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Method for calculating a global dynamic factor K_{AV} in gears subjected to variable velocity and loading conditions

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Abstract

The present paper aims to propose a method, in ISO Standard environment, in order to calculate a single global dynamic factor K_{AV}, replacing both K_A and K_V, in case of gears subjected to variable velocity and loading conditions.

This procedure, based on the Miner damage rule, allows to process a given load spectrum and to calculate a value of the equivalent tangential force that includes all dynamic effects; this force value is useful for bending and pitting calculation of the service life.

In this work a practical case for bending strength is presented, based on a recorded flight mission, referring to a gear box for aerospace applications. Obtained results in terms of equivalent forces and global dynamic factor values have been compared to those calculated by means of the classical ISO Standard formulae, based on the corresponding experimental data.

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Keywords: gear; internal dynamic factor; application factor; variable loads; cumulative damage.

1. Introduction

Classical design formulae available in literature to determine the load capacity of spur and helical gear drives are intended, both for pitting resistance and bending strength, to establish uniformly acceptable methods to assess the corresponding life estimation. The most common approach, widely described in ANSI/AGMA Standard 2001-D04 [1] and in ISO 6336-1 Standard [2-6], compares the calculated maximum stress values (tensile and contact, for

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bending and pitting respectively) to the permissible ones representing the limit value for stresses (tensile and contact) derived from material tests using meshing gears as test pieces.

Referring as an example to the bending case [1, 3], discussed in detail in the present paper, the permissible bending stress σ_{FP} provides damage curves characterized by the nominal stress number σ_{Flim} and by the life factor Y_{NT}, and then corrected using the relative influence factors for notch sensitivity, surface roughness, size.

For as concerns the calculated stress, the tooth root stress σ_F is the maximum tensile stress at the surface in the root and it can be obtained by multiplying the nominal tooth root stress (which is the maximum local principal stress produced at the tooth root) by the so called overload factors. Overload factors [1-4] are generally influence factors, independent each other, that aim to take into account uneven overloading conditions of gears. To this class of influence factors belong both application factor K_A (which takes into account load increments due to externally influenced variations of input or output torque), also called external dynamic factor, and dynamic factor K_V (which takes into account load increments due to internal dynamic effects), also called internal dynamic factor.

ISO Standard 6336 Part 1 [2] defines the dynamic factor K_V as the total mesh torque at operating speed respect to the mesh torque with "perfect" gears. Another definition is reported in [2] referring to the product between K_A and K_V factors, stating that it represents the total mesh torque at operating speed respect to the nominal transmitted (design) mesh torque. No other information than the above quoted is available in [2], and no methods or formulae are indicated to determine it.

The present paper aims to propose a method, in ISO Standard environment, in order to calculate a single global dynamic factor K_{AV}, replacing both K_A and K_V, in case of gears subjected to variable velocity and loading conditions.

This method represents the completion of a previously developed approach [10-11], referred to the determination of the in-operation service factor K_A, obtained for constant dynamic factor K_V.

The presented procedure, based on the Miner damage rule, allows to process a given load spectrum and to calculate a value of the equivalent tangential force that includes all dynamic effects; this force value is useful for bending and pitting calculation of the service life.

In other words, following the methodology already discussed in [10-11], the application factor has been calculated, but each tangential force level has been previously increased by the corresponding single dynamic factor value, considered in this particular case as an overload term.

In this work a practical case for bending strength is presented, based on a recorded flight mission, referring to a gear box for aerospace applications.

Obtained results in terms of equivalent forces and global dynamic factor values have been compared to those calculated by means of the classical ISO Standard formulae, based on the corresponding experimental data.

2. Theoretical background: method for calculating the global dynamic factor K_{AV}

The present section of the work describes in detail the theoretical background and the corresponding analytical procedure to calculate the global dynamic factor K_{AV}, referring only to the bending strength [4] for sake of brevity. Similar equations may be obtained also for pitting case [3].

As mentioned before, the method for obtaining the global dynamic factor K_{AV} aims to be a completion of that developed for the in-operation factor K_A [10-11], always in ISO environment; in particular, it concerns the velocity variation, as well as torques and tangential forces.

As in [10-11] where the service factor K_A has been calculated, the heart of this method involves the use of a fatigue curve of the component instead of the Woehler-damage curve of the material and, as also in ISO Standard [6], it utilizes the Miner damage rule and the corresponding exponent p of Woehler-damage curves (slope p) for bending case (and eventually for pitting one).

