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Evaluation of fatigue damage with an energy criterion of simple implementation

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Abstract

Many theoretical methods for multiaxial fatigue life prediction are present in literature, most of them based on their effectiveness on knowledge of the entire stress time history. This represents the great applicative limit. The incapacity to study real situations, not only deterministic one, let the authors to develop a simple and rigorous criterion, which helps the designer who works in this area. The criterion is presented focusing the attention on the basic premise, highlighting its applicability, its practicality and its computational power. To do that, the Authors take into account the deterministic or random character of the individual constraint components and their degree of correlation. In order to verify the method, simulations of multiaxial loads conditions, developed in the time domain, will be carried out with various correlation levels between the stress components on which the method will be applied.

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Keywords: Multi-Axial Fatigue; Fatigue Damage in Frequency Domain; Energy Method for Multi-Axial Fatigue; Deterministic or Random stress componet ; correlated or not-correlated stress compnent.

1. Introduction

Assessing the fatigue damage of stress on components by multiaxial phenomena is of fundamental importance for the designer today. The great variety and combinations of the stress components have made it clear that this phenomenon is extremely complex, Garud (1981). Defining the potential evolution over time of the stress tensor

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components and the correlation that these can have between them, is the starting point of this discussion. For this reason, particular multiaxial load conditions will be determined in time domains, with deterministic or random, correlated or uncorrelated components. The principal aim will be to treat the phenomenon in its most general case, as well as compare the fatigue power that these stress combinations have on mechanical components, Socie (1987). To do this, the authors will use a simple method that was widely discussed in a previous work Braccesi (2008) and that, in the present article, will be developed and expressed in its most extensive and explicit form. In this way, it will be possible to evaluate the contribution of the individual stress components and their correlation to one another. The proposed method can be included in the category of energy methods, Garud (1981), Park (2000), Elly (2007), Susmel (2013). This uses a frequency procedure to derive an alternating equivalent stress spectrum equivalent to be used as a simple monoaxial case. The use of a frequency domain procedure is useful for several reasons Bishop (1988). The first reason is its capacity to synthesize information, permitting rather reliable estimates very quickly. The second reason accounts for our inability to distinguish a stress load cycle in most real cases, Ellyin (2012), and the frequency domain allows us to properly determine useful parameters in defining equivalent stress. The critical phase of a procedure entirely developed in the domain frequency is, however, related to the determination of the fatigue damage. "Translating" the damage algorithms from the time domain into the frequency domain is, in fact, not easy to do. In the literature, there are several methods: Petrucci (2001), Benasciutti (2002), Zhao (1992), which, starting with the knowledge of the PSD signal, arrive at the value of fatigue damage. In this paper, for the estimation of damage, the Bendat (1964) method will be used to present and compare the value of damage from a series of test cases, either developed specifically by the authors or obtained from experimental tests taken from the literature, Papuga (2016). Such applications will attempt to demonstrate how the use of the proposed method is reliable; but, in particular, they will serve in the comparison of fatigue damage in a series of different multiaxial stresses. The authors would like to point out that, for this discussion, the simultaneous estimate of the alternating and mean stress distribution will be omitted, having deemed it sufficient to determine the single distribution of the amplitudes of alternating stress for the aim of the proposed work.

Nomenclature

t	time
$X(t)$	generic deterministic signal
$r(t)$	generic random signal
$\sigma_x(t)$	normal stress component
$\sigma_y(t)$	normal stress component
$\sigma_{xy}(t)$	shear stress component
$[\sigma(t)]$	stress tensor
S_{a_eq}	alternating equivalent stress frequency spectrum
f_o	frequency
S_i	frequency spectrum of the corresponding time signal $\sigma_i(t)$
$A\langle S_i\rangle$	auto-correlation operator of the signals S_i
$C\langle S_i, S_j\rangle$	cross-correlation operator of the signals S_i, S_j
$R[*]$	real operator
$k\ ,c$	coefficients of the wohler curve in the form $c = N * \sigma^k$
λ_o	zero order spectral moment
$\Gamma(*)$	gamma function
D	damage
R_m	Ultimate strength
S_n	Tensile fatigue limit
$[G]$	Cross spectral matrix
φ_x	phase-shift normal stress angle
φ_y	phase-shift normal stress angle
φ_{xy}	phase-shift shear stress angle

2. Multiaxiality and correlation of the stress components.

Generally, a multiaxial stress state is generated when multiple components of the stress tensor vary in time simultaneously. This implies a notable complication in fatigue design. In fact, in the case of multiaxiality, the mean and the alternating equivalent stresses are not easily predicted. The problems that arise from the complexity of analyses are many, though they mainly concern the trend over time of each stress component and its relationship in terms of phase-shift and amplitude.

