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Recent Developments on Sliding Mode Control of Complex Dynamical Systems - Research Article

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Stabilization computation for a kind of uncertain switched systems using non-fragile sliding mode observer method

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Abstract

A non-fragile sliding mode control problem will be investigated in this article. The problem focuses on a kind of uncertain switched singular time-delay systems in which the state is not available. First, according to the designed non-fragile observer, we will construct an integral-type sliding surface, in which the estimated unmeasured state is used. Second, we synthesize a sliding mode controller. The reachability of the specified sliding surface could be proved by this sliding mode controller in a finite time. Moreover, linear matrix inequality conditions will be developed to check the exponential admissibility of the sliding mode dynamics. After that, the gain matrices designed will be given along with it. Finally, some numerical result will be provided, and the result can be used to prove the effectiveness of the method.

Keywords

Sliding mode control, singular systems, non-fragile observer, time delay

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Introduction

Switched systems have received more and more attention because of its convenience to model lot of real-world systems, for example, chemical process systems, communication systems, and transportation systems. And switched systems consist of a family of subsystems. These subsystems are governed by a switching signal.[1] In fact, many fundamental results have been developed not only for switched systems of continuous-time but also for switched systems of discrete time.[2–6] Furthermore, singular systems also have played a significant role in practical application. Recently, more and more attentions have been given to the research of stability analysis and stabilization for switched singular systems. As in the previous studies,[7–9] the robust exponential admissibility problem is investigated. The problem is just for a kind of continuous-time uncertain switched singular systems in which the delay is interval time-varying; and the study by Zhou et al.,[10] which focused on switched linear continuous-time singular systems, addressed the stability analysis of the systems. For more references, readers can refer to previous literature.[11–14]

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Since its appearance, sliding mode control (SMC) has obtained increasing attention in practical applications for its good property of strong robustness which has a perfect performance against model uncertainties (including external disturbances and parameter variations), good order reduction and transient response in Choi.[15] In the past few years, researcher has put SMC into those application areas like robot control, electrical motors, and underwater vehicles. As to SMC of switched systems, also there exist relative works, such as the study by Wu et al.,[16] in which SMC of switched systems with stochastic perturbation is concerned and reduced-order sliding mode dynamics is derived; In the literatures,[17–19] SMC of switched delay systems with nonlinear perturbations has been studied by average dwell-time method. But notice that few works draw attention to SMC of switched singular systems, so the research on switched singular systems' SMC continues. Recently, in previous studies,[20–23] a weighted sum method with the input matrices has been applied for considering the uncertain switched systems' SMC, and one common sliding surface has been designed even though different input matrices exist in the subsystems. Motivated by this method and the fact mentioned aforehand, in this research, we will devote to discuss the SMC problem for a kind of switched singular systems.

However, the ideal assumption on the exact knowledge of the system state components does not always set up in real-world systems. In this case, it is impossible to realize the stability analysis and the implementation of controllers. And this aroused the issue of observer design. However, the variations in engineering application demonstrated are usually unavoidable. This may make the control systems' performance worse, or even make the systems to be instable. And this leads to an issue that non-fragile control has obtained much attention, and lot of constructive results have been developed with regard to non-fragile control of dynamic systems.[24–28]

With regard to above analysis, this article concerns the non-fragile SMC problem which focuses on a kind of uncertain switched singular time-delay systems, in which state is not available. First, a non-fragile observer has been designed for obtaining the estimated state components. Then by using the introduced weighted sum approach, an integral-type sliding surface will be developed with common input matrix even though each subsystem has different input matrices. Furthermore, this article synthesizes an observer-based SMC law which can prove the reachability of the specified sliding surface within a limited time. Moreover, the average dwell time algorithm will gain the sufficient conditions on the exponential admissibility of the sliding mode dynamics. The contributions are listed as follows: (1) to better accommodate the variability of dynamical systems, no constraints on the full column rank of input matrices are imposed on switching systems; (2) dynamics are often affected by nonlinearities, and the observer-based compensator is designed to attenuate the influence for stabilization purpose; and (3) the exponentially stable ability of singular switching system is achieved by solving the typical minimization problems on the observer space.

The rest of this article is structured as follows. In the second section, problem statement and preliminaries will be given. In next section, the steps of SMC method are realized by the proposed non-fragile observer, and linear matrix inequality (LMI) conditions are obtained to prove the exponential admissibility of the sliding mode dynamics. An illustrative example will be offered for demonstrating the effectiveness of the proposed approaches in the fourth section. Some conclusions will be given in last section.

Notations

Let matrices have compatible dimensions if their dimensions are not point out clearly. Let R^n stand for the -dimensional real space; let $R^{m\times n}$ be the $m\times n$ real matrix space; and let $\|\cdot\|$ be its induced matrix norm or the Euclidean norm of a vector. Given any real symmetric matrix, let $M>0(<0)$ denote that the matrix is positive (negative) definite. Give a vector $x=[x_1x_2\cdots x_n]^T$, let x^T be its transpose and let $sgn(x)$ be $[sgn(x_1)sgn(x_2)\cdots sgn(x_n)]^T$. Let I denote an identity matrix, let diag $(\cdot)$ to be a diagonal matrix, let the vector $1_n\in R^n$ is the one whose element is all ones, and let $e_i\in R^n$ to be the vector which is ith standard base. For a real symmetric matrix, let $\lambda_{max}(\cdot)$ to be the maximum eigenvalue and let $\lambda_{min}(\cdot)$ to denote the minimum eigenvalue. Given rank $(\cdot)$ to present the rank of a matrix, let $*$ stand for a part in a matrix which is induced by symmetry and let $\otimes$ to be the Kronecker product.

