Available online at www.sciencedirect.com

ScienceDirect

Energy Procedia 148 (2018) 1214–1221

Energy Procedia

www.elsevier.com/locate/procedia

73rd Conference of the Italian Thermal Machines Engineering Association (ATI 2018), 12–14 September 2018, Pisa, Italy

An analytical method to simulate the dynamic performances of truncated cone helix ground heat exchangers

Paolo CONTI*

Department of Energy, Systems, Territory, and Constructions Engineering (DESTEC), University of Pisa, Italy

Abstract

This paper proposes a dynamic analytical method to simulate the thermal performances of truncated cone helix ground heat exchangers (i.e., the so-called "energy baskets"). These ground-coupled devices are attractive solutions to reduce the initial cost of ground-coupled heat pump systems, as they require lower cost to be drilled and installed with respect to traditional boreholes. However, both design methodologies and performance assessment models are still not well developed, producing substantial uncertainties on final operative performances. This work presents a plain evaluation method based on the heat exchangers theory and the analytical solution of the truncated cone helix heat source in a semi-infinite medium. It can be advantageously used to simulate the thermal performance of truncated cone helix ground heat exchangers as a function of helix geometries and operative conditions evolution (e.g., inlet temperature, fluid flow rate, ground temperature…). Specifically, in this paper, we perform a sensitivity analysis of the thermal performances of a case study by varying the main geometrical parameters. Besides, we compare the heat transfer of the reference configuration with an equivalent cylindrical arrangment. The truncated coil configuration is more effective than cylindrical one as the cone aperture reduces the short-circuits between helix pitch and the equivalent thermal resistance with the ground surface. However, obtained results are notably affected by the assumption of an isothermal surface temperature, which leads to a shallow/plain helix/spiral as the best configuration: different conclusions are expected when a time dependent or adiabatic boundary condition will be accounted in the model.

* Corresponding author. Tel.: +39-050-221-7123; fax: +39-050-221-7160.
E-mail address: paolo.conti@unipi.it

10.1016/j.egypro.2018.08.009

Keywords: Ground-coupled heat exchangers; spiral coils; truncated cone helix; GSHP system; dynamic simulation

Nomenclature

Symbols

$\mathbf{r}$	generic position vector
$\mathbf{r}'$	position vector of the helix/heat generating points
$\mathrm{R_{th,p}}$	linear thermal resistance between the fluid and the duct-ground interface, Km/W
NTU	number of transfer units
c	heat capacity, J/kg/K
$\dot{\mathrm{m}}$	flow rate, kg/s
r_b	radius of the bottom base, m
r_t	radius of the top base, m
h_b	position of the bottom base, m
$\mathrm{h_{conv}}$	convective coefficient within the duct, W/(m^2 K)
h_t	position of the top base, m
$\dot{\mathrm{q}}_\mathrm{l}$	linear heat flux, W/m
t	time, s

Greek letters

Θ_g	dimensionless temperature solution of the *homogeneous* problem
α	thermal diffusivity, m^2/s
β	auxiliary variable
ϵ	heat transfer effectiveness
θ	temperature, °C or K
λ	thermal conductivity, W/(m K)
ψ	cone aperture, rad

Subscripts

b	bottom base
f	circulating fluid
g	ground
i	inner
o	outer
p	helix duct/pipe
t	top base

Superscripts

$\bar{\ }$	average
0	initial condition
i	generic *i-th* time step
n	current time step

Acronyms and abbreviations

GHEX	Ground-coupled heat exchangers
GSHP	Ground-source heat pump system
TCoGEHX	Truncated Cone Helix Ground Heat Exchangers

1. Introduction

Truncated cone helix ground heat exchangers (TCoGEHX) (i.e., the so-called "energy baskets") are attractive solutions to reduce the installation costs of ground-source heat pump systems (GSHP). They consist of a heat exchanging loop, buried at a shallow depth in the ground, forming a cylindrical or a truncated cone helix (see Fig. 1). Typical values for r_t, h_b, and h_t goes from 1 to 2 meters, 2 to 3 meters, 0.5 to 1 meters, respectively.

