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ASSESSMENT OF THE UNCERTAINTIES OF A CONCEPTUAL HYDROLOGIC MODEL BY USING ARTIFICIALLY GENERATED FLOWS

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ABSTRACT

Most of the studies that assess the performance of various calibration techniques have to deal with a certain amount of uncertainty in the calibration data. In this study we tested HBV model calibration procedures in hypothetically ideal conditions under the assumption of no errors in the measured data. This was achieved by creating an artificial time series of the flows created by the HBV model using the parameters obtained from calibrating the measured flows. The artificial flows were then used to replace the original flows in the calibration data, which was then used for testing how calibration procedures can reproduce known model parameters. The results showed that in performing one hundred independent calibration runs of the HBV model, we did not manage to obtain parameters that were almost identical to those used to create the artificial flow data without a certain degree of uncertainty. Although the calibration procedure of the model works properly from a practical point of view, it can be regarded as a demonstration of the equifinality principle, since several parameter sets were obtained which led to equally acceptable or behavioural representations of the observed flows. The study demonstrated that this concept for assessing how uncertain hydrological predictions can be applied in the further development of a model or the choice of calibration method using artificially generated data.

KEY WORDS

- *HBV model,*
- *Model calibration,*
- *Equifinality,*
- *Genetic algorithm,*
- *Harmony search.*

INTRODUCTION

Conceptual rainfall-runoff models are simplifications of the complex processes of runoff generation in a catchment. The particular components of these models often have to be described by empirical functions based on the observation of certain processes. The models therefore usually contain a number of parameters that do not represent directly measurable entities and thus must be estimated

through a process known as model calibration. In the calibration process the values of the parameters are gradually changed in order to adjust the behaviour of the model to mimic the behaviour of a real system. The process therefore requires measurements of a catchment's behaviour, usually in terms of the inputs (rainfall) and the outputs (e.g., the stream flow at the catchment's outlet). Calibration is a crucial part of identifying a model; therefore, a relatively great deal of attention is dedicated to this problem in

the literature, which leads to the development of many calibration procedures (see, e.g., Gupta et al., 2005; Vieux, 2004; Beven, 2004; Wagener, 2004; Kavetski et al. 2006a,b; Onwubolu and Babu, 2004; Weise, 2009; Rao, 2009).

Nowadays a hydrological modeller can choose from a large number of optimization algorithms and objective functions (see, e.g., Sorooshian and Dracup, 1980; Sorooshian et al., 1983; Kavetski et al., 2003) or calibration procedures (for a review, see Klemes, 1986). The testing of particular optimization algorithms or procedures can be peculiar since we do not know whether the found solution (the set of parameters) is a local or global maximum/minimum. This is mainly due to the fact that there is a certain amount of uncertainty in the measured data and model itself. Moreover, in hydrological modelling it is an acknowledged fact that in open systems a given end state can be reached by many potential means and that there are therefore many behavioural models of the same system (Beven, 2004; Gupta, Beven and Wagener, 2005) with different parameter sets leading to equally acceptable representations of the observed flows (the equifinality principle, Beven, 2004).

In this study we decided to test the equifinality principle in idealised conditions with the calibration of a lumped rainfall-runoff model. In order to remove all the uncertainties caused by errors in the data and the imperfect model structure, an artificial time series of flows was created with the model with a known set of parameters, which replaced the measured flows in the calibration. By doing this we expected to know the values of particular parameters which would lead to a global optimum. The calibration of the model using artificial data was then performed using a split sample test by splitting the input dataset into two halves, each of them calibrated separately. One hundred calibration runs were performed with the goal of recalibrating the model to its own known output and estimating the uncertainties in the model parameters.

1. METHODOLOGY

1.1 The rainfall runoff model

The Hron rainfall runoff model, which is a modified HBV model, was used here in a daily step (Bergstrom, 1976; 1992). The model requires the following input data: daily temperatures, precipitation and daily potential evapotranspiration, which can also be calculated using an index of the duration of the sunshine. The structure of the model can be divided into the following three components:

- snow submodel,
- soil submodel,
- runoff submodel.

