

Date of publication xxxx 00, 0000, date of current version xxxx 00, 0000.

Digital Object Identifier 10.1109/ACCESS.2017.Doi Number

On-Line Estimation Assessment of Power Systems Inertia with High Penetration of Renewable Generation

F. Allella¹, E. Chiodo², Member, IEEE, G.M. Giannuzzi¹, D. Lauria², F. Mottola³, Senior Member, IEEE

¹Terna S.p.A., Rome, I 00100 Italy

²Department of Industrial Engineering, University of Naples Federico II, Naples, I 80125 Italy

³ Department of Electrical Engineering and Information Technology, University of Naples Federico II, Naples, I 80125 Italy

Corresponding author: F. Mottola (e-mail: fabio.mottola@unina.it).

ABSTRACT Large-scale deployment of renewable energy sources in power systems is basically motivated by two universally recognized challenges: the need to reduce as far as possible the environmental impact of the massive increase of energy request and the dependency on fossil-fuel. Renewable energy sources are interfaced to the network by means of interfacing power converters which inherently exhibit zero inertia differently from the conventional synchronous generators. This matter jointly to the high level of time variability of the renewable resources involve dramatically frequency changes, recurrent frequency oscillations and high variability of frequency profile in general. The need of a fast estimation of time variability of the power system inertia arises at the aim of predicting critical conditions. Based on the analysis of some actual data of the Italian Transmission Network, in this paper the authors propose an autoregressive model which is able to describe the dynamic evolution of the power system inertia. More specifically, the inertia is modeled as the sum of a periodic component and a noise stochastic process distributed according a non-Gaussian model. The numerical results reported in the last part of the paper, demonstrating the efficiency and precision of estimation of inertia, allow justifying the assumptions of the above modeling.

INDEX TERMS Power Systems, Renewable Energy Sources, Rotational Inertia, Auto-Regressive Models, Statistical Inference, Stochastic process.

I. INTRODUCTION

The containment of frequency deviations within assigned ranges is of vital importance for electrical interconnected systems. Thanks to their stored kinetic energy, synchronous generators inherently contribute to the ability in counteracting the system frequency changes [1], [2]. The increasing power generation by renewable energy sources (RESs), such as wind and solar generation plant, is commonly associated to the reduction of the whole system inertia [2]-[6]. This kind of generation systems are interfaced to the electrical network by means of static converters which are in most cases controlled independently of the system frequency, this involving the unrequired effect of inertia lowering. This may be also interpreted as the result of the partial or full decoupling between the electrical machine speed and the network frequency. It is not trivial to put in evidence that RES production is random and intermittent by nature. It has to be highlighted the massive tendency in mitigating this phenomenon by improving the inertia characteristics through the optimal control of the power

converters, in such a manner that the subsystem constituted by the converter and the electrical generator behaves like a synchronous generator [7], [8].

In this contest the inertia estimation arises a major challenge for the secure operation of interconnected power systems. It is mandatory to introduce on-line tools able to monitor the system inertia in real time through methods which allow facing with the unavoidable uncertainties implied by renewable generation sources. In [9] a feasible technique is described in order to perform a robust estimation based on the information related to the production units and synchronous condensers. The estimation is performed in terms of kinetic energy instead of inertia and could be affected by some uncertainties. Estimation procedures are also presented in [10]-[13] again based on the measures of frequency transients provided by synchronized phasor measurement units. By taking into account the effect of the electrical distance between the measurement unit and the location of the event, methods for the accurate calculation of the frequency transient events have been then adopted, such

as a moving average filter [11], the analysis of the instantaneous nature of the inertia [8], [12], low-pass Butterworth filter [13], [14]. The method proposed in [15] uses the electromechanical oscillation response measured with phasor measurement units to estimate the equivalent inertia of the system on the basis of the classical swing equation. Extended Kalman filter-based method is also adopted for the inertia estimation of synchronous generators: in [16] the sensitivity of the method to the assumed time of disturbance is discussed. Based on the data provided by the phasor measurement units, on-line estimation methods are also proposed in [17] and [18]. A statistical approach is proposed in [19], where an on-line estimation method based on relatively small frequency variation was presented. A Bayes estimation method is proposed in [5]. In [20] an algorithm is proposed for the on-line inertia estimation through a first-order nonlinear aggregated power system model combined to dynamic regressor and mixing procedure. Based on a Gaussian mixture model with temporal dependence encoded as Markov chains, a method is proposed in [21] for the on-line estimation of the system inertia. In [22], an on-line estimation method for the inertia characteristics of power systems is proposed based on the synchronized measurements data of the change in frequency and active power. In [23] a methodology based on an autoregressive moving average exogenous input model is proposed, which includes the accurate time window identification, starting from the start of the disturbance. In [24] an on-line-estimation method is proposed which requires the injection of an additional probing signal used to identify in real time the time-varying and nonlinear equivalent inertia constant in power systems with complex heterogeneous components. In the approach proposed in [25], the real time estimation of power system inertia is obtained as time response of system identified models to a specific input and is based on the system identification. Some of the approaches discussed for the on-line estimation methods are summarized in Tab. I.