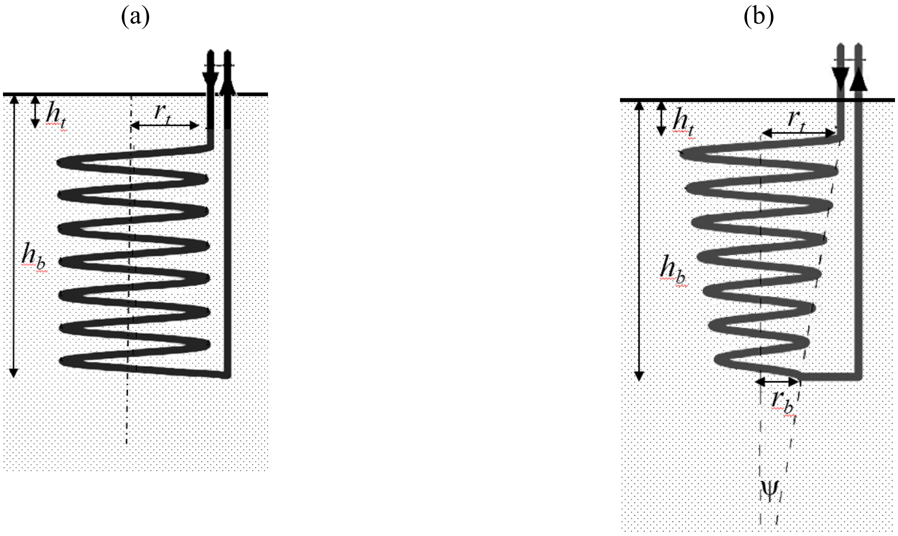


Fig. 1. (a) Cylindrical (or spiral) coil ground heat exchanger; (b) truncated cone helix ground heat exchangers.

To the best of our knowledge, there are no sufficiently plain and established methods to evaluate the energy performances of those GHEXs [1]. The evaluation process consists of a full 3D transient heat transfer problem with a complex geometry, different thermo-physical properties of materials (i.e., the fluid, the duct, and ground), transient operative and boundary conditions (e.g. surface temperature of the ground surface, inlet temperature to the GEHX and fluid flow rate). Except for nominal performance declared by manufactures or too simple and approximate rules-of-thumb (see, for instance, [2-3]), both research and professional operators lack handy design tools to assess the performance of different configurations and figure out the best solution in any specific case, even for a preliminary analysis.

Many works use a numerical approach to analyze TCoGEHXs [4-7]. However, numerical methods are characterized by a significant computational effort and do not represent a handy design tool, as they are strongly related to an accurate description of the specific case to be simulated. Moreover, the HVAC sector is experiencing an increasing employment of dynamic simulations and optimization routines which require simplified and fast routines to simulate the operative performance of the overall GSHP system [8-9]. Components models must therefore be based on a proper tradeoff between accuracy and computational effort.

In this perspective, analytical models represent an attractive alternative to full 3D transient numerical simulations because they can provide fast, practical and general indications with a reduced computational effort and flexibility in parametric evaluations. Besides, analytical formulations can be summarized in dimensionless maps and correlations, obtaining handy design, simulation and comparison tools for different types of GHEXs.

Several analytical models have been developed in recent years: among the others, we recall the models presented by [1,10-13] and based on the integration of Green's function [14] with respect to the time and assuming heat exchanging pipes as a series of linear ring-coils, as a linear helix, or as a surface heat source. The reader can refer to [15] or further details on available models for cylindrical coil ground heat exchangers.

The main drawback of the models presented in literature consists of considering the heat source as a constant homogenous heat source along the ducts: in other words, the linear heat flux, W/m, is assumed as a fixed boundary condition. This assumption does not account for the temperature distribution of the fluid over the GHEX and does not account for the influence of the geometry, mass flow rate, inlet temperature and ground temperature evolution on the actual heat exchange between the fluid and the ground. Moreover, the inlet temperature and flow rate are variables parameters that depend on the actual characteristics and operative conditions of the overall GSHP system [9].