The main task of the snow submodel is to simulate snow accumulation and melting in the catchment, which is done by using a very simple degree-day factor method. This method does not require a large amount of input data. Despite this, by using this method we can get comparable results with those obtained using advanced energy-based models (Turcan, 1982; Beven and Freer, 2001).

The soil submodel represents part of the hydrological cycle which occurs under the soil's surface. It includes various parts of the hydrological cycle, such as the infiltration of precipitation and melted snow from the soil surface to deeper soil layers, the distribution and accumulation of water in the soil layers, evapotranspiration, and the generation of the surface, subsurface and base flows. In the soil submodel the various soil layers are represented by two fictive reservoirs representing the accumulated soil and groundwater.

The runoff submodel is used to transform total runoff from the catchment comprising of the surface and subsurface runoffs and base flow. As a transformation function the model uses a simple triangular weighted function, which can be written as

$$Q = \begin{cases} \frac{(i-0.5) \cdot 4}{maxbas^2} \cdot q & \text{for the left side of the triangle} \\ \frac{(i-0.75) \cdot 4}{maxbas^2} \cdot q & \text{for the middle of the triangle} \\ \frac{(maxbas-i+0.5) \cdot 4}{maxbas^2} \cdot q & \text{for the right side of the triangle} \end{cases}$$

where *maxbas* is a parameter representing the total number of days into which the runoff is divided, $i = \{1, 2, \dots, maxbas\}$, and *q* is the total runoff from the whole catchment before any transformation. The triangular transformation function used in this study divides the runoff according to an equilateral triangle, where the greatest part is allocated to the middle day (Bergstrom, 1976; 1992). The transformation function redistributes the total runoff into several days, which results in the fact that a certain amount of precipitation falling on a catchment in a particular day affects the runoff from the catchment for more than one day.

The Hron model used in this study has 13 parameters that are described in Tab. 1.

1.2 Model calibration of the measured data

The model parameters were estimated by automatic calibration using two approaches: 1) genetic algorithm and 2) harmony search. Genetic algorithms (GA) belong to the family of evolutionary algorithms which are based on techniques that imitate evolutionary processes such as mutation, natural selection, crossing or heredity in searching for the best result. The principle of GA is the gradual creation of parameter generations comprising one or more

Tab. 1 Description of the Hron model parameters together with their upper and lower limits.

Parameter	Description and units	Range
fc	field capacity – represents the maximum amount of water that the upper part of the soil can hold [mm]	100 - 400
rc	coefficient influencing the amount of water contributing to the soil moisture and the upper reservoir [-]	0.1 - 4
uzl	upper zone limit – threshold value determining the occurrence of surface runoff q0 [mm]	10 - 40
tempRain	threshold temperature above which the entire precipitation is liquid [°C]	0.5 - 10
tempMelt	threshold temperature determining the start of the snow melting [°C]	-5 - 2
tempSnow	threshold temperature under which the entire precipitation is solid [°C]	-10 - 0
ddf	degree-day factor – determines the speed of the snow melting [mm]	0 – 3
perc	percolation – the amount of water percolating from the upper to the bottom reservoir [mm]	0.5 – 4
lpe	limit of potential evapotranspiration – used to estimate the potential evapotranspiration [-]	0.5 – 1
k0	empirical parameters influencing the surface (q0), subsurface (q1) and base (q2) flows [-]	1 – 50
k1		1 – 30
k2		10 – 100
maxbas	parameter determining the amount of days into which the catchment runoff is divided	1 - 6

populations with various numbers of different individuals (in our case, the set of parameters). At the beginning of the first generation a set of random individuals (sets of parameters) is generated from which some individuals (usually one or two) are directly included into the second generation, while some are selected for mutation and crossing. The rest of the individuals are then randomly generated to fill the number of individuals in the next generation. This procedure is repeated until the quality of the best individual meets the required criteria or till the algorithm reaches the maximum amount of generations. More about genetic algorithms can be found, e.g., in Sekaj (2005) or Mitchell (1996).