Problem statement and preliminaries

Give the uncertain switched singular systems as follows

$$\begin{cases} E\dot{x}(t)=(A_\sigma+\Delta A_\sigma))x(t)+(A_{\tau\sigma}+\Delta A_{\tau\sigma})x(t-\tau) \\ \qquad\quad +B_\sigma(u(t)+f_\sigma(x(t))) \\ y(t)=C_\sigma x(t), \\ x(s)=\varphi(t),\quad t\in[-\tau,0] \end{cases} \tag{1}$$

where $u(t)\in R^m$ is the control input, $x(t)\in R^n$ stands for the state, and $y(t)\in R^p$ stands for the system output. Let us assume that the rank$(E)=r\leqslant n$. $\{A_\sigma,A_{\tau\sigma},B_\sigma,C_\sigma:\sigma\in\Gamma\}$ stands for a family of known matrices and ΔA_σ and $\Delta A_{\tau\sigma}$ are the parameter uncertainties; all of them depend on the $\Gamma=\{1,2,\ldots,s\}$ which is a index set. For time t, let $\sigma(t):R\rightarrow\Gamma$ to be a piecewise constant function, which is said to be switching signal. And the admissible uncertainties $\Delta A_i, \Delta A_{\tau i}$ satisfy the following

$$[\Delta A_i \quad \Delta A_{\tau i}] = M_{red1i}F_{1i}(t)[H_{1i} \quad H_{2i}]$$

where M_{1i}, H_{1i}, and H_{2i} are constant matrices known before. $F_{1i}(t)$ stands for an unknown matrix function which satisfies $F_{1i}(t)^T F_{1i}(t) \leqslant I$. Moreover, it is assumed that the external disturbance $f_i(x(t))$ has a character of norm bounded, which means that $\| f_i(x(t)) \| \leqslant d_i$, where d_i is a positive scalar. Without lose of generality property, the matrix B_σ is assumed to be full column rank, which means that rank $(B_i) = m$.

In the sequel, let the parameters which are linked with the ith subsystem to be $A_\sigma \triangleq A_i, A_{\tau\sigma} \triangleq A_{\tau i}$, $B_\sigma \triangleq B_i, \Delta A_\sigma \triangleq \Delta A_i, \Delta A_{\tau\sigma} \triangleq \Delta A_{\tau i}, f_\sigma(x(t)) \triangleq f(x(t)), C_\sigma \triangleq C_i$, for each possible value $\sigma(t) = i, i \in \Gamma$. And the switching sequence $\{(i_o, t_o), (i_1, t_1), \ldots, (i_N, t_N) \,|i_k \in \Gamma\}$, in accordance with the switching signal $\sigma(t) = i_k$, intends that the i_kth subsystem is activated as $t \in [t_k, t_{k+1})$.

The content of this article could consist the following: non-fragile observer-based method is used to conduct the SMC problem of system (1). Then, exponential admissibility of the sliding mode dynamics will be guaranteed. Finally, the lemma and definition described as follows are introduced in this article.

Lemma 1. The H, D, and $F(t)$ stand for real matrices.[29] And let $F(t)$ has the character to make $F^T(t)F(t) \leqslant I$ set up. After that, for every $\epsilon > 0$, one can obtain

$$DF(t)H + H^T F^T(t) D^T \leqslant \epsilon^{-1} DD^T + \epsilon H^T H$$

Definition 1. Given any $T_2 > T_1 > 0$, we denote N_σ to be the switching number of $\sigma(t)$ among (T_1, T_2).[30] If

$$N_\sigma \leqslant N_0 + \frac{(T_2 - T_1)}{T_\sigma}$$

for $N_0 \geqslant 0$ and $T_\sigma > 0$, one can call T_σ to be the average dwell time. We let N_0 equal 0. This is widely applied in the former research.

Definition 2

1. Given any $i \in \Gamma$, when the pair (E, A_i) is impulse free and regular, we can say the following

$$E\dot{x}(t) = A_i x(t) + A_{\tau i} x(t - \tau) \tag{2}$$

to be impulse free and regular.

2. Given any scalars $\lambda > 0$ and $\rho \geqslant 1$, if the following

$$\| x(t) \| \leqslant \rho \| x(t_0) \| e^{-\lambda(t - t_0)}, \ \forall t \geqslant t_0$$

has a solution $x(t)$, then one can say that the equilibrium x^* equals 0 of system (2) is to be exponentially stable.

3. If singular switching system (2) is impulse free, regular, and exponentially stable, after that one can call it to be exponentially admissible.

Non-fragile observer-based SMC

In this part, first, we will design a non-fragile observer. Then, the first step of SMC is considered, that is, an integral-type sliding surface will be introduced, which is based on estimated state; moreover, in the second step of SMC, we synthesize an SMC law. This law has the ability to put the state trajectories arrive at the sliding surface predefined within limited time. Finally, the exponential admissibility of the sliding mode dynamics will be studied.

Non-fragile observer

First of all, we propose the non-fragile observer for equation (1) to be the formulation as follows

$$\begin{aligned} E\dot{\hat{x}} = {} & A_i\hat{x}(t) + A_{\tau i}\hat{x}(t - \tau) + B_i(u(t) + v(t)) \\ & + B_i(L_i + \Delta L_i)(y(t) - C_i\hat{x}(t)) \end{aligned} \tag{3}$$

where $\hat{x}(t)$ stands for the estimation of the state $x(t)$, external discontinuous feedback compensation control is presented by the robust term $v(t)$, which has a good ability to eliminate the effect of $f_i(x(t))$ that is the nonlinear term, and L_i is the observer gain to be determined. ΔL_i is a perturbed matrix, which is bounded in the following form

$$\Delta L_i = M_{2i} F_{2i}(t) H_{3i}$$

where $F_{2i}(t)$ is presented as an matrix which is time-varying, unknown, and satisfies $redF_{2i}^T(t)F_{2i}(t) \leqslant I$, and M_{2i} and H_{3i} stand for constant matrices.

Let $e(t) = x(t) - \hat{x}(t)$, we have the estimation error system as follows

$$\begin{aligned} E\dot{e}(t) = {} & (\bar{A}_{li} + \Delta A_i - B_i \Delta L_i) C_i e(t) + \Delta A_i \hat{x}(t) \\ & + (A_{\tau i} + \Delta A_{\tau i}) e(t - \tau) + \Delta A_{\tau i} \hat{x}(t - \tau) \\ & + B_i(f_i(x(t)) - v_i(t)) \end{aligned} \tag{4}$$

where $\bar{A}_{li} = A_i - B_i L_i C_i$.