The substantial difference is that the loading blocks to be used in the Miner damage rule correspond to tangential force levels taking singularly into account the internal dynamic factor entity, that is each tangential force value F_{ti} has to be already multiplied by the corresponding dynamic factor K_{Vi} before being introduced in the Miner equation. Then, following a similar procedure to that developed in [10-11], the tangential force level F_{tDV} for which the damage entity can be considered as zero (and both corresponding service K_A and dynamic K_V factor values are equal to 1) has been obtained.

The global dynamic factor K_{AV} may be considered, following the definition found in [2], as the total mesh torque at operating speed respect to the nominal (design) transmitted.

In the present approach, instead of the design operation factor [2] (corresponding to the design conditions), the in-operation application factor (corresponding to the operation conditions [11]) has to be considered and so the corresponding global dynamic factor K_{AV} has to be defined as the equivalent tangential force, corrected by the dynamic terms, respect to that involving nihil damage (for both bending and pitting cases).

According to [2], [4], the tooth root stress σ_F (as already quoted) is the maximum tensile stress at the surface in the root and it may be calculated by the following equation:

$$\sigma_F = \sigma_{FO} K_A K_V K_{F\alpha} K_{F\beta} \quad (1)$$

where K_A and K_V are respectively application and dynamic factors, $K_{F\beta}$ and $K_{F\alpha}$ face and transverse load factors, σ_{FO} is the nominal tooth stress expressed by:

$$\sigma_{FO} = \frac{F_t}{bm_n} Y_F Y_S Y_\beta Y_B Y_{DT} \quad (2)$$

where F_t is the nominal tangential load, b is the face width, m_n is the normal module, Y_F, Y_S, Y_β, Y_B, Y_{DT} are respectively form, stress correction, helix angle, rim thickness and deep tooth factors.

Equation (2) may be expressed in a compact form as:

$$\sigma_F = \frac{F_t}{bm_n} K_V K_A A_V \quad (3)$$

where:

$$A_V = K_{F\alpha} K_{F\beta} Y_F Y_S Y_\beta Y_B Y_{DT} \quad (4)$$

The permissible bending stress σ_{FP}, following [4] (method B), is given by:

$$\sigma_{FP} = \sigma_{F\lim} Y_{NT} \frac{Y_{ST} Y_{\delta relT} Y_{RrelT} Y_X}{S_{F\min}} \quad (5)$$

where σ_{Flim} is the nominal stress number for bending from reference test gears, Y_{ST} is the stress correction factor, Y_{NT} is the life factor for tooth root stress expressed as a function of the the number of load cycles N_L, S_{Fmin} is the minimum required safety factor for tooth root stress, $Y_{\delta relT}$, Y_{RrelT}, Y_X are respectively relative notch sensitivity, relative surface and size factors.

Also equation (5) may be expressed in a compact form as:

$$\sigma_{FP} = \sigma_{F\lim} Y_{NT} B_V \quad (6)$$

where:

$$B_V = \frac{Y_{ST} Y_{\delta relT} Y_{RrelT} Y_X}{S_{F\min}} \quad (7)$$

This procedure makes the tooth root stress corresponding to the permissible bending stress as:

$$\sigma_F = \sigma_{FP} \tag{8}$$

So, by substituting equations (3) and (6) respectively in equation (8), the following relationship may be obtained:

$$\frac{F_t}{bm_n} K_V K_A A_V = \sigma_{F\lim} B_V Y_{NT} \tag{9}$$

Equation (9) provides the basis of the procedure, only if the number of endurance limit cycles N_{Lref} is known. For the case of bending stress, N_{Lref} generally corresponds [2-6] to 3×10^6, so:

$$\frac{F_t}{bm_n} K_V K_A A_V = \sigma_{F\lim} B_V \left[\frac{3\times10^6}{N_L}\right]^{\exp} \tag{10}$$

where *exp* means the general term for slope *p* of the material fatigue curve. If N_L coincides with the number of endurance limit cycles $N_{Lref} = 3\times10^6$, also the tangential force F_t coincides with the load level for which the damage entity can be considered as zero, so $F_t = F_{tDV}$; in this way, the F_{tDV} force value can be easily obtained, by putting in Equation (9) the product $K_V K_A$ equal to 1:

$$F_{tDV} = \frac{\sigma_{F\lim} bm_n B_V}{A_V} \tag{11}$$

Once F_{tDV} is known for that gear, the Miner procedure may be run, by considering only the fatigue cycles that really are damaging the gear, constituted by tangential force levels F_{ti}^* already multiplied by the corresponding dynamic factor ($F_{ti}^*=K_{Vi}F_{ti}$).

