will open up the possibility to distinguish between the BM (LC), TBM/GRB, GRA and HG forms of $\tilde{U}_\nu$. Distinguishing between the TBM and GRB forms seems to require unrealistically high precision measurement of $\cos\delta$. Assuming that $|\cos\delta| < 0.93$, which means for 76% of values of δ, the error on δ, $\Delta\delta$, for an error on $\cos\delta$, $\Delta(\cos\delta) = 0.10\,(0.08)$, does not exceed $\Delta\delta \lesssim \Delta(\cos\delta)/\sqrt{1-0.93^2} = 16^\circ\,(12^\circ)$. This accuracy is planned to be reached in the future neutrino experiments like T2HK (ESSνSB) [5]. Therefore a measurement of $\cos\delta$ in the quoted range will allow one to distinguish between the

TBM/GRB, BM (LC) and GRA/HG forms at approximately 3σ C.L., if the precision achieved on $\sin^2\theta_{12}$, $\sin^2\theta_{13}$ and $\sin^2\theta_{23}$ is the same as in Figs. 3.

The results obtained in the studies performed in Refs. [29,30] (see also [90,91]) show, in particular, that the experimental measurement of the Dirac phase δ of the PMNS neutrino mixing matrix in the future neutrino experiments, combined with the data on the neutrino mixing angles, can provide unique information about the possible discrete symmetry origin of the observed pattern of neutrino mixing.

5. Outlook

The program of experimental research in neutrino physics extends beyond 2030 (see, e.g., Refs. [5,36,37,107]). It is stimulated by the fact that the existence of nonzero neutrino masses and the smallness of the neutrino masses suggest the existence of new fundamental mass scale in particle physics, i.e., of New Physics beyond that predicted by the Standard Theory. In the coming years we expect a wealth of new data that, it is hoped, will shed light on the fundamental aspects of neutrino mixing: the nature – Dirac or Majorana – of massive neutrinos, the status of CP symmetry in the lepton sector, the type of spectrum the neutrino masses obey, the absolute neutrino mass scale, the origin of the observed patters of the neutrino masses and mixing (new fundamental symmetry?), and, eventually, on the mechanism of neutrino mass generation. It is hoped that progress in the theory of neutrino mixing will also lead, in particular, to progress in the theory of flavour and to a better understanding of the mechanism of generation of the baryon asymmetry of the Universe. We are looking forward to the future exciting developments in neutrino physics.

I would like to conclude by wishing the Nobel laureates for Physics for the year 2015, Dr. T. Kajita and Prof. A. McDonald, further fundamental contributions to the understanding of the neutrino properties.

Acknowledgements

It is a pleasure to thank my Ph.D. students I. Girardi and A. Titov for the fruitful and enjoyable collaboration and the numerous illuminating discussions of the problems considered in the present article. This work was supported in part by the INFN program on Theoretical Astroparticle Physics (TASP), by the research grant 2012CPPYP7 (*Theoretical Astroparticle Physics*) under the program PRIN 2012 funded by the Italian Ministry of Education, University and Research (MIUR) and by the World Premier International Research Center Initiative (WPI Initiative, MEXT), Japan.

References

[1] B. Pontecorvo, Zh. Eksp. Teor. Fiz. 33 (1957) 549, Zh. Eksp. Teor. Fiz. 34 (1958) 247.
[2] Z. Maki, M. Nakagawa, S. Sakata, Prog. Theor. Phys. 28 (1962) 870.
[3] K. Nakamura, S.T. Petcov, K.A. Olive, et al., Particle Data Group, Chin. Phys. C 38 (2014) 090001.
[4] B. Pontecorvo, Zh. Eksp. Teor. Fiz. 53 (1967) 1717.
[5] A. de Gouvea, et al., arXiv:1310.4340.
[6] C. Giunti, Nucl. Phys. B (2016), in this Special Issue.
[7] S.M. Bilenky, J. Hosek, S.T. Petcov, Phys. Lett. B 94 (1980) 495.
[8] S.T. Petcov, Adv. High Energy Phys. 2013 (2013) 852987, arXiv:1303.5819.
[9] S.M. Bilenky, S.T. Petcov, Rev. Mod. Phys. 59 (1987) 671.
[10] F. Capozzi, et al., Phys. Rev. D 89 (2014) 093018.