Sliding surface design

The sliding mode function will be introduced as the following formulation

$$s(t) = G_i E\hat{x}(t) + G_i B \int_{t_0}^{t} K\hat{x}(\tau)d\tau \tag{5}$$

where G_i is selected to satisfy G_iB_i, which is nonsingular for every $i \in \Gamma$; the matrix K is introduced later in this article. Here, B is introduced as in Wang et al.[13] and described by

$$B \triangleq \sum_{i=1}^{n} \alpha_i B_i$$

where α_i stands for a parameter which satisfies $\underline{\alpha} \leqslant \alpha_i \leqslant \bar{\alpha}, i = 1, 2, \ldots, s$, in which $\underline{\alpha}$ and $\bar{\alpha}$ both are known scalars. Then, let

$$M \triangleq \tfrac{1}{2}[B - s\alpha_1 B_1 \quad B - s\alpha_2 B_2 \cdots B - s\alpha_n B_s]$$

$$V(i) \triangleq (I_s - 2e_ie_i^T) \otimes I_s, N \triangleq 1_s \otimes I_m$$

$$\beta \triangleq \max\{|s\underline{\alpha} - 1|, |s\bar{\alpha} - 1|\} \cdot \max_{1 \leqslant i \leqslant s}\{\| B_i \|\}$$

$$\mathcal{V}(i) \triangleq \begin{bmatrix} V(i) & 0 \\ 0 & \frac{1}{\beta}(1 - s\alpha_i)B_i \end{bmatrix}$$

$$\mathcal{M} \triangleq [M \quad \beta I_n], \mathcal{N} \triangleq \begin{bmatrix} N \\ I_m \end{bmatrix}$$

And we have that $B_i = B + \mathcal{M}\mathcal{V}(i)\mathcal{N}$ and $\mathcal{V}(i) \leqslant 1$.

Remark 1. For the introduced matrix B, in each subsystem, it is not necessary that the B_i which is input matrix is the same. And since B_i has full column rank, the assumption of G_iB_i is nonsingular could be easily satisfied if $G_i = B_iX(X>0)$. Furthermore, considering the sliding surface in equation (6), we could design a common sliding surface if we choose a common matrix G instead of G_i, and this way of selecting sliding surface is much more flexible than those in the study by Liu et al.[21]

According to equation (4), one can obtain

$$\dot{s}(t) = G_i[(A_i + BK)\hat{x}(t) + A_{\tau i}\hat{x}(t-\tau)] + G_iB_i(u(t) + v(t)) + G_iB_i(L_i + \Delta L_i)C_ie(t) \tag{6}$$

In view of sliding mode theory, if the state trajectories arrive at the sliding surface, then $s(t) = 0$ and $\dot{s}(t) = 0$. Thus, equivalent control could be gained consequently as follows

$$u_{eq}(t) = -(G_iB_i)^{-1}G_i[(A_i + BK)\hat{x}(t) + A_{\tau i}\hat{x}(t-\tau)] - v(t) - (L_i + \Delta L_i)C_ie(t) \tag{7}$$

After substituting system (8) into equation (4), one can gain the sliding mode dynamics

$$\begin{aligned} E\dot{\hat{x}} &= [A_i - B_i(G_iB_i)^{-1}G_i(A_i + BK)]\hat{x}(t) \\ &\quad + (I - B_i(G_iB_i)^{-1}G_i)A_{\tau i}\hat{x}(t-\tau) \\ &\triangleq \bar{A}_{ki}\hat{x}(t) + \bar{A}_{\tau i}\hat{x}(t-\tau) \end{aligned} \tag{8}$$

where $\bar{A}_{ki} = \bar{A}_i - \bar{B}K$, $\bar{A}_i = (I - \bar{G}_i)A_i$, $\bar{A}_{\tau i} = (I - \bar{G}_i)A_{\tau i}$, $\bar{B}_i = \bar{G}_iB$, and $\bar{G}_i = B_i(G_iB_i)^{-1}G_i$.

SMC law synthesis

In order to guarantee reachability of the sliding surface, we will synthesize an SMC law in this part. Finally, the sliding mode controller has been developed which has the following formulation

$$\begin{aligned} u(t) = &-(G_iB_i)^{-1}G_i[(A_i + BK)\hat{x}(t) + A_{\tau i}\hat{x}(t-\tau)] \\ &-L_i(y(t) - C_i\hat{x}(t)) \\ &-(\| E_{2i} \| \| H_{3i}C_ie(t) \| + \xi_i + d_i + \kappa_i)sgn((G_iB_i)^Ts(t)) \end{aligned} \tag{9}$$

and the $v(t)$, which is the robust term in equation (4), has been designed as follows

$$v(t) = (\xi_i + d_i)sgn(X_i(y(t) - C_i\hat{x}(t)) \tag{10}$$

where ξ_i and κ_i stand for positive scalars, as well as the X is to be introduced in Theorem 2 later.

The following theorem is to analyze the reachability of the sliding surface $s(t) = 0$.

Theorem 1. Let Assumption 1 setting up under the switched systems (1). When the SMC law is developed as equations (9)–(10) for the designed sliding function (6), the state trajectory could be arrived at the specified sliding surface $s(t) = 0$ within a limited time.

Proof 1. Select the following Lyapunov functional

$$V(t) = \tfrac{1}{2}s^T(t)s(t) \tag{11}$$

Therefore, it holds from system (6) to satisfy

$$\dot{V}(t) = s^T(t)[G_i(A_i + BK)\hat{x}(t) + G_iA_{\tau i}\hat{x}(t-\tau) + G_iB_i(u(t) + v(t)) + G_iB_i(L_i + \Delta L_i)C_ie(t)] \tag{12}$$

By substituting equations (9) and (10) into equation (12), one can obtain

$$\begin{aligned} \dot{V}(t) &= s^T(t)G_iB_i[-(\| E_{2i} \| \| H_{3i}C_ie(t) \| + \xi_i + d_i \\ &\quad + \kappa_i)sgn((G_iB_i)^Ts(t) + v(t) + \Delta L_iC_ie(t)] \\ &\leqslant -\kappa_i \| G_iB_i \| \| s(t) \| \end{aligned}$$

This implies that the state trajectories could arrive at the switching surface predefined within a limited time. It is completes this proof.