2.1. Deterministic or random components of Stress

An $X(t)$ signal is defined as deterministic, Luise (2009) if, at any moment in time, the value of the signal is already known. In other words, it is possible to define it as such if we are aware of the function that defines its evolution over time. Sinusoidal functions are the ones that are used the most to represent components of the stress tensor in multiaxial situations. Figure 1 shows the trend of one of the normal alternating stress components $\sigma_x(t)$. This signal was taken from the tenth experimental test conducted in a multiaxial state by Heidenreich (1984) on a cylindrical 34Cr4 specimen brought to indefinite life: $1.5\ 10^6$ cycles. An interval of one second has been reported here, and from this, it is possible to highlight how the amplitude of the stress remains constant over time and equal to 380 MPa.

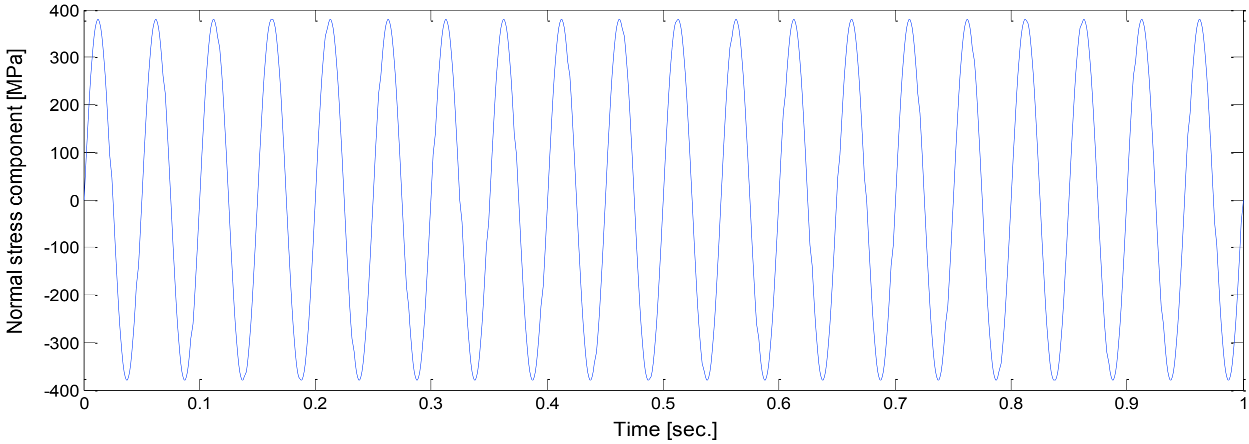

Fig.1. Deterministic sinusoidal signal

Random (or stochastic) signals $r(t)$ are of great importance as they represent, more so than deterministic signals, most real physical processes. Analyzing these types of signals is important for the study of the multiaxial phenomenon in its most general case. A signal is called random when its trend is not already known Luise (2009), the characteristic parameters of such signals can be studied, as is well known, only using the theory of probability. Figure 2 shows one portion of a random Gaussian distribution signal, with a duration of one second.

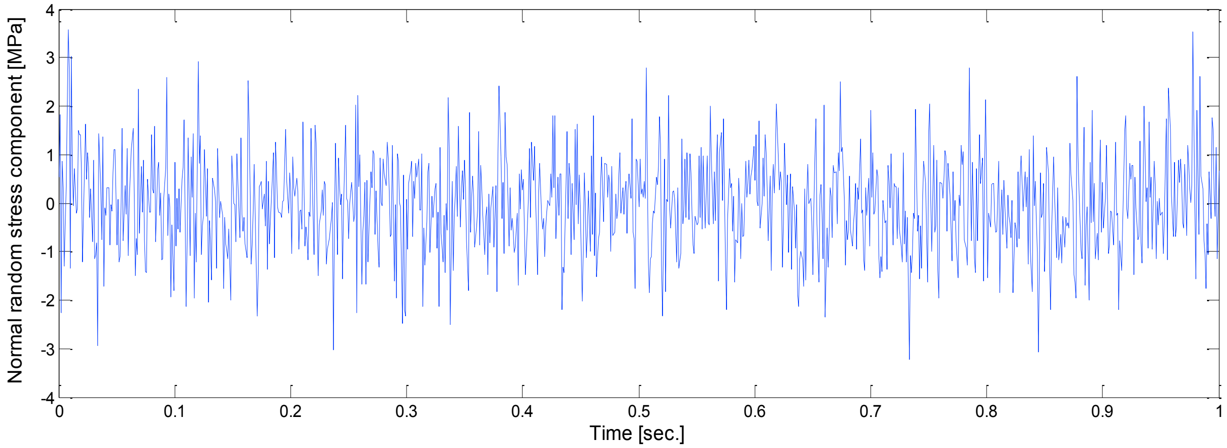

Fig.2 Random signal with Gaussian distribution

This signal could represent a large number of monoaxial stress inputs in a real system, such as the roughness of the ground input on a wheel, the vibrations of a machine tool or the pressure of a hydraulic system.