[11] M.C. Gonzalez-Garcia, M. Maltoni, T. Schwetz, J. High Energy Phys. 1411 (2014) 052.
[12] K. Abe, et al., Phys. Rev. Lett. 112 (2014) 061802.
[13] F.P. An, et al., Phys. Rev. Lett. 112 (2014) 061801.
[14] P. Adamson, et al., NOvA Collaboration, arXiv:1601.05022 [hep-ex].
[15] F. Capozzi, et al., arXiv:1601.07777 [hep-ph].
[16] F.P. An, et al., Daya Bay Collab, Phys. Rev. Lett. 108 (2012) 171803;
F.P. An, et al., Daya Bay Collab, Chin. Phys. C 37 (2013) 011001;
F.P. An, et al., Phys. Rev. Lett. 112 (2014) 061801;
F.P. An, et al., Daya Bay Collab, arXiv:1505.03456.
[17] J.K. Ahn, et al., Phys. Rev. Lett. 108 (2012) 191802;
W. Choi [for the RENO Collab.], Talk at the WIN2015 International Workshop.
[18] Y. Abe, et al., Double Chooz Collab, Phys. Rev. Lett. 108 (2012) 131801;
Y. Abe, et al., Double Chooz Collab, Phys. Lett. B 723 (2013) 66;
Y. Abe, et al., Double Chooz Collab, J. High Energy Phys. 1014 (2014) 086.
[19] S. Pascoli, S.T. Petcov, A. Riotto, Nucl. Phys. B 774 (2007) 1;
S. Pascoli, S.T. Petcov, A. Riotto, Phys. Rev. D 75 (2007) 083511.
[20] R.N. Cahn, et al., arXiv:1307.5487.
[21] S.T. Petcov, M. Piai, Phys. Lett. B 533 (94) (2002) 94;
S. Pascoli, S.T. Petcov, Phys. Lett. B 544 (2002) 239;
S.T. Petcov, S. Palomares-Ruiz, Nucl. Phys. B 712 (2005) 392;
S.T. Petcov, T. Schwetz, Nucl. Phys. B 740 (2006) 1.
[22] F. Perrin, C. R. 197 (1933) 868;
E. Fermi, Nuovo Cim. 11 (1934) 1.
[23] Ch. Kraus, et al., Eur. Phys. J. C 40 (2005) 447.
[24] V. Lobashev, et al., Nucl. Phys. A 719 (2003) 153c.
[25] V.N. Aseev, et al., Phys. Rev. D 84 (2011) 112003.
[26] K. Eitel, et al., Nucl. Phys. B, Proc. Suppl. 143 (2005) 197.
[27] K.N. Abazajian, et al., Astropart. Phys. 35 (2011) 177.
[28] P.A.R. Ade, et al., Planck Collab, arXiv:1502.01589.
[29] S.T. Petcov, Nucl. Phys. B 892 (2015) 400.
[30] I. Girardi, S.T. Petcov, A.V. Titov, Nucl. Phys. B 894 (2015) 733;
I. Girardi, S.T. Petcov, A.V. Titov, Int. J. Mod. Phys. A 30 (2015) 1530035.
[31] I. Girardi, et al., J. High Energy Phys. 1402 (2014) 050, and references quoted therein.
[32] N. Cabibbo, Phys. Lett. B 72 (1978) 333.
[33] V.D. Barger, K. Whisnant, R.J.N. Phillips, Phys. Rev. Lett. 45 (1980) 2084.
[34] P.I. Krastev, S.T. Petcov, Phys. Lett. B 205 (1988) 84.
[35] C. Jarlskog, Z. Phys. C 29 (1985) 491.
[36] R. Acciarri, et al., DUNE Collaboration, arXiv:1512.06148;
R. Acciarri, et al., DUNE Collaboration, arXiv:1601.05471;
R. Acciarri, et al., DUNE Collaboration, arXiv:1601.02984.
[37] K. Abe, et al., Prog. Theor. Exp. Phys. 053C02 (2015).
[38] L. Wolfenstein, Phys. Rev. D 17 (1978) 2369;
L. Wolfenstein, in: E.C. Fowler (Ed.), Proc. of the 8th Int. Conference on Neutrino Physics and Astrophysics – "Neutrino'78", Purdue University Press, West Lafayette, 1978, p. C3;
S.P. Mikheev, A.Y. Smirnov, Sov. J. Nucl. Phys. 42 (1885) 913;
see also: V. Barger, et al., Phys. Rev. D 22 (1980) 2718.
[39] P. Langacker, et al., Nucl. Phys. B 282 (1987) 589.
[40] M. Freund, Phys. Rev. D 64 (2001) 053003.
[41] A.D. Dziewonski, D.L. Anderson, Phys. Earth Planet. Inter. 25 (1981) 297.
[42] M. Maris, S.T. Petcov, Study performed in December of 1966, unpublished, cited in S.T. Petcov, Phys. Lett. B 434 (1998) 321, Phys. Lett. B 444 (1998) 584 (E).
[43] S.K. Agarwalla, Y. Kao, T. Takeuchi, J. High Energy Phys. 1404 (2014) 047.
[44] P. Minkowski, Phys. Lett. B 67 (1977) 421;
T. Yanagida, in: Proc. of the Workshop on Unified Theory and Baryon Number of the Universe, KEK, Japan, 1979;
M. Gell-Mann, P. Ramond, R. Slansky, Sanibel talk, CALT-68-709, Feb. 1979;
M. Gell-Mann, P. Ramond, R. Slansky, Supergravity, North Holland, Amsterdam, 1979;