Sliding mode dynamics analysis

Theorem 2. Let switched singular systems (1) satisfy Assumption 1. Let $\gamma>0$, if $P_i>0$, $0<Q_j<\beta I$ ($j=1,2,\beta>0$), X_i, W_i with appropriate dimensions as well as positive scalars $\epsilon_{ji}>0(j=1,2)$ and $c_i>0$, so the following LMI is satisfied for every $i \in \Gamma$

$$\begin{bmatrix} \Theta_{1i} & Y_i\bar{A}_{\tau i} + \epsilon_{1i}H_{1i}^T H_{2i} & 0 & 0 & 0 & 0 \\ * & -Q_1 + \epsilon_{1i}H_{2i}^T H_{2i} & 0 & 0 & 0 & 0 \\ * & * & \Theta_{2i} & Y_i A_{\tau i} + \epsilon_{1i}H_{1i}^T H_{2i} & Y_i M_{1i} & Y_i B_i M_{2i} \\ * & * & * & -Q_2 + \epsilon_{1i}H_{2i}^T H_{2i} & 0 & 0 \\ * & * & * & * & -\epsilon_{1i}I & 0 \\ * & * & * & * & * & -\epsilon_{2i}I \end{bmatrix} < 0 \tag{13}$$

with

$$B_i^T Y_i^T = c_i X_i C_i \tag{14}$$

where $R \in R^{n\times(n-r)}$ stands for any matrix with full column which satisfies $E^T R = 0$, $Y_i = E^T P_i + W_i R^T$ and

$$\Theta_{1i} = Y_i\bar{A}_{ki} + \bar{A}_{ki}^T Y_i^T + \gamma E^T P_i E + \tau\gamma\beta I + \epsilon_{1i}H_{1i}^T H_{1i}$$
$$\Theta_{2i} = Y_i\tilde{A}_{li} + \tilde{A}_{li}^T Y_i^T + \gamma E^T P_i E + \tau\gamma\beta I + \epsilon_{1i}H_{1i}^T H_{1i} + \epsilon_{2i}C_i^T H_{3i}^T H_{1i} C_i$$

Then, the closed-loop system is called exponentially admissible having average dwell time $T_\sigma > \ln\mu/\gamma$ and the parameter

$$\mu = \max_{i,j\in\Gamma, i\neq j}\frac{\lambda_{\max}(P_i)}{\lambda_{\min}(P_j)} \tag{15}$$

Moreover, the norm of $\eta(t) = [x^T \quad e^T(t)]^T$ obeys

$$\| \sigma(t) \| \leqslant \eta e^{-\lambda t} \| \zeta(t_0) \| \tag{16}$$

where

$$\lambda = \frac{1}{2}\left(\gamma - \frac{\ln\mu}{T_\sigma}\right), \eta = \sqrt{\frac{b+\tau\beta}{a}} \geqslant 1$$
$$a = \min_{i\in\Gamma}\{\lambda_{\min}(E^T P_i E)\}, b = \max_{i\in\Gamma}\{\lambda_{\max}(E^T P_i E)\} \tag{17}$$

Proof 2. First, we will prove that the sliding mode dynamics (8) and the error system (4) are impulse free and regular. Because $\text{rank}(E) = r \leqslant n$, two nonsingular matrices L and H exist, which satisfy

$$LEH = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} \tag{18}$$

Then

$$R = L^T \begin{bmatrix} 0 \\ I \end{bmatrix} N$$

Now, denote

$$L\bar{A}_{ki}H = \begin{bmatrix} A_{1i} & A_{2i} \\ A_{3i} & A_{4i} \end{bmatrix}, L^{-T}P_i L^{-1} = \begin{bmatrix} P_{1i} & P_{2i} \\ & P_{3i} \end{bmatrix}$$
$$H^T W_i = \begin{bmatrix} W_{1i} \\ W_{2i} \end{bmatrix}$$

It can be found from $\Theta_{1i} < 0$ that

$$(E^T P_i + W_i R^T)A_{ki} + A_{ki}^T(E^T P_i + W_i R^T)^T < 0 \tag{19}$$

After that, pre-multiplying equation (19) by H_i^T and post-multiplying equation (19) by H_i, one can obtain

$$A_{4i}^T N W_{2i}^T + W_{2i} N^T A_{4i} < 0 \tag{20}$$

which means A_{4i} is nonsingular. Thus, the pair (E_i, A_i) is impulse free and regular. By definition 2, the sliding mode dynamics is impulse free and regular. It is similar to the error system (4).

Next, we will prove the considered systems' exponential stability. According to the closed-loop system, we select the following Lyapunov functional

$$V_i(t) = \hat{x}^T(t)E^T P_i E\hat{x}(t) + \int_{t-\tau}^{t} \hat{x}(s)Q_1\hat{x}(s)ds + e^T(t)E^T P_i E e(t) + \int_{t-\tau}^{t} e(s)Q_2 e(s)ds \tag{21}$$

Thus, along the solution of systems (4) and (8), we have

$$\begin{aligned} \dot{V}_i(t) = {} & 2\hat{x}^T(t)(E^T P_i + W_i R^T)[\bar{A}_{ki}\hat{x}(t) + \bar{A}_{\tau i}\hat{x}(t-\tau)] \\ & + \hat{x}^T(t)Q_1\hat{x}(t) - \hat{x}^T(t-\tau)Q_1\hat{x}(t-\tau) \\ & + 2e^T(t)(E^T P_i + W_i R^T)[(\bar{A}_{li} + \Delta A_i - B_i\Delta L_i C_i)e(t)] \\ & + \Delta A_i\hat{x}(t) + (A_{\tau i} + \Delta A_{\tau i})e(t-\tau) \\ & + \Delta A_{\tau i}\hat{x}(t-\tau) + B_i(f_i(x(t)) - v_i(t)) \\ & + e^T(t)Q_2 e(t) - e^T(t-\tau)Q_2 e(t-\tau) \end{aligned} \tag{22}$$