To overcome this issue, [16] coupled the classical heat exchanger theory (i.e., ε-NTU) [17] to the dynamic thermal response of the ground-coupled apparatus, obtaining a direct correlation among time, supply temperature from HP unit(s), materials and geometrical properties of the GHEX and ground source. The evolution of all quantities is evaluated through the time and space superposition techniques [14, 17], preserving the analytical nature of the methods. The validity range of space and time scales for using the ε-NTU method has been investigated in [19].

In Section 2, we described how the same methodology presented [14, 18] can be used to analyze the thermal performance of different TCoGEHX geometries. In Section 3, we present a case study to show the results obtainable by the proposed methodology. Besides, we perform a preliminary sensitivity analysis on the long-term performances varying one geometrical parameter at time to identify which quantity mainly affect the exchange. Finally, we compare the heat capacity of the reference case with an equivalent cylindrical configuration.

2. The application of the ε-NTU method to TCoGEHXs.

In this work, we refer to the truncated cone helix heat source model presented in [1]. In fact, cylindrical helix configurations can be seen as a limit case of the TCoGEHX, assuming $\psi = 0$. As mentioned in the Introduction, [1] presented the analytical expression of the temperature evolution in the ground, assuming a constant and homogeneous helix heat source, which represents the heat exchanging coil: we refer to this problem as the *homogeneous* one[†]. The dimensionless solution of that problem reads:

$$\Theta_g = \frac{1}{4\pi}\int_{2\pi H_t/B}^{2\pi H_b/B} \sqrt{\left[1+\left(H_{mi}-\frac{B}{2\pi}\beta\right)\right]+\frac{B^2}{4\pi^2}\left(1+(\tan\psi)^2\right)}\left[\frac{1}{D_p}erfc\left(\frac{D_p}{2\sqrt{Fo_{r_{mi}}}}\right)-\frac{1}{D_n}erfc\left(\frac{D_n}{2\sqrt{Fo_{r_{mi}}}}\right)\right]d\beta \tag{1}$$

where:

$$D_p = \sqrt{R^2+\left[1+\left(H_{mi}-\frac{B}{2\pi}\beta\right)\tan\psi\right]^2-2R\left[1+\left(H_{mi}-\frac{B}{2\pi}\beta\right)\tan\psi\right]\cos(\varphi-\beta)+\left(Z-\frac{B}{2\pi}\beta\right)^2}$$

$$D_n = \sqrt{R^2+\left[1+\left(H_{mi}-\frac{B}{2\pi}\beta\right)\tan\psi\right]^2-2R\left[1+\left(H_{mi}-\frac{B}{2\pi}\beta\right)\tan\psi\right]\cos(\varphi-\beta)+\left(Z+\frac{B}{2\pi}\beta\right)^2}$$

$$\Theta_g = \lambda_g(\theta-\theta^0)/\dot{q}_l \quad H_{mi} = h_{mi}/r_{mi} \quad B = b/r_{mi} \quad R = r/r_{mi} \quad Z = z/r_{mi} \quad Fo_{r_{mi}} = \alpha_g t/r_{mi}^2$$

$$r_{mi} = (r_t+r_b)/2 \quad h_{mi} = (h_t+h_b)/2$$

In an engineering perspective, the actual linear heat flux $\dot{q}_l$ is unknown and it must be determined as a function of helix geometry, thermo-physical properties of the media, and operative parameters (e.g., supply temperature, $\theta_{f,in}$, and flow rate, $\dot{m}_f$). Moreover, $\dot{q}_l$ varies in space and time depending on local temperature of the duct-ground interface according to the energy equation of the fluid:

[†] The dimensionless solution of the *homogeneous problem* represents the evolution of the thermal field within the ground source due to a constant and homogeneous heat source. In the GSHP context, that solution is typically named “G-function”.