2.2. Correlation or uncorrelation between stress components

At this point, the authors consider it appropriate to define and analyze the issues that derive from the phase-shift and amplitude variation between stress components. Some or all components of the stress tensor may be correlated to each other. In other words, it is possible to affirm that, in some situations, the alternating stress components, whether deterministic or random, may differ only from multiplication or addition factors that remain constant over time. A scheme of this concept is shown in Figure 3, where blue boxes are time-constant linear transformations, also called transfer functions.

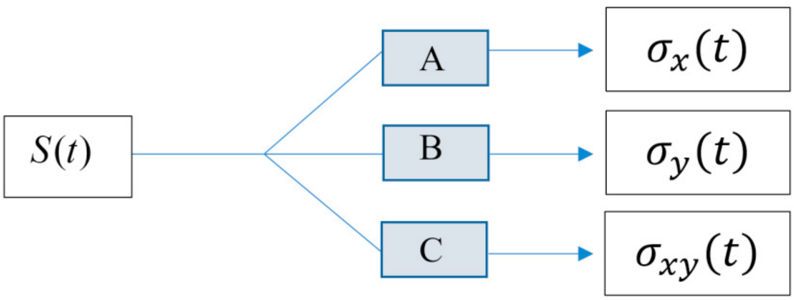

Fig 3. Conceptual scheme for the generation of correlated signals

In this generic multiaxial case, the stress tensor is composed of the three components $\sigma_x(t), \sigma_y(t), \sigma_{xy}(t)$ all correlated with one another. In mathematical terms, as well as for signal theory, two or more signals are correlated Schijve (2008) when their cross-correlation functions are not null. This statement will remain useful for analyzing future results better. Conversely, two signals are uncorrelated if their cross-correlation signal is null.

3. Definitions of some possible multiaxiality cases

It is clear from the previous chapter that additional combinations of stress can occur by varying the evolution over time of each stress component and its relationship in terms of phase-shift. The following chapter will illustrate three particular real stress cases to demonstrate how the proposed method is useful in the analysis of the influence that this variability has on fatigue damage.

3.1. Multiaxial stress with correlated sinusoidal deterministic components.

Assuming a triaxial stress state composed of two normal stress components $\sigma_x(t)$, $\sigma_y(t)$ and a shear one $\sigma_{xy}(t)$, each represents a pure alternating stress. By definition, these sinusoidal deterministic components are always correlated to one another. In reality, it is very difficult for alternating stress components to assume a deterministic form without any approximation. One of the significant applications of sinusoidal signals, however, is the one related to multiaxial fatigue experimental tests. In this academic case, the alternating stress components are actually sinusoidal. The three alternating components of the same experimental multiaxial fatigue test carried out by Troost (1987) at a frequency $f_o = 20$ [Hz], will be taken as representative of this load condition. Figure 4 shows the signal for a duration of one second.

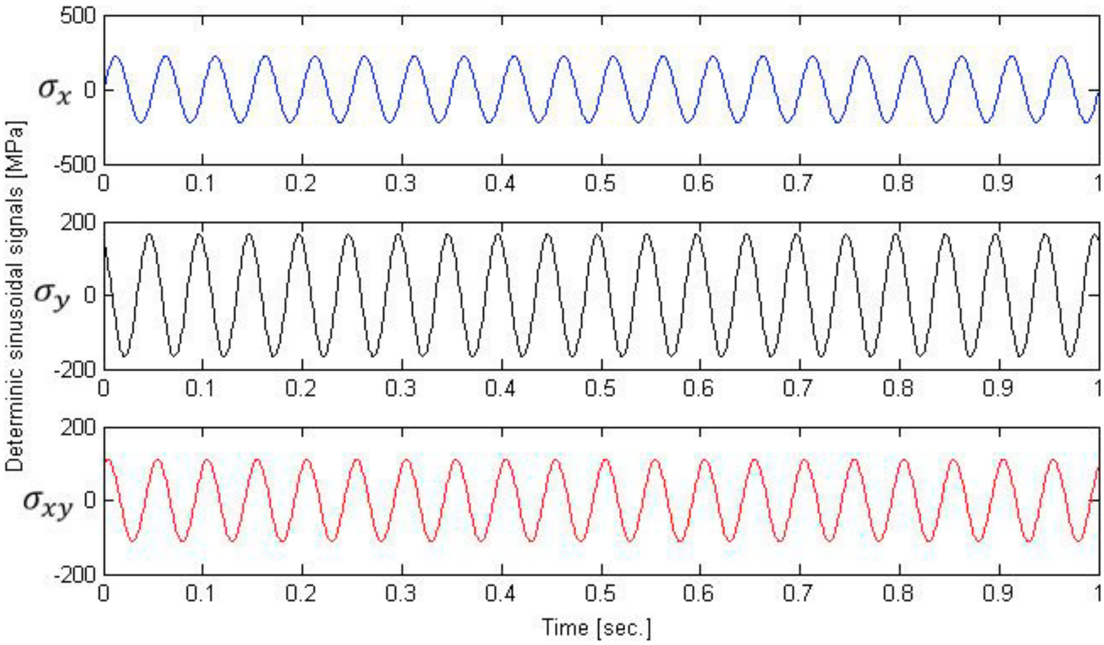

Fig.4. Sinusoidal deterministic components of the multiaxial stress.