In view of Lemma 1, one further has for $\epsilon_{i1}>0$ and $\epsilon_{i2}>0$

$$\begin{aligned} & 2e^T(t)(E^T P_i + W_i R^T)[\Delta A_i e(t) + \Delta A_i\hat{x}(t) \\ & + \Delta A_{\tau i}e(t-\tau) + \Delta A_{\tau i}\hat{x}(t-\tau)] \\ & \leqslant \epsilon_{1i}^{-1}e^T(t)(E^T P_i + W_i R^T)M_{1i}M_{1i}^T(E^T P_i + W_i R^T)^T e(t) \\ & + \epsilon_{1i}[H_{1i}\hat{x}(t) + H_{2i}\hat{x}(t-\tau) \\ & + H_{1i}e(t) + H_{2i}e(t-\tau)]^T[H_{1i}\hat{x}(t) + H_{2i}\hat{x}(t-\tau) \\ & + H_{1i}e(t) + H_{2i}e(t-\tau)] \end{aligned} \tag{23}$$

$$\begin{aligned}&-2e^T(t)(E^TP_i + W_iR^T)B_i\Delta L_iC_ie(t)\\&\leqslant \epsilon_{2i}^{-1}e^T(t)(E^TP_i + W_iR^T)B_iM_{2i}M_{2i}^TB_i^T(E^TP_i + W_iR^T)^Te(t)\\&+\epsilon_{2i}e^T(t)C_i^TH_{i2}^TH_{i2}C_ie(t)\end{aligned} \tag{24}$$

By means of equations (10) and (14), we have that

$$\begin{aligned}&e^T(t)(E^TP_i + W_iR^T)B_i(f_i(x(t)) - v_i(t))\\&= e^T(t)C_i^TX_i(f_i(x(t)) - v_i(t))\\&\leqslant -(1 + c_{2i})\xi_i \parallel X_i((y(t)) - C_i\hat{x}(t)) \parallel \quad < 0\end{aligned} \tag{25}$$

So combining equations (22)–(25), it yields

$$\dot{V}_i(t) + \gamma V_i(t) \leqslant \zeta^T(t)\Psi_i\zeta(t) \tag{26}$$

where $\zeta(t) = [\hat{x}^T(t) \quad \hat{x}^T(t-\tau) \quad e^T(t) \quad e^T(t-\tau)]^T$ and

$$\Psi_i = \begin{bmatrix}\tilde{\Theta}_{1i} & (E^TP_i + W_iR^T)A_{\tau i} + \epsilon_{1i}H_{1i}^TH_{2i} & 0 & 0\\ * & -Q_1 + \epsilon_{1i}H_{2i}^TH_{2i} & 0 & 0\\ * & * & \tilde{\Theta}_{2i} & (E^TP_i + W_iR^T)A_{\tau i} + \epsilon_{1i}H_{1i}^TH_{2i}\\ * & * & * & -Q_2 + \epsilon_{1i}H_{2i}^TH_{2i}\end{bmatrix}$$

$$\tilde{\Theta}_{1i} = (E^TP_i + W_iR^T)\bar{A}_{ki} + \bar{A}_{ki}^T(E^TP_i + W_iR^T)^T + \gamma E^TP_iE + \tau\gamma\beta I + \epsilon_{1i}H_{1i}^TH_{1i}$$

$$\begin{aligned}\tilde{\Theta}_{2i} &= (E^TP_i + W_iR^T)\bar{A}_{li} + \bar{A}_{li}^T(E^TP_i + W_iR^T)^T + \gamma E^TP_iE + \tau\gamma\beta I + \epsilon_{1i}H_{1i}^TH_{1i} + \epsilon_{2i}C_i^TH_{3i}^TH_{1i}C_i\\&+ \epsilon_{1i}^{-1}Y_iM_{1i}M_{1i}^TY_i^T + \epsilon_{2i}^{-1}Y_iB_iM_{1i}M_{1i}^TB_i^TY_i^T\end{aligned}$$

Then, employing Schurs' complement, it can be seen that X_i is nonsingular and $\Psi_i < 0$ is implied by equation (13). According to equation (26), one can obtain

$$\dot{V}_i(t) \leqslant -\gamma V_i(t)$$

Therefore, there holds

$$V_i(t) \leqslant e^{-\gamma(t-t_0)}V_i(t_0) \tag{27}$$

which has a mean that each closed-loop system's subsystem has the exponentially stable character.

Let t_k, where $k \in \{1, 2, \ldots, N_\sigma\}$ is the switching instant. Therefore, one can obtain $\sigma(t_k^-) = j$ and $\sigma(t_k^+) = i$. From equations (15) and (27), one can get

$$V_i(t) \leqslant e^{-\gamma(t-t_k)}V_i(t_k), \quad \text{and} \quad V_i(t_k) \leqslant \mu V_j(t_k^-) \tag{28}$$

Let $N_\sigma(t_0, t) \leqslant N_0 + (t - t_0)/T_\sigma$. From equation (28), one can obtain

$$\begin{aligned}V_i(t) &\leqslant e^{-\gamma(t-t_k)}\mu V_j(t_k^-)\\&\vdots\\&\leqslant e^{-\gamma(t-t_0)}\mu N_\sigma(t_0,t)V_{\sigma(t_0)}(t_0)\\&\leqslant e^{(-\gamma-\ln\mu/T_\sigma)(t-t_0)}V_{\sigma(t_0)}(t_0)\end{aligned} \tag{29}$$

Considering equation (21), we know that

$$a \parallel \eta(t)\|^2 \leqslant V_i(t), \quad \text{and} \quad V_{\sigma(t_0)}(t_0) \leqslant (b + \tau\beta) \parallel \eta(t_0)\|^2 \tag{30}$$

which together with equations (29) and (30) yields

$$\parallel \eta(t)\|^2 \leqslant \frac{1}{a}V_i(t) \leqslant \frac{b+\beta}{a}e^{(-\gamma-\ln\mu/T_\sigma)(t-t_0)} \parallel \eta(t_0)\|^2 \tag{31}$$

According to equations (17) and (31), one can see that the closed-loop system has the exponentially stable character. This proof is completed.