$$\dot{m}_f c_f \frac{\partial \theta_f}{\partial l} = \frac{2\pi r_{p,o}[\theta_f - \theta_p(\mathbf{r}',t)]}{R_{th,p}} = \dot{q}_l(\mathbf{r}',t) \tag{2}$$

where $\mathbf{r}'$ is the considered position over the helix, $\theta_p(\mathbf{r}',t)$ is the local temperature of the duct-ground interface, and $R_{th,p}$ is the thermal resistance between the fluid and the ground (see Eq. 9). Eq. 2 is the constitutive equation of the classical heat exchanger theory; thus, we can write the overall thermal performance of the GHEX, $\dot{Q}_l$, using the $\epsilon - NTU$ method [17], namely:

$$\dot{Q}_l(t) = \bar{q}_l(t) L_{helix} = \dot{m}_f c_f \left(\theta_{f,in} - \theta_{f,out}(t)\right) = \epsilon_p \dot{m}_f c_f \left(\theta_{f,in} - \bar{\theta}_p(t)\right) \tag{3}$$

where $\bar{q}_l$ is the average linear heat flux over the helix length, ϵ_p is the heat transfer effectiveness, and $\bar{\theta}_p$ is the average temperature of the duct-ground interface over the helix length.

Eq. 3 relates $\dot{Q}_l(t)$ and $\theta_{f,out}(t)$ to $\bar{\theta}_p$ evolution; on the other hand, $\bar{\theta}_p$ evolves according to Eq. 1 depending on $\bar{q}_l$ value. The set of the two equations can be solved through the Duhamel's Theorem [14], i.e. the time superposition techniques.

A time variant $\bar{q}_l(t)$ introduces a nonhomogeneous time-dependent boundary condition for the problem of the heat diffusion in the ground. The solution of that *nonhomogeneous* problem can be derived from the *homogeneous* one (i.e., Eq. 1) as:

$$\theta(\mathbf{r},t) - \theta(\mathbf{r},0) = \frac{1}{\lambda_g} \int_0^t \bar{q}_l(\mathbf{r}',\beta) \frac{\partial \Theta_g(r,r',t-\beta)}{\partial \beta} d\beta \tag{4}$$

If $\dot{q}_l$ is assumed as a series of step changes $\Delta\dot{q}_l$, occurring at tines $t = i\Delta t$, Eq. 4 becomes:

$$\theta(\mathbf{r}, t = n\Delta t) - \theta(\mathbf{r},0) = \frac{1}{\lambda_g} \sum_{i=1}^{n} \Theta_g(\mathbf{r}, i\Delta t)\left[\bar{q}_l^{n-i+1} - \bar{q}_l^{n-i}\right] \tag{5}$$

For the sake of readability, in Eq 5 and the next ones, we use the subscripts to indicate the time at which the quantity is referred.

$\bar{\theta}_p$value cannot be calculated directly through Eq. 1 as it presents a singularity in $\mathbf{r} = \mathbf{r}'$. Thus, we assume $\bar{\theta}_p$ as the average ground temperature, $\bar{\Theta}_p$, over a circular surface having the helix as extrusion line and radius equal to $r_{p,o}$. The evaluation method of $\bar{\theta}_p$ is already used and presented in [1]. Following the just-mentioned hypothesis, as shown in [17], Eqs. 3 and 5 can be rearranged as:

$$\bar{\theta}_p^n = \left[\bar{\Theta}_p^1 \frac{\epsilon_p \dot{m}_f c_f}{L_{helix}\lambda_g}\left(\theta_{f,in}^n + \bar{\theta}_p^{n-1} - \theta_{f,in}^{n-1}\right) + \sum_{i=2}^{n} \frac{\bar{\Theta}_p^i}{\lambda_g}\left(\bar{q}_l^{n-i+1} - \bar{q}_l^{n-i}\right) + \bar{\theta}_p{}^0\right] / \left(1 + \bar{\Theta}_p^1 \frac{\epsilon_p \dot{m}_f c_f}{L_{helix}\lambda_g}\right) \tag{6}$$

$$\dot{Q}_l^n = \bar{q}_l^n l = \epsilon_p \dot{m}_f c_f \left(\theta_{f,in}^n - \bar{\theta}_p^n\right) \tag{7}$$

The heat transfer effectiveness ϵ_p and the thermal resistance of the pipe can be evaluated as:

$$\epsilon_p = 1 - exp\left(-NTU_p\right) = 1 - exp\left(-\frac{L_{helix}}{m_f c_f R_{th,p}}\right) \tag{8}$$

$$R_{th,p} = R_{th,cond} + R_{th,conv} = \frac{ln\left(r_{p,o}/r_{p,i}\right)}{2\pi\lambda_p} + \frac{1}{2\pi r_{p,i} h_{conv}} \tag{9}$$

where h_{conv} can be evaluated through the classical Gnielinski formula for turbulent flow [17]. We invite the reader to pay attention to the subscript t in the $\theta_{f,in}^{t}$ expression. This means that any profile of $\theta_{f,in}$ evolution can be simulated through the present method.