So, the equivalent tangential force F_{teqV} may be obtained, where only the tangential force levels F_{ti}^* higher than F_{tDV} are taken into account (respectively running for n_{iw} cycles).

$$F_{teqV} = \left[\frac{\sum_{i=1}^{L} n_{iw}\left(F_{ti}^*\right)^{\frac{1}{\exp}}}{\sum_{i=1}^{L} n_{iw}}\right]^{\exp} \tag{12}$$

The numbers of cycles n_{iw} means both a number of cycles (as in [6]) or, depending on the available data, an already weighted (subscript *w*) number of cycles, as described in the following paragraph related to a practical case.

Finally, the global dynamic factor K_{AV} can be obtained as:

$$K_{AV} = \frac{F_{teqV}}{F_{tDV}} \tag{13}$$

3. Practical case: accessories gearbox for aerospace application

The practical case analyzed in the present work refers to the global dynamic factor K_{AV} determination of a gear for aerospace application. This gear belongs to an accessories gearbox for which are available both power and speed values during the phases of a complete flight mission.

More in detail, the gearbox consists in a series of cylindrical and conic gears and it distributes the requested powers from an input source (INPUT, see Figure 1) to the accessories (OIL PUMP, FUEL PUMP, ….., see Figure 1).

In particular, the global dynamic factor has been calculated referring to the OIL PUMP gear that engages with the INPUT gear (on the left) and with the IDG gear (on the right). The OIL PUMP gear geometry is shown in Figure 2.

Gears data are: modulus $m_n = 2.54$, pressure angle α=20°, number of teeth z_{IN} = 52 (INPUT), z_{OP} = 111 (OIL PUMP), z_{IDG} = 52 (IDG). Material has been chosen as hardening steel.

Experimental data are available on this gear in terms of power, rotating speed and duration of each phase of a flight mission, referring to the INPUT gear of the gearbox.

Table 1 reports the following data for each flight phase (Taxi out, take off, ….): duration t, rotation speed ω_{OP} of OIL PUMP gear, number of cycles n_{iw} (already weighted respect to the complete flight mission). Tables 1 shows also: tangential forces values F_{tOP1} and F_{tOP2} of OIL PUMP gear involving respectively the contact with INPUT and IDG gears, internal dynamic factor values K_{Vi} for OIL PUMP gear (calculated following ISO Standard formula [2], Method B), tangential forces values F_{tOP1} and F_{tOP2} already multiplied by the corresponding K_{Vi} and then expressed as F_{tOP1}^* and F_{tOP2}^*. Tangential forces values has been calculated by the corresponding equilibrium equations, here omitted for sake of brevity.

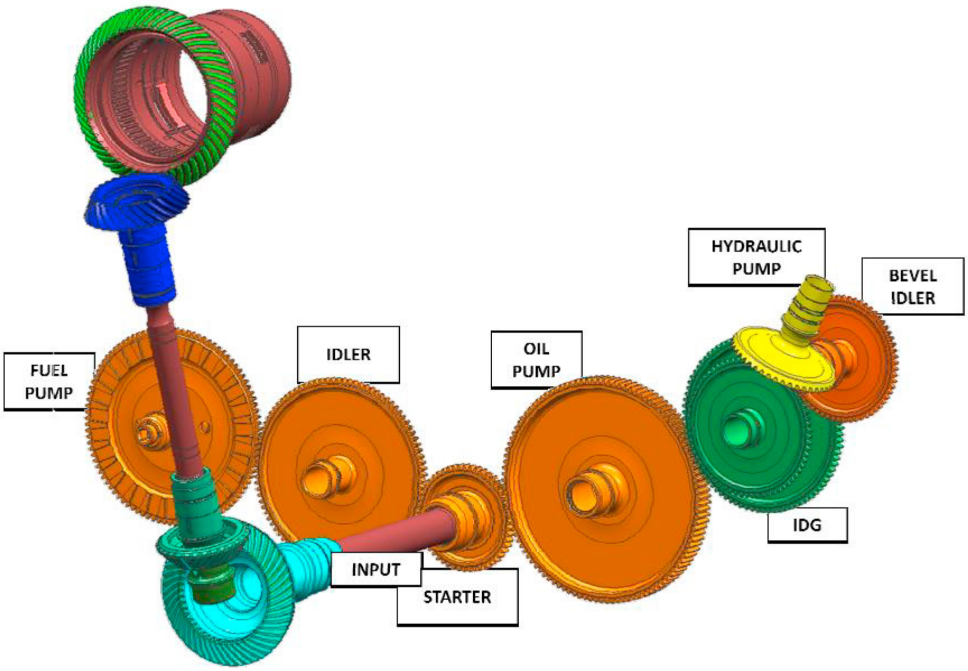

Fig. 1. Accessories gear box.