In order to develop sliding mode controller and non-fragile observer, the observer gain L_i in equation (3) and the sliding mode gain in equation (5) should be given in advance. To this end, we draw out the results as follows.

Theorem 3. Consider the switched systems (1)–(2) satisfy Assumption 1. Let scalar $\gamma>0$, if there exist $P_i>0$, $0<Q_j<\beta I(j = 1, 2)$, X_i, $\bar{K}_l$ $\bar{L}_i$, W_i having appropriate dimensions and positive scalars $\epsilon_{ji}>0, j = 1, 2$ and $c_i>0$, such that the LMIs as follows set up for every $i \in \Gamma$

$$\begin{bmatrix}\Phi_{1i} & Y_i\bar{A}_{\tau i} + \epsilon_{1i}H_{1i}^TH_{2i} & 0 & 0 & 0 & 0\\ * & -Q_1 + \epsilon_{1i}H_{2i}^TH_{2i} & 0 & 0 & 0 & 0\\ * & * & \Phi_{2i} & Y_iA_{\tau i} + \epsilon_{1i}H_{1i}^TH_{2i} & Y_iM_{1i} & Y_iB_iM_{2i}\\ * & * & * & -Q_2 + \epsilon_{1i}H_{2i}^TH_{2i} & 0 & 0\\ * & * & * & * & -\epsilon_{1i}I & 0\\ * & * & * & * & * & -\epsilon_{2i}I\end{bmatrix} < 0 \tag{32}$$

and

$$B_i^TY_i^T = c_iX_iC_i \tag{33}$$

where

$$\Phi_{1i} = Y_i\bar{A}_i + \bar{A}_i^T Y_i^T - \bar{K}_l - \bar{K}_l^T + \gamma E^T P_i E + \tau\gamma\beta I + \epsilon_{1i} H_{1i}^T H_{1i}$$

$$\Phi_{2i} = Y_i A_i + A_i^T Y_i^T - \bar{L}_i C_i - C_i^T \bar{L}_i^T + \gamma E^T P_i E + \tau\gamma\beta I + \epsilon_{1i} H_{1i}^T H_{1i} + \epsilon_{2i} C_i^T H_{3i}^T H_{3i} C_i$$

and the average dwell time

$$T_\mu > \frac{\ln \mu}{\gamma} \tag{34}$$

For arbitrary switching signal $\sigma(t)$, the closed-loop system has a exponentially stable ability. The other notions are defined in Theorem 2. Moreover, the sliding mode gain is presented by $K = \bar{B}^+ Y_l^{-1} \bar{K}_l$, and the non-fragile observer gain is denoted by $L_i = B_i^+ Y_i^{-1} \bar{L}_i$, where B^+ and B_i^+ are the Moore–Penrose inverse of matrices B and B_i, respectively.

Proof 3. In the proof of Theorem 2, noting that $l \in \Gamma$ such that $Y_l \bar{B} K \leqslant Y_i \bar{B} K$ for any $i \in \Gamma$. Also, we have confirmed that Y_i is nonsingular in Theorem 1, so letting $Y_l \bar{B} K = \bar{K}_l$ and $Y_i B_i L_i = \bar{L}_i$, we could easily have that equation (32) guarantees equation (13) holds.

Remark 2. Noting that the conditions proposed in above theorems exist linear matrix equality, which could not get a solution using MATLAB's LMI-Toolbox directly. In order to solve these problems, equation (14) is replaced by the equivalent forms as follows:

$$\mathrm{trace}[(B_i^T Y_i^T - c_i X_i C_i)^T (B_i^T Y_i^T - c_i X_i C_i)] = 0 \tag{35}$$

So, there is a positive scalar $d>0$ to satisfy

$$(B_i^T Y_i^T - c_i X_i C_i)^T (B_i^T Y_i^T - c_i X_i C_i) < dI \tag{36}$$

By Schur complement, equation (36) is equal to

$$\begin{bmatrix} -dI & (B_i^T Y_i^T - c_i X_i C_i)^T \\ * & -I \end{bmatrix} < 0 \tag{37}$$

So, the non-fragile observer-based SMC control problem has been transformed into solving a global solution of the following problem, which is a typical minimization problem

$$\min d, \quad \text{subject to equations (32) and (37)}$$

Numerical simulation

Let the switched singular system (1) with $N = 2$ as well as parameters as follows.

For $i = 1$, the system's dynamics are introduced as follows

$$A_1 = \begin{bmatrix} 2.5 & 0 \\ -0.1 & -4.5 \end{bmatrix}, A_{\tau 1} = \begin{bmatrix} 1.1 & 1.3 \\ 0.8 & 0.7 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1.5 \\ 1 \end{bmatrix}, C_1 = [0.5 \quad -1]$$

$$M_{11} = \begin{bmatrix} 0.1 & 0.3 \\ 0 & -0.1 \end{bmatrix}, H_{11} = \begin{bmatrix} -0.1 & 0 \\ 0.1 & -0.3 \end{bmatrix},$$

$$H_{21} = \begin{bmatrix} 0 & 0.5 \\ 1 & 0.2 \end{bmatrix}, M_{21} = 0.5, H_{31} = -0.1$$

For $i = 2$, the system' dynamics are introduced as

$$A_2 = \begin{bmatrix} 5.5 & -1.3 \\ 0.5 & -2.3 \end{bmatrix}, A_{\tau 2} = \begin{bmatrix} -1.2 & 0.9 \\ 0.5 & 1.1 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} -1.3 \\ 2 \end{bmatrix}, C_2 = [0.5 \quad 0.8]$$

$$M_{12} = \begin{bmatrix} -0.1 & 0 \\ 0.2 & -0.1 \end{bmatrix}, H_{12} = \begin{bmatrix} -0.1 & -0.1 \\ 0.1 & -0.1 \end{bmatrix},$$

$$H_{22} = \begin{bmatrix} 0.3 & -0.2 \\ 0 & 0.2 \end{bmatrix}, M_{22} = 0.2, H_{32} = 0.1$$