As described in Table 1, the test was performed using a stochastic method with 15 cylindrical specimens of 25CrMo4 material having $R_m = 801$ [MPa] e $S_n = 340$ [MPa]. These specimens, subjected to a triaxial sinusoidal stress state, with constant amplitudes and phases over time, led to an indefinite lifetime. Table 1 shows the RMS values for each stress component as representative of the fatigue power of the single stress.

Table 1. Multiaxial stress with correlated sinusoidal deterministic components

Material 25CrMo4		σ_x	σ_y	σ_{xy}
Amplitude	[MPa]	A=222.1	0.75 A	0.5 A
Phase-shift	[degree]	0	+90	+45
RMS	[MPa]	156.97	117.839	78.542

From the elements provided, it is possible to obtain the Wöhler curve, Schott (1996), of the material in the form $c = N * \sigma^k$, and these parameters will be useful later in the discussion.

3.2. Multiaxial stress with random correlated stress components

A multiaxial state, with random correlated components, is a representative model of many real situations. In this case, the stress components could be seen as outputs of a real system, the input of which is represented by a single random signal. Therefore, we presume a random signal $r(t)$ statistically defined by a zero mean and Gaussian distribution of the amplitudes. This signal, entering any mechanical system, will be filtered and amplified on the frequencies of this system and will break out in the various components of stress. For the sake of simplicity, it is possible to assume that these random stress tensor components, assuming that there are three $(\sigma_x, \sigma_y, \sigma_{xy})$, have the same statistical characteristics as the input random signal: they will all be monomodal signals at the same frequency $f_o = 20$ [Hz] (because they are filtered by the same mechanical system) and will maintain a constant relationship between the amplitudes and the phases. An example may be seen in Figure 5, which shows part of the temporal trend of the three hypothetical components of the stress: two normal stresses $\sigma_x(t)$, $\sigma_y(t)$, and a shear one $\sigma_{xy}(t)$

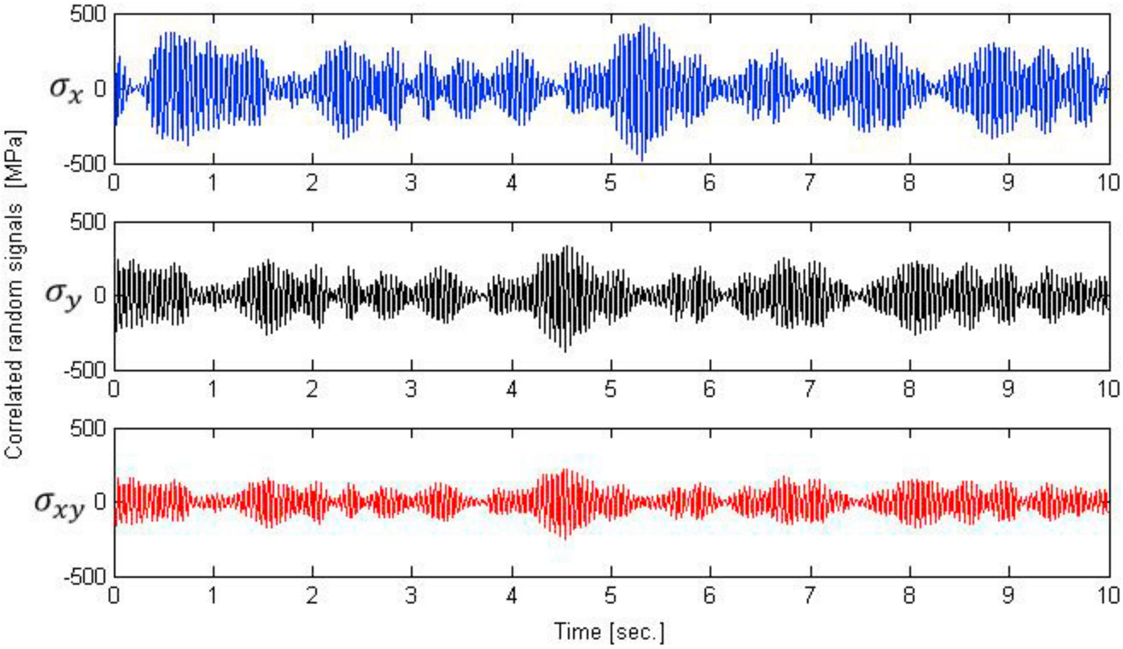

Figure 5 Random correlated components of the multiaxial stress.

As can be seen from Table 2, in order to be able to compare the results obtained from different types of stresses in the best possible way, this multiaxial random stress state will:

- Be applied on the same specimen of 25CrMo4 material used in sinusoidal experimental testing and will thus have the same Wöhler curve.
- Have amplitudes of the normal component σ_x random, but will keep in constant proportion the time of the other amplitudes of two signals with a factor equal to that used for the three components in the sinusoidal stress.
- Have three components with a phase-shift equal to the previous sinusoidal cases.
- Have RMS of each component equal to the corresponding sinusoidal signals.