Harmony search methods (HS) are relatively new optimization algorithms which also belong to the group of evolutionary algorithms. They are phenomenon-mimicking algorithms which are inspired by the improvisation process of jazz musicians. HS algorithms exist in many specifications that are suitable for solving various scientific or engineering problems. Here we give only the description of the basic structure of the HS algorithm. The optimization process used in the HS algorithm can be divided into the following steps: 1) problem formulation, 2) algorithm parameter setting, 3) random tuning for memory initialization, 4) harmony improvisation, 5) memory update, 6) performance of termination and 7) the cadenza. After the formulation of the optimization problem, the parameters of the HS algorithm itself have to be set. The algorithm includes these parameters: harmony memory size (HMS), harmony memory considering rate (HMCR), pitch adjusting rate (PAR), maximum improvisation (MI) and fret width (FW). In the third step a number of random harmonies (possible solutions) are generated from which the top HMS harmonies (based on the value of the optimization

function) are selected to fill the harmony memory (HM). In the harmony improvisation process new HMS harmonies are created, where a member of a harmony is either picked from within the value range or is taken from the HM with the probability of HMCR. In the latter case the value can also be adjusted (with the probability of PAR) by adding a certain amount to the value. If the new harmony is better in terms of objective function than the worst harmony stored in the HM, the new harmony replaces the worst harmony in the HM. However, it is also important to take into account the diversity of harmonies in HM as well as the maximum number of identical harmonies in HM. This process is repeated until the HS satisfies the termination criteria, when the HS algorithm that returns the best harmony is stored in the HM. For further information about the HS algorithm, see, Geem (2009).

In both cases the Nash-Sutcliffe (NS) coefficient was selected as an optimization criterion (Nash and Sutcliffe, 1970). The NS coefficient was calculated using the following formula

$$NS = 1 - \frac{\sum_{i=1}^n (Q_{sim}^i - Q_{obs}^i)^2}{\sum_{i=1}^n (Q_{obs}^i - \bar{Q}_{obs})^2}$$

where Q_{sim} and Q_{obs} represent the simulated and observed flows, respectively, and \bar{Q}_{obs} is the average of the observed flow. The NS coefficient can obtain values between $-\infty$ and 1, where 1 represents an absolute compliance between the observed and calibrated data. Both calibration methods were used to run 100 independent calibrations using the original measured input data. The initial starting values of the parameters were generated randomly from a uniform distribution based on the range of admissible values

(see Tab. 1). The initial conditions of the model in the terms of the values of snow water equivalent (swe), storage in the upper zone (suz), storage in the lower zone (slz) and soil moisture (sm) were set to the following values: swe = 80, suz = 15, slz = 60 and sm = 40. In the case when as the calibration period was used the second half of the data the starting values were taken from the previous day. The values in the previous day were calculated using the best set of parameters when calibrating on the whole dataset using genetic algorithm and harmony search optimization algorithms. The NS coefficients were calculated for each calibration run and used to produce box plots showing their distribution for both algorithms. Based on the assessment of the box plots, the algorithm with the better overall performance was selected and used in the next stages of the study.

1.3 Generation of artificial data for testing the model

In this study we suggest applying the equifinality principle for evaluating the performance of the model calibration procedure with respect to the uncertainties of the parameters. In order to remove all uncertainties caused by errors in the data and the imperfect model structure from the calibration, an artificial time series of flows was created with the Hron model using a known set of parameters. This replaced the measured flows in the calibration data set. By doing this we expect to exactly know the values of the particular parameters of the system and have the perfect model of the system. We also expect that an efficient calibration procedure should lead to a global optimum and arrive at the set of the known parameters. The best set of parameters (in terms of the NS coefficient) out of the set of 100 calibrations produced using both optimization algorithms was subsequently used to calculate the simulated runoff from the catchment for the artificial runoff series. Hereinafter this new input file, with the replaced observed and simulated flows, is referred to as the generated data.