S.L. Glashow, Cargese Lectures, 1979;
R.N. Mohapatra, G. Senjanovic, Phys. Rev. Lett. 44 (1980) 912.
[45] R. Mohapatra, et al., Rep. Prog. Phys. 70 (2007) 1757.
[46] S.T. Petcov, Phys. Lett. B 110 (1982) 245.
[47] S.M. Bilenky, S. Pascoli, S.T. Petcov, Phys. Rev. D 64 (2001) 053010.
[48] S. Pascoli, S.T. Petcov, L. Wolfenstein, Phys. Lett. B 524 (2002) 319.
[49] V. Barger, et al., Phys. Lett. B 540 (2002) 247.
[50] S. Pascoli, S.T. Petcov, W. Rodejohann, Phys. Lett. B 549 (2002) 177.
[51] S. Pascoli, S.T. Petcov, T. Schwetz, Nucl. Phys. B 734 (2006) 24.
[52] A. De Gouvea, B. Kayser, R. Mohapatra, Phys. Rev. D 67 (2003) 053004.
[53] A. Giuliani, A. Poves, Adv. High Energy Phys. 2012 (2012) 857016.
[54] W. Rodejohann, Int. J. Mod. Phys. E 20 (2011) 1833.
[55] S. Dell'Oro, et al., arXiv:1601.07512.
[56] L. Wolfenstein, Phys. Lett. B 107 (1981) 77;
S.M. Bilenky, et al., Nucl. Phys. B 247 (1984) 61;
B. Kayser, Phys. Rev. D 30 (1984) 1023.
[57] S. Pascoli, S.T. Petcov, Phys. Lett. B 544 (2002) 239.
[58] A.S. Barabash, Phys. At. Nucl. 74 (2011) 603.
[59] H.V. Klapdor-Kleingrothaus, et al., Mod. Phys. Lett. A 16 (2001) 2409.
[60] H.V. Klapdor-Kleingrothaus, et al., Phys. Lett. B 586 (2004) 198.
[61] K.-H. Ackermann, et al., Phys. Rev. Lett. 111 (2013) 122503.
[62] H.V. Klapdor-Kleingrothaus, et al., Nucl. Phys. Proc. Suppl. 100 (2001) 309.
[63] C.E. Aalseth, et al., Phys. At. Nucl. 63 (2000) 1225.
[64] A. Barabash, et al., J. Phys. Conf. Ser. 173 (2009) 012008.
[65] C. Amaboldi, et al., Phys. Rev. C 78 (2008) 035502.
[66] M. Auger, et al., Phys. Rev. Lett. 109 (2012) 032505.
[67] A. Gando, et al., Phys. Rev. Lett. 110 (2013) 062502.
[68] S.M. Bilenky, et al., Phys. Rev. D 56 (1996) 4432.
[69] G.L. Fogli, et al., Phys. Rev. D 84 (2011) 053007.
[70] S.M. Bilenky, S.T. Petcov, arXiv:hep-ph/0405237.
[71] J. Vegados, H. Ejiri, F. Simkoviĉ, Rep. Prog. Phys. 75 (2012) 106301.
[72] A. Faessler, et al., Phys. Rev. D 83 (2011) 113003;
A. Meroni, S.T. Petcov, F. Simkoviĉ, J. High Energy Phys. 1302 (2013) 025.
[73] S. Pascoli, S.T. Petcov, Phys. Rev. D 77 (2008) 113003.
[74] E. Molinaro, S.T. Petcov, Phys. Lett. B 671 (2009) 60.
[75] M. Fukugita, T. Yanagida, Phys. Lett. B 174 (1986) 45.
[76] V.A. Kuzmin, et al., Phys. Lett. B 155 (1985) 36.
[77] P.A.R. Ade, et al., Planck Collab, Astron. Astrophys. 571 (2014) A16.
[78] W. Buchmuller, et al., Ann. Phys. 315 (2005) 305.
[79] J.A. Casas, A. Ibarra, Nucl. Phys. B 618 (2001) 171.
[80] A. Abada, et al., J. Cosmol. Astropart. Phys. 0604 (2006) 004;
A. Abada, et al., J. High Energy Phys. 0609 (2006) 010.
[81] E. Nardi, et al., J. High Energy Phys. 0601 (2006) 164.
[82] R. Barbieri, et al., Nucl. Phys. B 575 (2000) 61.
[83] S.T. Petcov, et al., Nucl. Phys. B 739 (2006) 208.
[84] E. Molinaro, S.T. Petcov, Eur. Phys. J. C 61 (2009) 93.
[85] A. de Gouvea, H. Murayama, Phys. Lett. B 747 (2015) 479;
A. de Gouvea, H. Murayama, Phys. Lett. B 573 (2003) 94;
L. Hall, H. Murayama, N. Weiner, Phys. Rev. Lett. 84 (2000) 2572.
[86] S.F. King, C. Luhn, Rep. Prog. Phys. 76 (2013) 056201.
[87] S.F. King, et al., New J. Phys. 16 (2014) 045018.
[88] S.F. King, T. Neder, A.J. Stuart, Phys. Lett. B 726 (2013) 312;
C. Hagedorn, A. Meroni, E. Molinaro, Nucl. Phys. B 891 (2015) 499;
C.C. Li, G.J. Ding, arXiv:1503.03711;
P. Ballett, S. Pascoli, J. Turner, Phys. Rev. D 92 (2015) 093008.
[89] D. Marzocca, et al., J. High Energy Phys. 1305 (2013) 073.