3. Application example

In this work, we employ the method presented in Section 2 to analyze the thermal performance of a TCoGEHXs. The geometrical parameters and the operative conditions are shown in Table 1. Besides, we perform a sensitivity analysis of the GEHX performances as a function of the main geometrical parameters, also comparing the steady-state heat exchange with an equivalent cylindrical coil ground heat exchanger. The term "equivalent" refers to a cylindrical coil with the same mean depth, h_{mi}, and mean radius, r_{mi}. The circulating fluid is assumed as pure water.

Table 1. Geometrical and thermo-physical parameters of the case study. The reference values are in bold.

Varied parameters	Value(s)	Fixed Parameters	Value
Minor radius, r_b	2, 1.5,**1**, 0.5	Installation depth, *m*	0.5
Helix depth, *m*	0.5, 1, **2**, 4	Duct outer/inner diameter, *m*	0.032/0.026
Helix pitch, *m*	0.08, **0.16**, 0.32, 0.48	Thermal conductivity of the duct, *W/m/K*	0.4
Mass flow rate, *kg/s*	**0.25**, 0.5, 1	Thermal conductivity of the ground, *W/m/K*	1.78
		Thermal diffusivity of the ground, *m^2/s*	7.12 x 10^{-7}
		Major radius, *m*	2
		Inlet fluid temperature, *°C*	5
		Undisturbed ground temperature, *°C*	15

4. Results and discussion

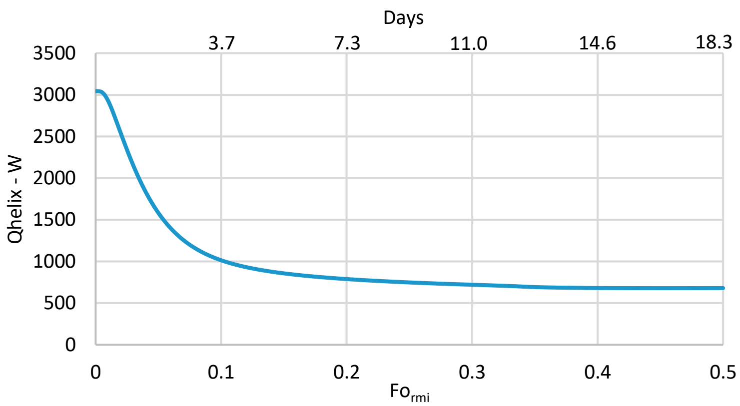
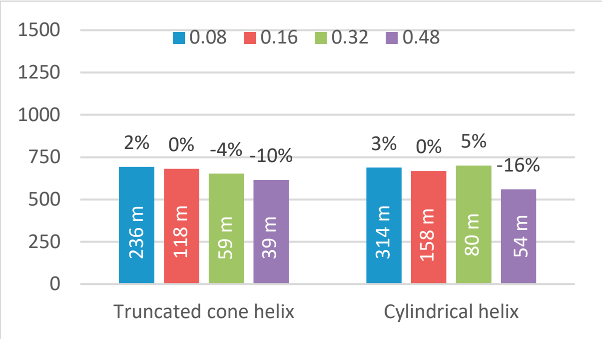


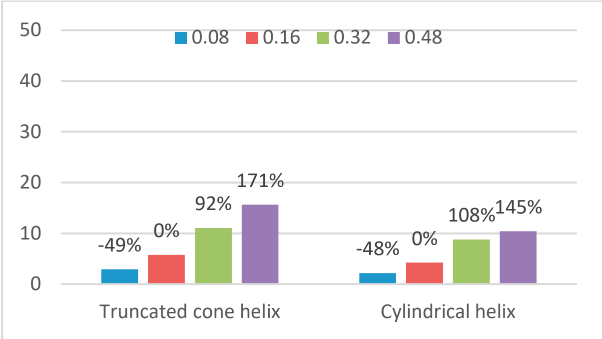
Fig. 2. Evolution of the heat exchanged by the circulating fluid, $\dot{Q}_l$, for the reference configuration (see Table 1).