The calibration of the model using the generated data was then performed separately on the two halves of the input dataset. One hundred calibration runs were performed on both sets with the goal of recalibrating the model to its own known output. The uncertainties in the estimated model parameters were evaluated.

2. DATA

As a pilot basin for the case study, we used the Upper Hron River catchment (the catchment outlet at the city of Banska Bystrica) with an area of 1766.48 km² and an altitude ranging from 340 to 2043 m.a.s.l. (the average altitude is 805 m.a.s.l.). The average annual precipitation rate for the whole catchment is 800 mm, while in the lower parts of the catchment it is 600 mm, and goes up to 1600 mm in the upper parts. The average annual evapotranspiration ranges from 300 to 600 mm.

Input data in a daily step in the period between 1 January 1981 and 31 December 2000 was used in this study. The input data consisted of (1) average daily flows in the River Hron – Banska Bystrica section, (2) average daily temperatures in the catchment, (3) the catchment's average daily precipitation rate, and (4) an index of the duration of sunshine for each month. Fig. 1 shows the measured flows together with the corresponding precipitation during the whole period.

3. RESULTS AND DISCUSSION

The calibration of the model for the measured time series was performed using two optimization algorithms: HS and GA with the NS coefficient as the optimisation criterion. One hundred independent calibrations were performed with both algorithms. Fig. 2 shows a comparison of the performance of both algorithms

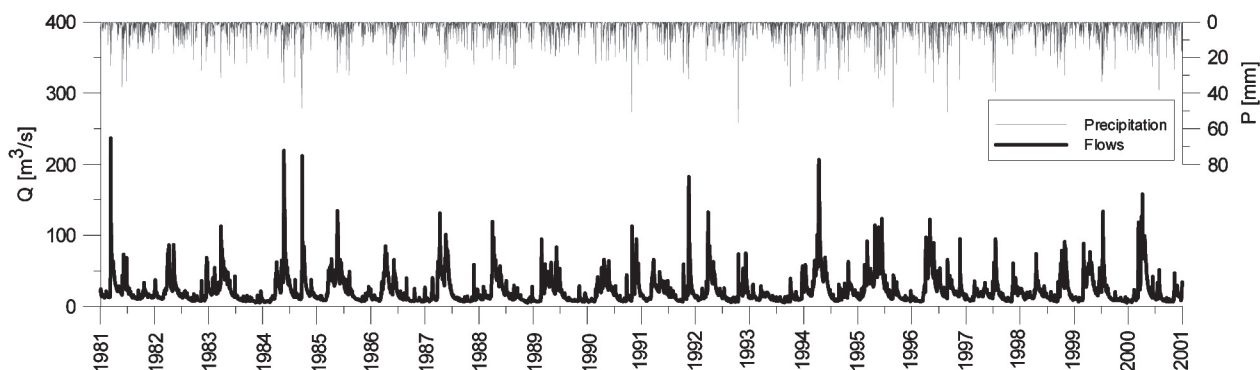


Fig. 1 The observed flows together with corresponding average precipitation of the catchment in the Upper Hron River catchment in the period between 1981 and 2000.