[90] I. Girardi, S.T. Petcov, A.V. Titov, Eur. Phys. J. C 75 (2015) 345.
[91] I. Girardi, et al., Nucl. Phys. B 902 (2016) 1.
[92] J. Turner, Phys. Rev. D 92 (2015) 116007.
[93] H. Ishimori, et al., Prog. Theor. Phys. Suppl. 183 (2010) 1.
[94] P.F. Harrison, D.H. Perkins, W.G. Scott, Phys. Lett. B 530 (2002) 167;
Z.z. Xing, Phys. Lett. B 533 (2002) 85;
see also: L. Wolfenstein, Phys. Rev. D 18 (1978) 958.
[95] F. Vissani, arXiv:hep-ph/9708483;
V.D. Barger, et al., Phys. Lett. B 437 (1998) 107.
[96] A. Datta, F.S. Ling, P. Ramond, Nucl. Phys. B 671 (2003) 383;
L.L. Everett, A.J. Stuart, Phys. Rev. D 79 (2009) 085005;
Y. Kajiyama, M. Raidal, A. Strumia, Phys. Rev. D 76 (2007) 117301.
[97] W. Rodejohann, Phys. Lett. B 671 (2009) 267;
A. Adulpravitchai, A. Blum, W. Rodejohann, New J. Phys. 11 (2009) 063026.
[98] C.H. Albright, A. Dueck, W. Rodejohann, Eur. Phys. J. C 70 (2010) 1099;
J.E. Kim, M.S. Seo, J. High Energy Phys. 1102 (2011) 097.
[99] P.H. Frampton, S.T. Petcov, W. Rodejohann, Nucl. Phys. B 687 (2004) 31.
[100] G. Altarelli, F. Feruglio, Rev. Mod. Phys. 82 (2010) 2701.
[101] G. Altarelli, F. Feruglio, L. Merlo, J. High Energy Phys. 0905 (2009) 020.
[102] F. Feruglio, C. Hagedorn, Y. Lin, L. Merlo, Nucl. Phys. B 775 (2007) 120.
[103] D. Marzocca, et al., J. High Energy Phys. 1111 (2011) 009;
S. Antusch, et al., Nucl. Phys. B 866 (2013) 255;
J. Gehrlein, et al., Nucl. Phys. B 890 (2014) 539;
A. Meroni, S.T. Petcov, M. Spinrath, Phys. Rev. D 86 (2012) 113003;
M.C. Chen, K.T. Mahanthappa, Phys. Lett. B 681 (2009) 444;
M.C. Chen, et al., J. High Energy Phys. 1310 (2013) 112.
[104] Y. Wang, PoS Neutel 2013 (2013) 030.
[105] C. Zhang, et al., arXiv:1501.04991.
[106] P. Coloma, H. Minakata, S.J. Parke, Phys. Rev. D 90 (2014) 093003.
[107] S. Ahmed, et al., INO Collab, arXiv:1505.07380.