Fig. 2 shows the evolution of the thermal performance of the reference TCoGEHX, evaluated through Eq. 6. We note that after a few days the heat transfer reaches a steady-state value, which can be intended as the nominal capacity of the heat exchanging apparatus.

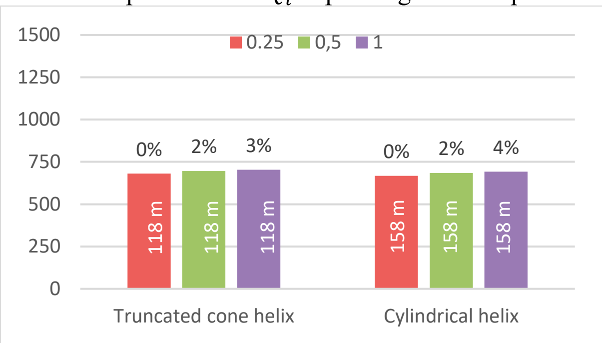
Figs. 3 show the variation of that nominal capacity and linear heat flux in all tested configurations. We note that increasing the helix pitch results in a higher average linear heat flux, $\bar{q}$, due to a reduced thermal short circuit among helix turns. The reduction of the overall performance, $\dot{Q}_l$, is lower than 10% even if the coil length is reduced by a factor 0.7. Both flow rate and cone aperture slightly affect the overall $\dot{Q}_l$, but we found an optimal ψ value around 50°. Further investigations are required on this point to test the generality of that value. We note also that cylindrical configuration has lower performances with respect to the truncated cone one as it has a larger equivalent thermal resistance between the coil and the ground surface. This conclusion was achieved also by [1]. We find out the same conclusion investigating the depth of the coil: higher performances are not obtained with longer ducts, but when the helix has an average shallow position.



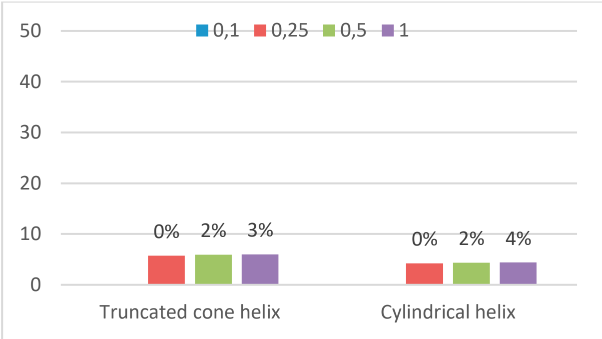
Thermal performance $\dot{Q}_l$ depending on helix pitch



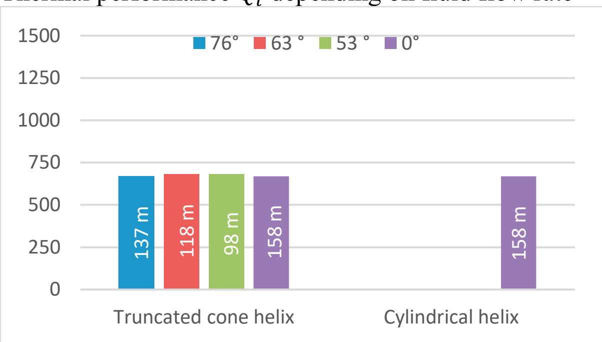
Liner thermal flux $\bar{q}_l$ depending on helix pitch