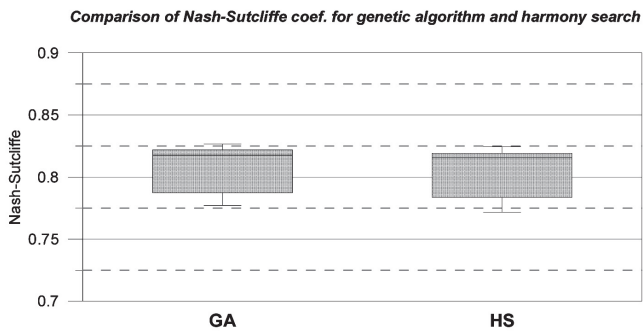


Fig. 2 Comparison of the variability of NS coefficients for 100 calibrations using GA and HS.

in terms of the variability of the NS coefficient values obtained. It shows that both algorithms gave almost identical results, with the GA performing marginally better and also supplying the best set of parameters with the highest NS coefficient. We have also produced box plots comparing variations in the particular parameters obtained by the GA and HS algorithms. These plots are displayed in Fig. 3 and show that the variations of the particular parameters are similar in both cases (GA and HS). The parameters with relatively high variations such as *k0* or *uzl* are those that are not very sensitive and therefore do not have a substantial effect on the quality of the simulation. This plot can also be regarded as an illustration of the equifinality principle, which not only shows, that there is no unique parameter set, but also demonstrates, that different optimisation methods may not supply equal sets of parameters and that these also differ in their variability (despite using the same optimisation criterion). This fact has consequences for the practical application of rainfall-runoff models and the uncertainties associated with their predictions.

Despite the fact that both optimization algorithms gave very similar results, the GA was preferred for further simulations, even though it did not prove to be computationally more effective with the Hron model. The best set of parameters, in terms of the NS coefficient, was selected from the set of parameters obtained by GA (the highest NS was 0.826). The values of the parameters used to generate the artificial flows are listed in Tab. 2.

In the next part of this study the observed flows in the input file were substituted by the simulated flows. This new input file was then used to test the performance of the calibration algorithm of the Hron model with the expectation that the algorithm should be able to obtain the same set of parameters repeatedly with the values of the NS coefficient very close to 1 (a perfect fit). In order to test the influence of the calibration period on the results, the ‘generated data’ was split into two parts, and each one was separately used for

Tab. 2 The Hron model parameter set used for generating the artificial flows *f* or the model’s calibration.

Parameter	Value
fc	162.6
rc	1.003
uzl	10.174
tempRain	7.422
tempMelt	-1.521
tempSnow	-8.974
ddf	0.757
perc	2.670
lpe	0.504
k0	48.567
k1	4.192
k2	22.798
maxbas	3

calibrating the model. The results of the calibration are shown in Fig. 4 and Tab. 3.

Fig. 4 illustrates the variability of particular parameters after 100 calibrations using GA, whereas black the box plots in Fig. 4 represent the results obtained when calibrating the first half of the dataset and the white boxes the second half. The figures show that the variability of all the parameters is substantially higher in the first case, where the parameters *perc*, *k1*, *k2* and *lpe* especially take the values from within the larger part of their range. The large degree of the variability of a particular parameter usually indicates its insensitivity and thus its inability to substantially influence the quality of the simulation. The black box plots in Fig. 4 would suggest that the insensitive parameters are *perc*, *k1*, *k2* and *lpe* and that the very sensitive ones are *fc*, *rc* and *ddf*. However, these results could not be confirmed with the outputs from the calibration of the second half of the data, where all of the parameters show a very small degree of the variability (white box plots).