TABLE I
SOME APPROACHES FOR ON-LINE ESTIMATION OF THE INERTIA

Ref.	Method	Ref.	Method
[5]	Bayes based Estimation	[21]	Gaussian mixture model with temporal dependence encoded as Markov chains
[9][17] [18][19] [22]	Measurement based method	[23]	Autoregressive moving average exogenous input model and accurate time window identification
[15]	Electromechanical oscillation response measured with phasor measurement unit	[24]	Injection of additional probing signals used to identify the time-varying equivalent inertia constant
[20]	First-order nonlinear aggregated power system model combined to dynamic regressor and mixing procedure	[25]	Time response of system identified models to specific inputs based on suitable model identification

As shown in the literature, the direct estimation method is the one based upon the calculation of the rate of change of frequency which, in case of reduced synchronous inertia could activate disconnection of generation systems and loads, even jeopardizing the grid stability. The assessment based upon the rate of change of frequency could be unsatisfactory since it requires the precise knowledge of the instant at which the contingency starts. Furthermore, the swing equation provides useful information only over a restricted time interval, since they do not allow to take properly into account the governor actions of the power system synchronous generators [20]. Another crucial aspect is related to the diversity of the frequency behavior according to the location of the measurement system. In this regard, it is easy to argue that a reliable estimate of the power system inertia cannot be based upon a single measurement, and that the most proper choice is the combination of various measurements, evaluating the dynamic behavior of the center of inertia of the frequency. To overcome the issues of the methods discussed, in this paper it is proposed a method which uses a stochastic process to describe the system inertia. The process is obtained by a statistical analysis of the system inertia data which can be recursively updated in view of an on-line estimation. At this aim, efforts have been devoted to the proposal of a novel method based on the statistical estimation of the characteristics of the system inertia by focusing on the case of high penetration of RESs. In particular, the proposed method can capture the dynamic evolution of the power system inertia on the basis of the analysis of actual data. Starting from the identification of the parameters, the inertia is modeled as the sum of periodic components and a noise stochastic process distributed according to a non-gaussian model.

The contributions in this work are as follows:

- the proposal of a new statistical method for the on-line estimation of the power system inertia based on the recursive observation data of the selected area network;
- based on the classical time series theory, the proposed method uses a probabilistic model to characterize the system inertia with high variations caused by the presence of large penetration of RESs;
- the spectral analysis is performed with respect to the actual data, thus allowing the identification of the case of time-varying harmonics which permits to describe the inertia through a simple stochastic model;
- the statistical analysis on the actual data allows representing the inertia dynamics as the sum of periodic components and a noise stochastic process distributed according to a non-gaussian model.

The rest of the paper is organized as follows. Theoretical background is reported in Section II. In Section III and IV the dynamic estimation model is presented, focusing on the peculiarities of both low and high penetration of RESs, respectively. In Section V the numerical application is presented. Our conclusions are drawn in Section VI.

II. PRELIMINARY CONCEPTS

The inertia constant is a parameter related to the ability in counteracting the frequency changes due to the unavoidable power unbalances occurring in power systems. The mechanism of opposition is due to the kinetic energy gained by the rotating masses of the synchronous machines and the rotating loads. The rotational energy E_{kin} of a single synchronous generator is given by the well-known formula:

$$E_{kin} = \frac{1}{2} J \Omega_n^2 \quad (1)$$

where J is the moment of inertia of the group constituted by the synchronous machine and the turbine and Ω_n is the rated rotor angular frequency. The inertia constant H , expressed in s, for a single synchronous machine is currently defined as:

$$H = \frac{E_{kin}}{S} \quad (2)$$

where S is the rated power of the generator and inertia constant H denotes the time duration for which the group can provide its rated power solely with its stored kinetic energy.

The dynamic of a single synchronous group is properly described by the swing equation:

$$2H \frac{d}{dt} \left(\frac{f}{f_n} \right) = \frac{f_n}{f} \frac{P_m - P_e}{S} \quad (3)$$

with f (f_n) the frequency (nominal frequency) and P_e (P_m) the electric (mechanic) power.