Table 2. Multiaxial stress with correlated random components

Material 25CrMo4		σ_x	σ_y	σ_{xy}
Amplitude	[MPa]	A random	0.75 A	0.5 A

Phase-shift	[degree]	0	+90	+45
RMS	[MPa]	156.97	117.839	78.542

3.3. Multiaxial stress with random uncorrelated stress components

In this case, real applications are difficult to find, but they represent a limiting state of considerable academic interest. Each of the three components of multiaxial random stress $(\sigma_x, \sigma_y, \sigma_{xy})$ in Figure 6 are derived from three different random signals. It is possible to assume that three random stresses which are input in the mechanical system, contribute separately to defining the time functions of the normal and shear components of the stress tensor. The ratio between the amplitudes and the ratio between the phase-shift of the corresponding stress components in this case, are random. However, even in this case, if we exit the same system, each signal of the stress component is monomodal (at the same frequency $f_o = 20$ [Hz]). Statistically, each of the $(\sigma_x, \sigma_y, \sigma_{xy})$ components, whose trend is shown in Figure 6, will be defined by a zero mean and Gaussian distribution of the amplitudes as in the previous case with correlated signals.

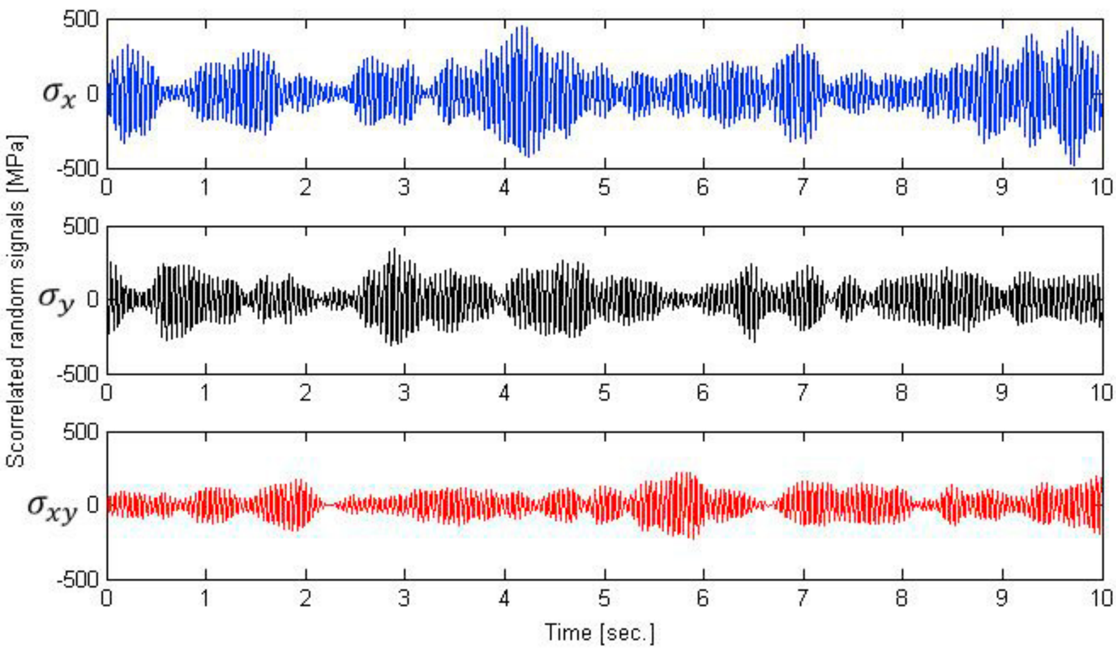

Fig. 6. Multiaxial stress with random uncorrelated stress components.

As can be seen from Table 3, random signals were created by maintaining the same RMS of the deterministic sinusoidal signals in this situation so as to compare them with other stress types at a later time.

Table 3. Multiaxial stress with uncorrelated random components

Material 25CrMo4		σ_x	σ_y	σ_{xy}
Amplitude	[MPa]	random	random	random
Phase-shift	[degree]	random	random	random
RMS	[MPa]	156.97	117.839	78.542

4. The model for multiaxial fatigue

4.1. Determination of the equivalent alternating stress

The variety, and in particular, the complexity of the signal types introduced in the previous paragraph, have led the authors to develop a method that is fast and simple to use for the multiple combinations of stress signals that can occur in multiaxial cases. The authors' criterion, developed from their previous work, Braccesi (2008), is included in that category of methods that evaluate the damage of a component subjected to multiaxial fatigue stress using the Energy Density Thomas (1999). This simple criterion, developed in the frequency domain, reduces a multiaxial stress state to a monoaxial equivalent. The result is, in fact, the defining of an alternating stress spectrum S_{a_eq} that can be used as a monoaxial one. It is reasonable to imagine having a generic single-mode multiaxial stress state, and it is therefore possible to define the corresponding stress tensor $[\sigma(t)]$ as follows:

$$[\sigma(t)] = \begin{bmatrix} \sigma_x(t) & \sigma_{xy}(t) & \sigma_{xz}(t) \\ \sigma_{yx}(t) & \sigma_y(t) & \sigma_{yz}(t) \\ \sigma_{zx}(t) & \sigma_{zy}(t) & \sigma_z(t) \end{bmatrix} \tag{1}$$