The E is assumned to be

$$E = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

Moreover, τ and γ are selected as $\tau = 0.4$ and $\gamma = 4.5$, and $f_1(t) = f_2(t) = e^{-t}\sin t$. The tuning scalars $c_i = 1 (i = 1, 2)$. By solving LMI (32) and (37) using Matlab, one feasible solution is given as follows with $G_i = B_i^T (i = 1, 2)$, $\alpha_1 = \alpha_2 = 0.5$

$$\epsilon_{11} = 0.4619, \epsilon_{21} = 5.2108, \epsilon_{12} = 1.3772, \epsilon_{22} = 5.1765$$

$$\beta = 2.0322$$

$$Q_1 = \begin{bmatrix} 1.3782 & -0.0092 \\ -0.0092 & 1.3304 \end{bmatrix}$$

$$Q_2 = \begin{bmatrix} 0.7603 & -0.0138 \\ -0.0138 & 0.7583 \end{bmatrix}$$

$$P_1 = \begin{bmatrix} 4.2143 & -2.0166 \\ -2.0166 & 1.1895 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 4.2182 & -2.0088 \\ -2.0088 & 1.2050 \end{bmatrix}$$

$$K_l = \begin{bmatrix} 6.3412 & 57.7274 \\ -50.0193 & 17.0393 \end{bmatrix}$$

$$Y_1 = \begin{bmatrix} -2.1726 & 3.3949 \\ -0.7685 & 1.3085 \end{bmatrix}$$

$$Y_2 = \begin{bmatrix} -0.5558 & -0.6125 \\ 0.1697 & 0.3243 \end{bmatrix}$$

$$L_1 = \begin{bmatrix} 2.8205 \\ -9.0601 \end{bmatrix}, L_2 = \begin{bmatrix} 7.3497 \\ 18.4432 \end{bmatrix}$$

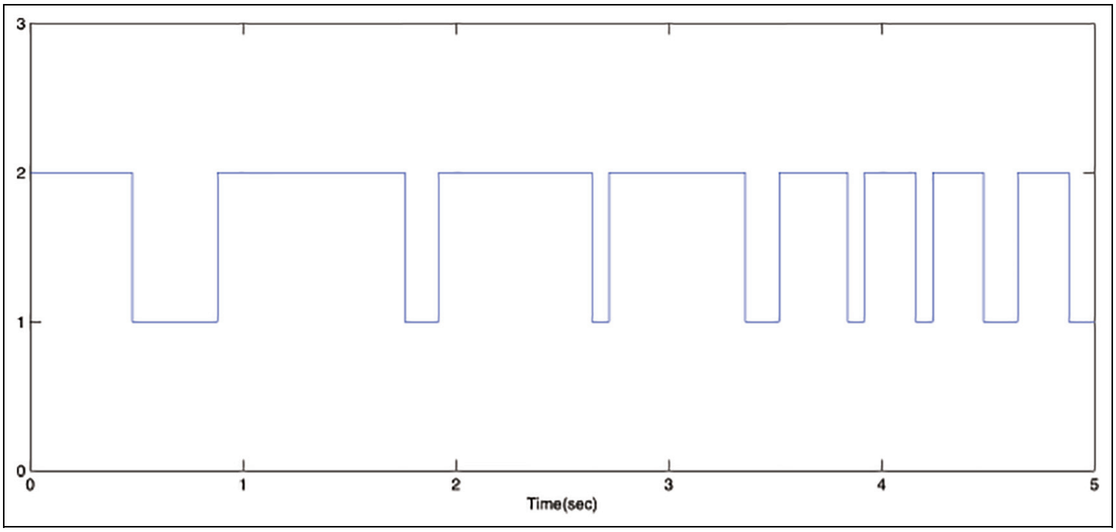

Figure 1. Switching signal with average dwell time $T_\mu \geqslant 0.25$.

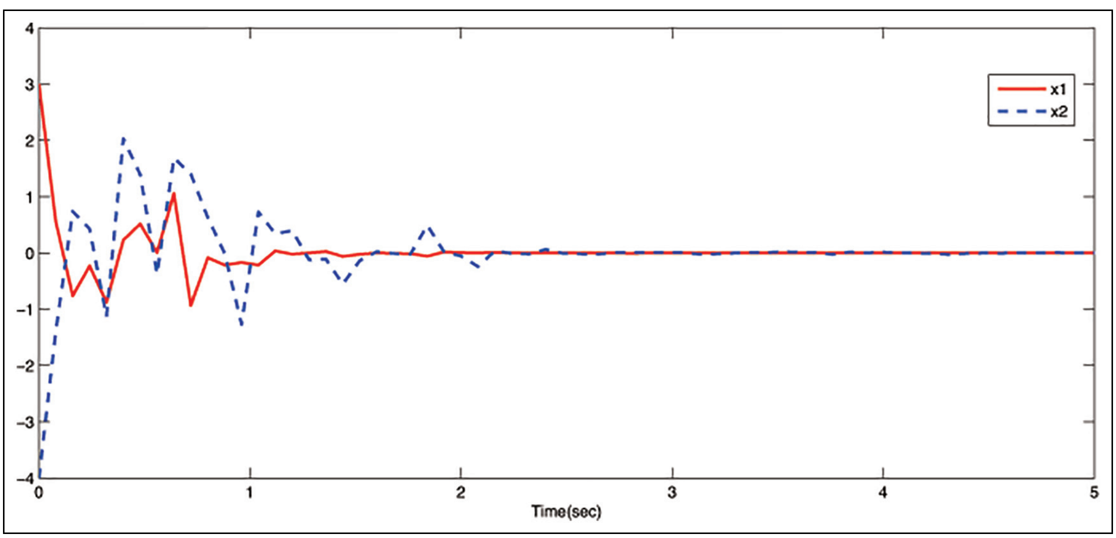

Figure 2. State response of $x(t)$.

So according to Theorem 1, we could have

$$\mu = \max_{i,j\in\Gamma, i\neq j} \frac{\lambda_{\max}(P_i)}{\lambda_{\min}(P_j)} = 28.8223$$

Furthermore, the average dwell time

$$T_\mu > \frac{\ln \mu}{\gamma} = 0.2076$$

Thus, we could choose the average dwell time $T_\mu = 0.25$.