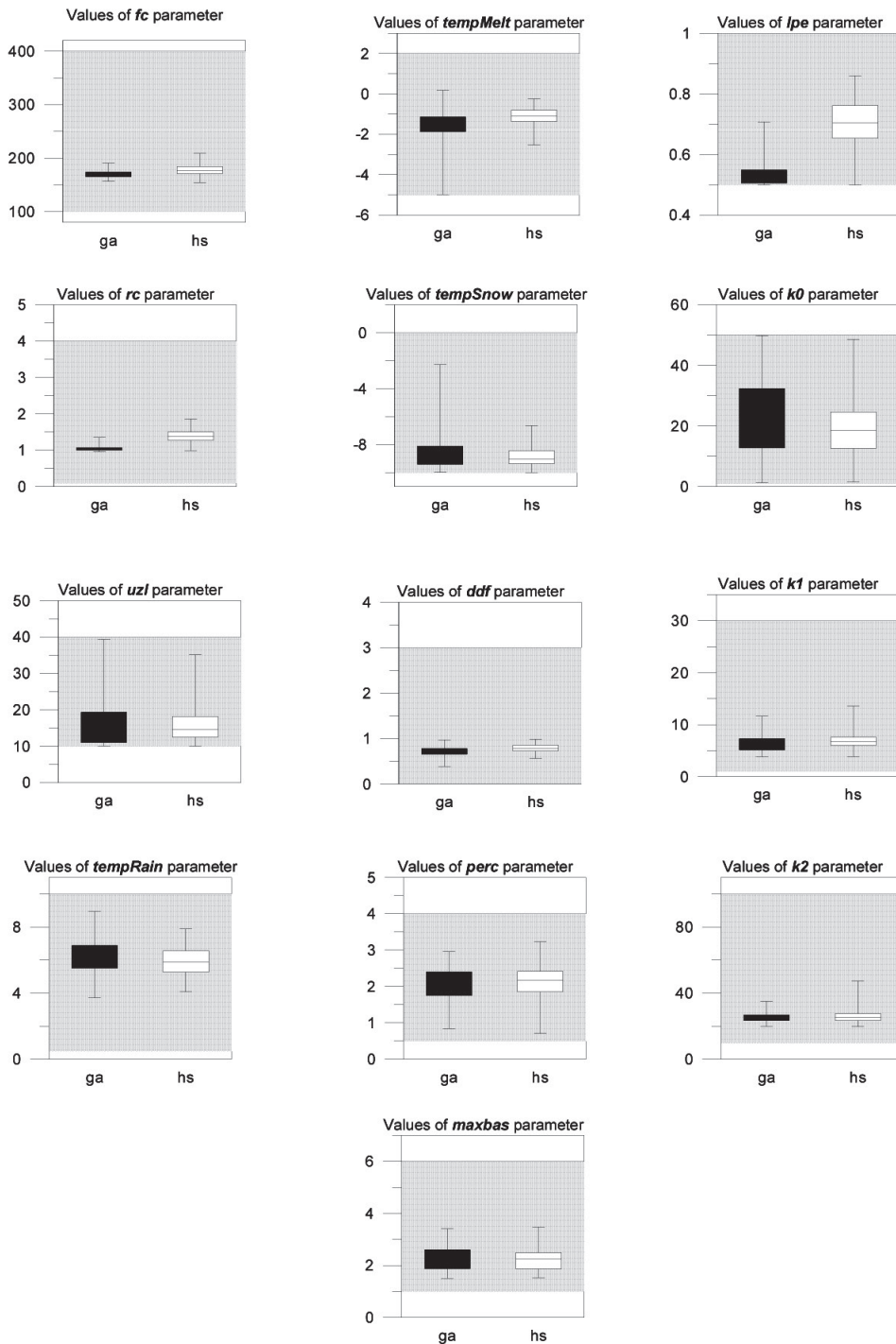


Fig. 3 Comparison of the variability of the parameters of the Hron model for GA and HS optimization algorithms after 100 calibrations of the observed data and period between 1981 and 2000.