In the transition to the frequency domain (FFT) of the individual stress component, and using f_o for the working frequency, it is possible to affirm that in the pure alternating multiaxial state, as defined above, the cross-spectral matrix, Munier (1991) can be associated:

$$[G] = \begin{bmatrix} \begin{bmatrix} A\langle S_x\rangle & C\langle S_{x,}S_y\rangle & C\langle S_{x,}S_z\rangle \\ C\langle S_{y,}S_x\rangle & A\langle S_y\rangle & C\langle S_{y,}S_z\rangle \\ C\langle S_{z,}S_x\rangle & C\langle S_{z,}S_y\rangle & A\langle S_z\rangle \end{bmatrix} & \begin{bmatrix} C\langle S_x, S_{xy}\rangle & C\langle S_{x,}S_{xz}\rangle & C\langle S_{x,}S_{yz}\rangle \\ C\langle S_{y,}S_{xy}\rangle & C\langle S_y, S_{xz}\rangle & C\langle S_{y,}S_{yz}\rangle \\ C\langle S_{z,}S_{xy}\rangle & C\langle S_{z,}S_{xz}\rangle & C\langle S_z, S_{yz}\rangle \end{bmatrix} \\ \begin{bmatrix} C\langle S_{xy}, S_x\rangle & C\langle S_{xy}, S_y\rangle & C\langle S_{xy}, S_z\rangle \\ C\langle S_{xz}, S_x\rangle & C\langle S_{xz}, S_y\rangle & C\langle S_{xz}, S_z\rangle \\ C\langle S_{yz}, S_x\rangle & C\langle S_{yz}, S_y\rangle & C\langle S_{yz}, S_z\rangle \end{bmatrix} & \begin{bmatrix} A\langle S_{xy}\rangle & C\langle S_{xy,}S_{xz}\rangle & C\langle S_{xy,}S_{yz}\rangle \\ C\langle S_{xz,}S_{xy}\rangle & A\langle S_{xz}\rangle & C\langle S_{xz,}S_{yz}\rangle \\ C\langle S_{yz,}S_{xy}\rangle & C\langle S_{yz,}S_{xz}\rangle & A\langle S_{yz}\rangle \end{bmatrix} \end{bmatrix} \tag{2}$$

In formula (2), the generic value S_i represents the frequency spectrum of the corresponding time signal $\sigma_i(t)$. The operator $A\langle S_i\rangle$ represents the spectral auto-correlation of the signal S_i with itself, and the operator $C\langle S_{i,}S_j\rangle$ instead indicates the spectral cross-correlation of the two different signals S_i, S_j. The latter (as is well known), applied on non-real signals generates values that are not real. At this point, it will be possible to develop the formula of S_{a_eq} which authors had concluded in their previous work to arrive at an alternating stress spectrum equal to:

$$S_{a\_eq} = \sqrt{trace([R] * [G] * [R]^T)} \tag{2}$$

In this expression, [R] is a matrix of constant terms and $[R]^T$ corresponds to its transposed and conjugated form. By developing this expression, we obtain the explicit formulation of S_{a_eq} which, as shown below, has an analogous form from Von Mises (1913) definition.

$$S_{a\_eq} = \sqrt{A\langle S_x\rangle + A\langle S_y\rangle + A\langle S_z\rangle - \{R[C\langle S_{x,}S_y\rangle] + R[C\langle S_{x,}S_z\rangle] + R[C\langle S_{y,}S_z\rangle]\} + 3 * \{A\langle S_{xy}\rangle + A\langle S_{xz}\rangle + A\langle S_{yz}\rangle\}} \tag{3}$$

The operator $R[C\langle S_{i,}S_j\rangle]$reports the real value of the cross-correlation of the two signals. At this point it is helpful to analyze the contribution that the individual parts of the equation (3) make to the definition of S_{a_eq}. In the first analysis, it is evident that the presence of shear and normal stress always makes a positive contribution. In other words, if these increase, they will have an increase of the S_{a_eq} value. It should be noted, however, that the presence of shear stress weighs substantially more than normal stress. For the cross-correlation terms, it is possible to affirm that: if those that derive from shear stresses make no contribution (and are not present in the formula (3)), those derived from the correlation of normal stress could make negative, positive or null contributions; it depends on the phase-shift value between the normal stress components. More precisely, the cross-correlation function is proportional to the cosine phase-shift angle between the two involved signals. Simply, the formula can be written as: $CC = -\{R[C\langle S_{x,}S_y\rangle]\}$:

$$\begin{cases} CC < 0 & se \quad -90° < \varphi < +90° \\ CC = 0 & se \quad \varphi = +90° \ \vee \ \varphi = +270° \\ CC > 0 & se \quad +90° < \varphi < +270° \end{cases} \tag{4}$$