Then, controller in equation (9) is given by

$$u(t) = \begin{cases} [180.8189 \quad 12.2915]x(t) \\ \quad -[0.7538 \quad 0.8154]x(t-0.4) \\ \quad +96.7937(y(t) - [0.5 \quad -1]x(t)) \\ \quad -(\| -0.05(y(t) - [0.5 \quad -1]x(t)) \| \\ \quad + \xi_1 + d_1 + \kappa_1) sgn((G_1B_1)^T s(t)); \quad i = 1; \\ [179.6441 \quad 12.2117]x(t) \\ \quad -[0.4499 \quad 0.1801]x(t-0.4) \\ \quad -93.9272(y(t) - [0.5 \quad 0.8]x(t)) \\ \quad -(\| 0.02(y(t) - [0.5 \quad 0.8]x(t)) \| \\ \quad + \xi_2 + d_2 + \kappa_2) sgn((G_2B_2)^T s(t)). \quad i = 2 \end{cases}$$

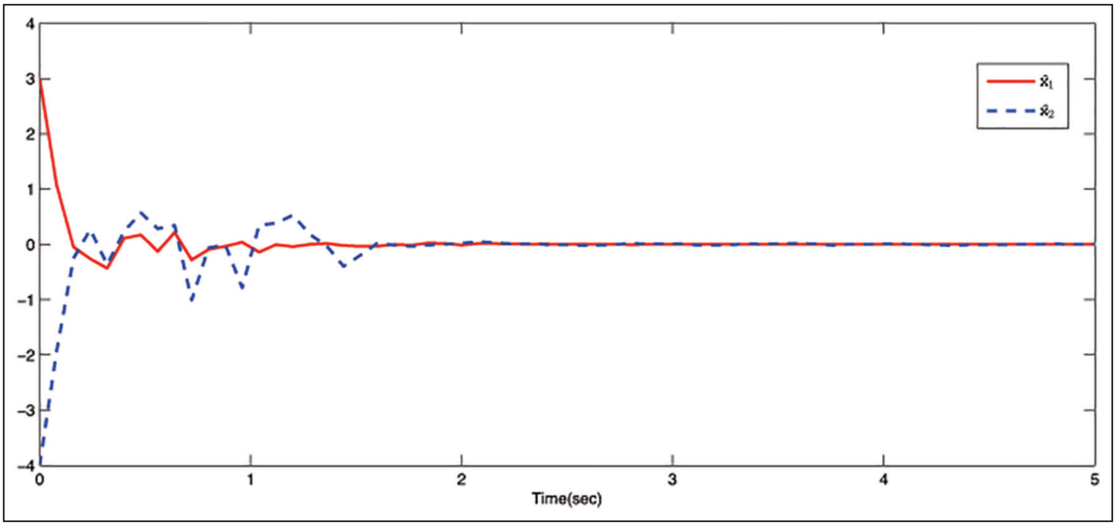

Figure 3. State response of $\hat{x}(t)$.

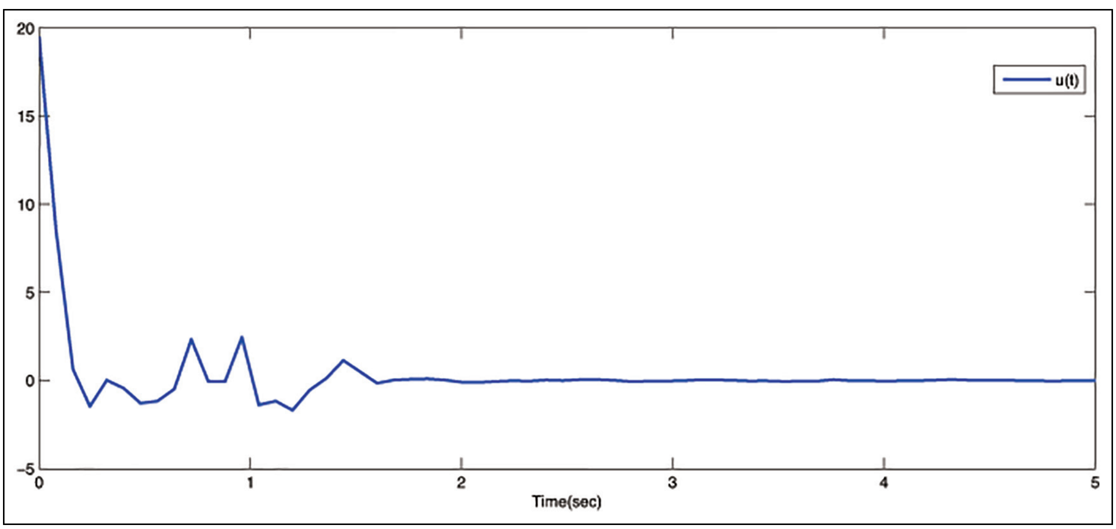

Figure 4. Control input $u(t)$.

For simulation purpose, let the initial conditions $x(t) = \hat{x}(t) = [3 \quad -4]^T$ for all $t \in [-0.4, 0]$. And the adjustable scalars $\xi_1 = \xi_2 = 0.8$ and $\kappa_1 = \kappa_2 = 1$. The simulation numerical results can be obtained from Figures 1 to 4. Figure 1 plots the switching signals; Figures 2 and 3 give the state response of the original system and the observer system, respectively; and the control input is shown by Figure 4.

Conclusion

This research has discussed the non-fragile SMC of uncertain switched singular systems which is with time delay. Since system states are not available, we have designed a non-fragile observer at first; second, a switched integral-type sliding surface has been constructed according to the estimated states, and this method could be extended to develop other common sliding surface for the benefit of the introduced input matrices' weighted sum. Third, an SMC law has been synthesized in order to make sure the reachability of sliding surface. Furthermore, the exponential admissibility of the closed-loop system could be proved using a minimization problem along with LMI conditions. Finally, a numerical result has been presented to demonstrate the effectiveness of the method.

Author's Note

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