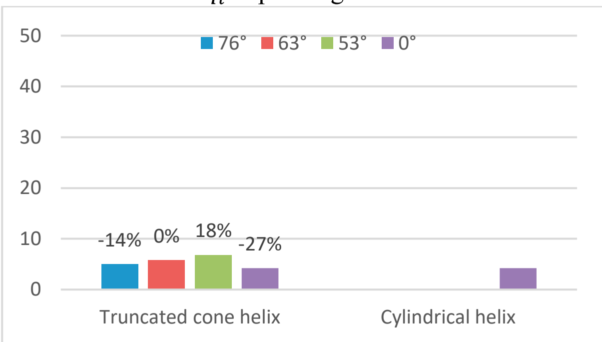
Thermal performance $\dot{Q}_l$ depending on fluid flow rate



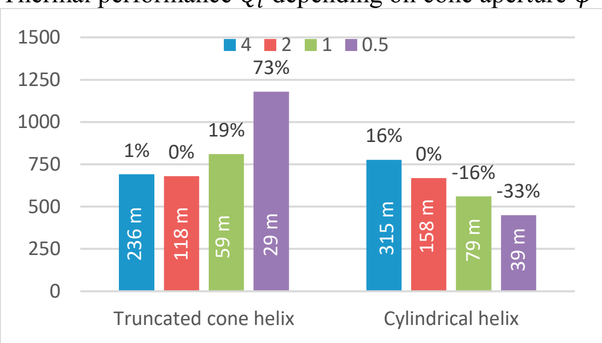
Liner thermal flux $\bar{q}_l$ depending on fluid flow rate



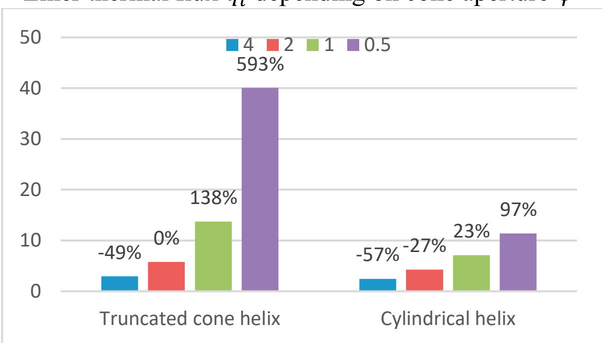
Thermal performance $\dot{Q}_l$ depending on cone aperture ψ



Liner thermal flux $\bar{q}_l$ depending on cone aperture ψ



Thermal performance $\dot{Q}_l$ depending on cone depth



Liner thermal flux $\bar{q}_l$ depending on cone depth

Fig. 3. Sensitivity analysis of the truncated cone helix ground heat exchanger performance depending on geometrical variables and flow rate.

5. Conclusions and future developments

This work presented an analytical method to simulate the thermal performance of TCoGEHXs depending on operative conditions (i.e., inlet temperature and flow rate), thermo-physical properties of the materials (circulating fluid, duct, and ground), and geometrical parameters of the truncated cone helix. We presented an illustrative case study to show how the proposed method can be used to analyze the performance of different helix geometries. Besides, we compared different configurations varying one parameter at time between fluid flow rate, helix dimension and aperture. We found that the cone arrangement is more effective than the cylindrical one as the aperture angle reduces the short-circuits between helix pitch and the average depth of the heat exchanger. However, the latter finding reveals a criticism of the employed model: specifically, assuming an isothermal ground surface always leads to the shallow configurations as the best ones; different conclusions are expected when a time dependent (e.g. the evolution of the outdoor climate), or adiabatic, boundary condition will be accounted in the model.

Further development and ongoing activities concerns the validation of the proposed method though full transient 3D numerical simulation and/or experimental data. The model will be used to develop some general dimensionless maps and correlations to fast compare the thermal performance of TCoGEHXs with other shallow ground-coupled heat exchangers (e.g., energy piles). Last, but not least, the simulation method described in Section 2 is suitable to be employed in a comprehensive dynamic simulation of a GSHP system in order to analyze and optimize the sizing and the operative performance of the overall system.

References

[1] Huang, Guangqin, Yang, Xiaofeng, Liu, Yajiao, Zhuang, Chunlong, Zhang, Hongyu, and Lu, Jun. "A novel truncated cone helix energy pile: Modelling and investigations of thermal performance". *Energy and Buildings* 158 (2018): 1241–1256.