4.2. Determination of the fatigue damage

To date, in the literature, there are several methodologies. First of all, it is Bendat (1964) who, under the assumption of pure alternating and single-mode stress signals (i.e. having Gaussian FFT, PSD concentrating on single frequency and PDF distributed as a Rayleg function) provides a valid equation for the evaluation of fatigue damage per time unit both in the presence of random and deterministic stress cases. In particular, the Bendat formula can be expressed as follows:

$$D_t = \left(\frac{f_o}{c} * 2^{\frac{k}{2}} * \lambda_o{}^{\frac{k}{2}}\right) * \Gamma\left(\frac{k}{2}+1\right) \tag{5}$$

In it, the coefficients k and c derive from the material characteristics and are attributable to the coefficients of the Wöhler curve expressed in the $c = N * \sigma^k$ form. In the equation, the first factor is also a function of the working frequency f_o and of the spectral zero order moment λ_o of the PSD signal which is the area underlying the PSD function, Lori (2003). The second factor, on the other hand, is represented by the gamma function $\Gamma(*)$, Abramowitz (1964), which accounts for the uncertainty of the phenomenon. In fact, this term, which derives from the presence of a signal pdf, disappears for deterministic stresses. The gamma function, however, has no-zero values for random stresses, and in this way, the Bendat formula takes into account the random character of the phenomenon.

5. Applicability of the method and valuation

In this chapter, the method will be used to estimate fatigue damage in the case that the same component is subject to different multiaxial load situations. In this way, we will try not only to verify its applicability, but above all, we will highlight how the nature and phase-shift of the different stress components affect fatigue damage.

5.1. Analysis of the sinusoidal deterministic components of stress

Table 4 summarizes the values, in terms of S_{a_eq} and damage, that emerge from several cases of multiaxial stress by the variation of the phase-shift angle between the deterministic sinusoidal components. In particular, the first case of multiaxiality refers to the experimental test introduced in the second chapter. The successive ones, generated from this, will leave the values of the amplitudes, and therefore the RMS signals, unchanged, but will vary the phase-shift angle between the components of the stress.

Table 4 Phase-shift of the sinusoidal deterministic stress components influence on the fatigue damage.

Material 25CrMo4	σ_x Amplitude fix [MPa]	σ_y Amplitude fix [MPa]	σ_{xy} Amplitude fix [MPa]	φ_x [deg]	φ_y [deg]	φ_{xy} [deg]	CC	S_{a_eq} [MPa]	Γ	D
Case 1	A=222.1	0.75 A	0.5 A	0	+90	+45	0	337.76	0	0.96
Case 2	A=222.1	0.75 A	0.5 A	0	+90	∀	0	337.76	0	0.96
Case 3	A=222.1	0.75 A	0.5 A	0	+40	∀	$-1.42\ 10^4$	292.80	0	0.38
Case 4	A=222.1	0.75 A	0.5 A	0	0	∀	$-1.85\ 10^4$	277.63	0	0.27
Case 5	A=222.1	0.75 A	0.5 A	0	-60	∀	$-0.93\ 10^4$	309.16	0	0.54
Case 6	A=222.1	0.75 A	0.5 A	0	-90	∀	0	337.76	0	0.96
Case 7	A=222.1	0.75 A	0.5 A	0	+130	∀	$+1.19\ 10^4$	371.30	0	1.76
Case 8	A=222.1	0.75 A	0.5 A	0	+180	∀	$+1.85\ 10^4$	388.69	0	2.36
Case 9	A=222.1	0.75 A	0.5 A	0	+220	∀	$+1.42\ 10^4$	377.39	0	1.95

From the results of the first load condition (experimental case), compared with the characteristics of the material described in paragraph 3.1, it can be deduced that the method developed by the authors correctly evaluates the value of S_{a_eq} as a consequence of the material damage. Therefore, in making this experimental test a reference, it may be seen from case 2, and for those that follow, that any change of the φ_{xy} shear stress angle does not affect the damage (confirmation of this is shown in chapter 4). The variation of the value of S_{a_eq} is, in this case, only due to the change of the angle φ_y.

5.2. Analysis of the correlated random components of stress

Assuming that the same material is used (thus keeping the Wöhler curve constant), Table 5 shows the results of several multiaxial stress cases generated from random correlated stress components. The first of these is the one discussed in the second chapter. It can be seen, even in this case, that any change of the phase-shift of the shear stress components does not correspond to any variation in S_{a_eq} or damage: further confirmation that a phase variation of the shear components does not cause any damage. However, if we want to make a direct comparison with sinusoidal evaluations, the same S_{a_eq} value does not correspond to the same damage. This is mainly due to the presence of the gamma factor $\Gamma\left(\frac{k}{2}+1\right)$, the value of which is reported in Table 5. This means that in the same RMS condition and same phase-shift between the corresponding components, random stress with correlated components has a power fatigue damage greater than the sinusoidal multiaxial stress state.