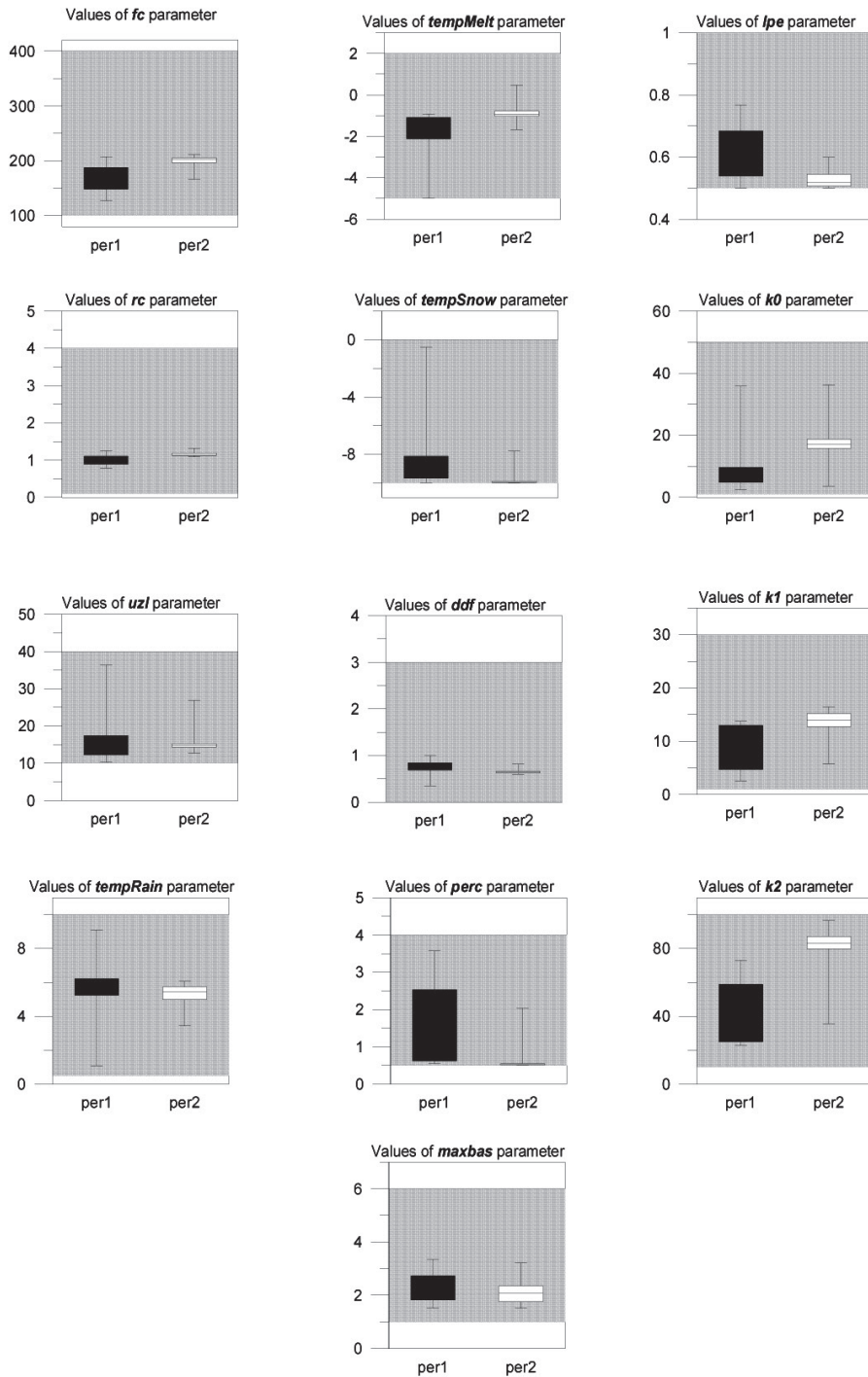


Fig. 4 The variability of the parameters of the Hron model for the GA optimization algorithm after 100 calibrations of the artificial flows. The black box plots represent the results of the calibration obtained from the first calibrations period (from 1981 to 1990) and the white from the second period (from 1990 to 2000).

Tab. 3 Summary table with the best sets of parameters obtained for the calibration of the original and generated data (for both halves of the record). The last two columns show the range in the parameter space in which a particular parameter was searched.