[2] BetaTherrm. "Geothermal Energy Basket-Manual". *BetaTherm GmbH & Co. KG* (2009) Wittwaisstraße, Wangen (D).

[3] Mächler, Evelyn. "An investigation of environmental and economic impacts of the use of geothermal energy baskets in Switzerland". Term Paper – Energy Economics and Policy (2011), D-MTEC ETH Zürich (CH).

[4] Zarrella, Angelo., and De Carli, Michele. "Heat transfer analysis of short helical borehole heat exchangers". *Applied Energy* 102 (2013): 1477-1491.

[5] Zarrella, Angelo., Emmi, Giuseppe, and De Carli, Michele. "Analysis of operating modes of a ground source heat pump with short helical heat exchangers ". *Energy Conversion and Management* 97 (2015): 351-361.

[6] Dehghan, B. Babak. "Effectiveness of using spiral ground heat exchangers in ground source heat pump system of a building for district heating/cooling purposes: Comparison among different configurations". *Applied Thermal Engineering* 130 (2018): 1489–1506.

[7] Dehghan, B. Babak, Sisman, Altug, and Aydin, Murat. "Parametric investigation of helical ground heat exchangers for heat pump applications". *Energy and Buildings* 127 (2016): 999–1007.

[8] Fadejev, Jevgeni, Simson, Raimo, Kurnitski, Jarek, Hand aghighat, Fariborz. "A review on energy piles design, sizing and modelling". *Energy* 112 (2017):390–407.

[9] Grassi, Walter, Conti, Paolo, Schito, Eva, Testi, Daniele. "On sustainable and efficient design of ground-source heat pump systems" *Journal of Physics: Conference Series* 655 (2015):12003.

[10] Man, Yi, Yang, Hongxing, Diao, Nairen, Liu, Junhong, and Fang, Zhaohong. "A new model and analytical solutions for borehole and pile ground heat exchangers". *International Journal of Heat and Mass Transfer* 53 (2010): 2593–2601.

[12] Cui, Ping, Li, Xin, Man, Yi, and Fang, Zhaohong. "Heat transfer analysis of pile geothermal heat exchangers with spiral coils ". *Applied Energy*, 88(11) (2011): 4113–4119.

[12] Man, Yi, Yang, Hongxing, Diao, Nairen, Cui, Ping, Lu, Lin, and Fang, Zhaohong. "Development of spiral heat source model for novel pile ground heat exchangers". *HVAC&R Research* 17(6) (2011): 1075–1088.

[13] Li, Min, and Lai, CK. Alvin. "Heat-source solutions to heat conduction in anisotropic media with application to pile and borehole ground heat exchangers". *Applied Energy* 158 (2012): 451–458.

[14] Ozisik, M. Necati. "Heat conduction" (1993) John Wiley & Sons, Inc., Hoboken (NJ).

[15] Lei, Fei, Hu, Pingfang, and Huang, and XingPu. "Hybrid analytical model for composite heat transfer in a spiral pile ground heat exchanger". *Applied Thermal Engineering* 137 (2018): 555–566.

[16] Conti, Paolo, Testi, Daniele, and Grassi, Walter. "Revised heat transfer modeling of double-U vertical ground-coupled heat exchangers". *Applied Thermal Engineering* 106 (2016) 1257–1267.

[17] Bergman, L. Theodore, Lavine, S. Adrienne, Incropera, P. Frank, and DeWitt, P. David. "Heat exchangers, in: Fundamentals of Heat and Mass Transfer" (2011) 7th ed., John Wiley & Sons, Inc., Hoboken (NJ).

[18] Conti, Paolo. "Dimensionless Maps for the Validity of Analytical Ground Heat Transfer Models for GSHP Applications". *Energies* 9(12) (2016): 890.

[19] Conti, Paolo, Schito, Eva, and Testi Daniele. "Thermal characterization of energy pile dynamics". *Proceedings of the International Symposium on Energy Geotechnics (SEG)*, 25 – 28 September 2018, Lausanne (CH).