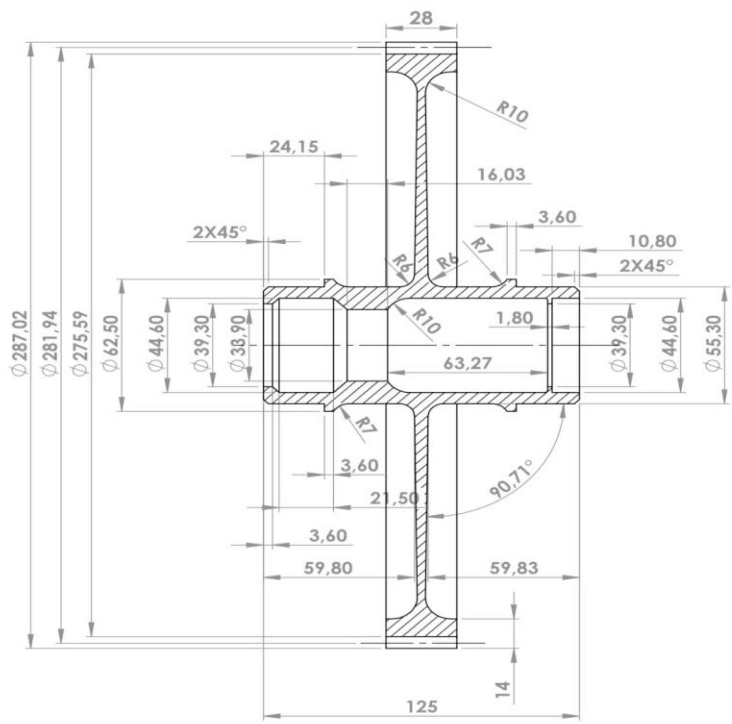

Fig. 2. Oil Pump gear geometry (all dimensions are in mm).

Table 1. Flight mission.

Flight phase	Taxi out	Take off	Climb	Cruise	Top descent	Descent	Landing	Deceleration	Taxi in
t [s]	540	120	1380	19920	10	1560	240	5	300
ω_{OP} [rpm]	3600	6820	6820	6110	4777	4034	4034	4777	3600
n_{iw} %	0.0224	0.0050	0.0574	0.8273	0.0004	0.0648	0.0100	0.0002	0.0125
F_{tOP1} [N]	6822	4124	4458	4380	6174	7480	7292	5317	6822
F_{tOP2} [N]	6549	3813	4147	4144	6015	7292	7048	5111	6549
K_{Vi}	2,438	2,860	2,895	2,890	2,355	2,213	2,365	2,245	2,438
F_{tOP1}^* [N]	16633	11795	12905	12658	14540	16554	17246	11937	16633
F_{tOP2}^* [N]	15966	10905	12004	11977	14166	16138	16669	11474	15966

4. Results and discussion

Table 2 reports the coefficients values related to OIL PUMP gear obtained following the ISO Standard formulae (method B) [2, 4]. The exponent of the Woehler-damage line is generally indicated as *exp*. Coefficients have been obtained following the indications of the manufacturer about material, surface finish, and so on.

Table 3 summarizes the obtained results in terms of global dynamic factor K_{AV} for OIL PUMP gear, following the method proposed in the present work. Calculations have been done referring to two different values of the length width, respectively 18 and 25 mm (first column). In particular, second and third columns show respectively the values of constant A_V and B_V (equations (4) and (7)), fourth column the force F_{tDV} (equation (11)), fifth column indicates the damaging phases, sixth column the equivalent force F_{teqV} (equation (12)) calculated by considering only tangential force levels F_{ti}^* higher than F_{tDV}, finally the last column shows the global dynamic factor K_{AV} values (equation (13)).