Table 5 Phase-shift of the random correlated stress components influence on the fatigue damage.

Material 25CrMo4	σ_x Amplitude random [MPa]	σ_y Amplitude [MPa]	σ_{xy} Amplitude [MPa]	φ_x [deg]	φ_y [deg]	φ_{xy} [deg]	CC	S_{a_eq} [MPa]	Γ	D
Case 10	A	0.75 A	0.5 A	0	+90	+45	0	336.12	7.53	7.22
Case 11	A	0.75 A	0.5 A	0	+90	∀	0	336.12	7.53	7.22
Case 12	A	0.75 A	0.5 A	0	+40	∀	$-1.41\ 10^4$	293.35	7.53	3.02
Case 13	A	0.75 A	0.5 A	0	0	∀	$-1.85\ 10^4$	277.63	7.53	2.03
Case 14	A	0.75 A	0.5 A	0	-60	∀	$-0.86\ 10^4$	311.38	7.53	4.43
Case 15	A	0.75 A	0.5 A	0	-90	∀	0	335.90	7.53	7.19
Case 16	A	0.75 A	0.5 A	0	+130	∀	$+1.23\ 10^4$	372.28	7.53	13.89
Case 17	A	0.75 A	0.5 A	0	+180	∀	$+1.82\ 10^4$	388.13	7.53	18.14
Case 18	A	0.75 A	0.5 A	0	+220	∀	$+1.42\ 10^4$	377.43	7.53	15.17

5.3. Analysis of the random uncorrelated components of stress

A final analysis was performed on random uncorrelated stress components. Again, in this case, the RMS values of the stress components are the same as those generated for the corresponding sinusoidal components. The randomness of the phenomenon leads to the definition of S_{a_eq} equal to those in cases 11 or 15 reported in Table 5.

Table 6 Analysis on random uncorrelated stress components.

Material 25CrMo4	σ_x Amplitude random [MPa]	σ_y Amplitude [MPa]	σ_{xy} Amplitude [MPa]	φ_x [deg]	φ_y [deg]	φ_{xy} [deg]	CC	S_{a_eq} [MPa]	Γ	D
Caso 19	rand	rand	rand	rand	rand	rand	0	338.12	7.53	7.28

6. Conclusion

At the end of this work, it can be stated that when using the proposed method, we are now able to evaluate the influence that the typology and the phase-shift of the individual stress tensor components have on the fatigue damage. In particular, Figure 7 shows and compares, changing the phase-shift angle between the normal stress components φ_y, the damage values for three multiaxial load conditions: with related sinusoidal deterministic components and with correlated or uncorrelated random components.

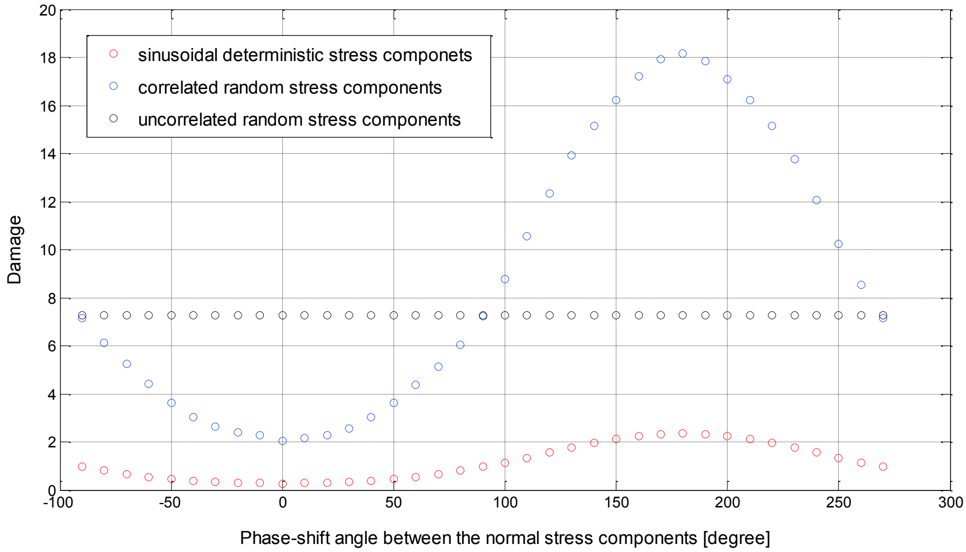

Fig. 7. Correlation of damage with the variation of φ_y.

Under the same RMS, the damage trend is the same when comparing correlated cases. The influence of the random correlated components amplifies the damage according to the gamma factor. For uncorrelated random components, it is possible to affirm that under the same RMS value of the corresponding stress components, damage will always be constant and equal to the damage value reached when random correlated components have phase-shift equal to ± 90° between the normal stress components.

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