Parameter	Calib. Orig.	Calib per. 1	Calib. Per. 2	Lower bound.	Upper bound.
fc	162.6	164.585	162.018	100	400
rc	1.003	1.026	1.023	0	4
uzl	10.174	14.432	30.859	10	40
tempRain	7.422	7.197	7.204	1	10
tempMelt	-1.521	-1.524	-1.529	-5	2
tempSnow	-8.974	-8.765	-8.756	-10	0
ddf	0.757	0.757	0.755	0	3
perc	2.670	2.672	2.713	1	4
lpe	0.504	0.519	0.517	1	1
k0	48.56 7	27.924	18.677	1	50
k1	4.192	4.175	4.017	1	30
k2	22.798	22.808	22.897	10	100
maxbas	3	3	3	1	6
NS	0.826	0.802	0.858		

4. CONCLUSIONS

When calibrating a rainfall-runoff model most of the uncertainties come from two sources: the model itself, which is caused by the simplifications of the very complex runoff generating process, and from errors in the measured data, which are mainly caused by problems associated with the temporal and spatial variability of the measured variables. Moreover, it has been repeatedly stated in the literature that the potential for multiple acceptable models (behavioural models) as representations of hydrological and other environmental systems (the equifinality thesis) should be given more serious consideration when calibrating models than has been done until the present (e.g., Beven, 2004).

In this paper, a version of a lumped HBV model developed at the Department of Land and Water Resources Management at the Faculty of Civil Engineering of the Slovak University of Technology in Bratislava was tested in order to assess the effectiveness of currently used parameter estimation techniques.

As a case study we used the Upper Hron catchment until the Banska Bystrica station, which gave us 20 years of daily observed data between 1 January 1981 and 31 December 2000.

In the first calibration exercise the model was calibrated 100 times with the whole dataset using two calibration algorithms: Harmony Search and a Genetic Algorithm. The results of the calibration are displayed in Fig. 2. These show that both of the calibration algorithms performed similarly, with most of the calibrations achieving the values of the NS coefficient close to 0.82. Fig. 3 also shows that even though variations in the parameters may look similar in both cases, there are differences (in some cases significant) in their

range. These results can be seen as a demonstration of equifinality across the optimisation methods applied. They also show that an assessment of the uncertainty of a parameter from applying this principle may also be dependent on the particular optimisation method used and should be considered in serious model building and uncertainty assessment studies.

Since the main objective of this study was to assess the calibration procedure of the Hron model, we decided to place ourselves into ideal conditions where the measurement errors and imperfections of the model are not present. This was achieved by constructing an artificial generated time series of flows, which was then used to replace the observed flows in the Hron model's input file. The artificial time series of the simulated flows was constructed with the Hron model using the best parameter set from the first calibration exercise. In doing this we expected to achieve the same set of parameters which were used to generate the data and hoped that we could exactly reconstruct these data.

For the second calibration experiment, the model was calibrated 100 times using only the GA alternatively on two equally long halves of the generated dataset. The results of these calibrations are displayed in Fig. 4. The best sets of parameters are listed in Tab. 3., which shows the values of the NS coefficient a being very close to 1 in both calibrations.

From a theoretical point of view, the results from both datasets again exhibited equifinality and also indicate that the properties of both calibration datasets played a significant role. The values of the best parameters are very similar to those used to construct the artificial flows with the two exceptions of *k0* and *uzl* (moreover, it can be shown that these parameters are not very sensitive). However, the

variability of the two parameter sets is different: one set is closer to the ideal values and has a lower variability, and the second is less acceptable. This indicates that for successful model calibration, much more attention may need to be given to the choice of the calibration period than is generally expected. As a practical result, we could also say that the calibration procedure of the Hron model used would work satisfactorily when a conducting large number of calibrations; in general, we did not manage to calibrate the model on itself in one run.

This paper also shows some of the shortcomings of the calibration process that should be focused on in further studies and in the

development of a model. The fact that not all the model parameters are sensitive could be considered, and attempts should be undertaken to reduce the number of calibrated parameters. This could be done by doing a sensitivity analysis of all the model parameters, which will be the goal of the next paper.

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