It may be observed that in the calculation of the equivalent force F_{teqV} both forces F_{tOP1}^* and F_{tOP2}^* have been considered, referring to each of them half of the corresponding total number of cycles (percent) for that mission phase.

Tables 4 and 5 report, as a comparison, the in-operation service factor K_A values, independently obtained, following the method described in [10-11], similar to that presented in this paper; these values have been obtained by considering as a constant the internal dynamic factor K_V, the maximum one (climb phase) for Table 4 and the minimum one (descent) for Table 5 respectively.

More clearly, both global dynamic factor K_{AV} and in-operation service factor K_A have been obtained as the ratio between the equivalent tangential force and the load level for which the damage entity can be considered as zero.

The substantial difference is that, in the first case (Table 3), the equivalent tangential force has been obtained by processing each single force level that has been previously multiplied by the corresponding overload dynamic factor and the load level for which the damage entity can be considered as zero has been in turn obtained by setting both internal and external dynamic factors (K_V and K_A respectively) equal to one.

In the second case (Tables 4 and 5), only the in-operation service factor K_A has been obtained by this procedure (that is by ratio between the equivalent tangential force and the load level for which the damage entity can be considered as zero, obtained by putting only the service factor equal to one) and the dynamic factor has been considered as constant and calculated referring to an established velocity value, following ISO indications [1-2].

From the analysis of these results, it may be observed that the in-operation factor K_A singularly calculated (Tables 4 and 5) is strongly influenced by the velocity and, once multiplied by the chosen dynamic value K_V, it may bring to overestimate the tangential force and the corresponding maximum tensile stress (at the surface in the bending case).

Table 2. ISO bending strength coefficients.

$K_{f\beta}$	$K_{f\alpha}$	Y_F	Y_S	Y_β	Y_B	Y_{DT}	σ_{Flim} [N/mm^2]	S_{Fmin}	Y_{ST}	$Y_{\delta relT}$	Y_{RrelT}	Y_X	N_{Lref}	*exp*
1.209	1.065	1.128	2.704	1	1	1	525	1	1.4	0.997	1.004	0.99	3x10^6	0.115

Table 3. Global dynamic factor values.

b [mm]	A_V	B_V	F_{tDV} [N]	Damaging phases	F_{teqV} [N]	K_{AV}
18	3.9273	1.4426	8817	all	13520	1.533
25	3.9273	1.4426	12246	1,5,6,7,9 3,4 (only for F_{tOP1}^*)	13260	1.083

Table 4. In-operation service factor values (K_{Vmax}).

b [mm]	K_{Vmax}	A_V	B_V	F_{tDV} [N]	Damaging phases	F_{teqV} [N]	K_A
18	2.895	11.3695	1.4426	3046	all	5638	1.851
25	2.895	11.3695	1.4426	4230	1,5,6,7,8,9 3,4 (only for F_{tOP1})	5680	1.343

Table 5. In-operation service factor values (K_{Vmin}).

b [mm]	K_{Vmin}	A_V	B_V	F_{tDV} [N]	Damaging phases	F_{teqV} [N]	K_A
18	2,213	8,691	1.4426	3984	all (for F_{tOP1}) 1,3,4,5,6,7,8,9 (for F_{tOP3})	5638	1.415
25	2,213	8,691	1.4426	5534	1,5,6,7,9	5584	1.009

5. Conclusions

The method proposed in the present paper aims to complete the ISO Standard approach for calculate the application factor [6] following two fundamental aspects.

Firstly, from the loading condition point of view, instead of considering the torque values that traditionally are involved in the design phase, the in-operation conditions have been taken into account, related to the tangential force levels obtained once pitch diameters are known for that gear.

Secondly, basing on a similar procedure, all dynamic effects have been considered, both local for the gear and global for the transmission.

In other words, the actual working conditions of the gear box have been taken into account, related to the load amplitude levels and to the variable velocity regimes.

The presented approach has been developed in detail from the theoretical point of view for the bending case.

Obtained formulae have been applied to an aerospace transmission, for which experimental data were available.

From the analysis of these results, it may be observed that the procedure to obtain the global dynamic factor K_{AV} provides overload conditions more tuned on the actual operating conditions of the whole transmission.

On the contrary, if the dynamic factor is considered as a constant, it depends on the speed entity and it may bring to design substantially over dimensioned gears. In this case, the so obtained dynamic factor value has also to be multiplied by the application factor, leading to overload conditions that may substantially vary the corresponding calculated service life, both for bending strength and pitting resistance.

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