The definition of the inertia constant can be directly extended to a power system with n_G machines, once the inertia constants H_k ($k = 1, \dots, n_G$) and the corresponding rated powers $S_{B,k}$ ($k = 1, \dots, n_G$) have been assigned; in this case the system inertia constant (H_{sys}) can be evaluated as:

$$H_{sys} = \sum_{k=1}^{n_G} \frac{H_k S_{B,k}}{S_B} \quad (4)$$

with S_B the total rated power of the n_G generators, that is:

$$S_B = \sum_{k=1}^{n_G} S_{B,k} \quad (5)$$

In the case of actual power networks that consist of many interconnected synchronous generators, a powerful concept is the center of inertia frequency, f_{COI} , defined as follows [26]:

$$f_{COI} = \frac{\sum_{k=1}^n H_k f_k}{\sum_{k=1}^n H_k} \quad (6)$$

with f_k the frequency of the k th generator. The variable f_{COI} is introduced by the observation that the inertia estimation cannot be performed on the basis of a single frequency measurement. However, it is often considered more useful to avoid the need of making recourse to the frequency measurements, by exploiting the knowledge of the total

kinetic energy, $E_{kin,T}$, of the whole power system expressed in MWh and given by:

$$E_{kin,T} = H_{sys} S_B = \sum_{k=1}^{n_G} H_k S_{B,k} \quad (7)$$

$E_{kin,T}$ can be directly evaluated by knowing the position of the circuit breaker of each production unit. As clearly described in [27], this calculation requires that a SCADA system gives information about the synchronous machines status, thus permitting the direct estimation of the overall system kinetic energy.

With the aim of defining the inertia of a power system comprehending RESs, in the following it is assumed that the renewable-based generators interfaced to the network do not contribute to the kinetic energy. Hence, the inertia of the whole system can be coherently defined as:

$$H_{sys} = \frac{\sum_{k=1}^{n_G} H_k S_{B,k}}{S_B + S_R} \quad (8)$$

being S_R the total amount of active power injected by the renewable energy generators. This quantity can be considered a stochastic process (SP), i.e., it exhibits the characteristic of random variability and time variability.

III. LOW RENEWABLE PENETRATION: A KALMAN FILTER-BASED ESTIMATION METHOD

In the previous section, based upon the general theory of the power system inertia, it has been derived that the estimation of the inertia is linked to the ratio between the installed power of renewable sources and that corresponding to the rotational generators. As discussed in [5] and [27], by introducing the parameter ρ :

$$\rho = \frac{S_R}{S_B} \quad (9)$$

the inertia can be directly evaluated through a linear approximation as a function of the above ratio ρ . More in depth, the inertia of the system at the time instant $t = j\Delta t$, $H_{sys,j}$, can be directly related to the value to the parameter $\rho_j = S_{R,j}/S_{B,j}$, through the approximated linear relationship:

$$H_{sys,j} = a - b\rho_j \quad (10)$$

where a and b are positive constants that depend on the system parameters, such that $H_{sys,j}$ is maximum (e.g., $H_{sys,j} = 3.5$ s) when $\rho_j = 0$, and, theoretically, $H_{sys,j}$ is minimum (e.g., $H_{sys,j} = 0$) when $\rho_j = 1$.

Indeed, (10) can be justified by introducing the weighted mean value of the constant inertia values H_k of the rotating machines, $H = \sum_{k=1}^{n_G} H_k S_{B,k} / S_B$ (which is the same of (4) in the case all the generators are rotating machines), thus resulting:

$$H_{sys} = \frac{\sum_{k=1}^n H_k S_{B,k}}{S_B + S_R} = \frac{H S_B}{S_B + S_R} = \frac{H}{1 + \rho} \quad (11)$$

By adopting the Mc Laurin expansion in (11), i.e., $(1+x)^{-1} \approx (1-x)$ as $x \rightarrow 0$, it is deduced that for low values of the ρ , the inertia of the whole system H_{sys} can be described, as a first order approximation, as:

$$H_{sys} \cong H(1 - \rho) \quad (12)$$

It is worth to remark that when $S_R \ll S_B$, $\rho \rightarrow 0$ and the total system inertia constant can be approximated by (12).

The proposed recursive estimation exploits the probabilistic knowledge of the RES and its evolution in time, which is generally available, especially in very short time operation. With reference to a power system having both synchronous generators and RES generators, the SP of their generation powers are denoted by $\{S_{B,j}; j = 1,2, \dots\}$ and $\{S_{R,j}; j = 1,2, \dots\}$, respectively.

On the basis of actual data of typical transmission systems, it can be assumed that the succession of the logarithms of each SP – i.e. of both sequences $\{S_{B,j}; j = 1,2, \dots\}$ and $\{S_{R,j}; j = 1,2, \dots\}$ – follows a dynamic linear model [28], which is often adopted, with good accuracy, for the SPs of electrical load in very short time applications [5]. This model is supposed to generate the generic “load” values – that in this application refer to $S_{B,k}$ and $S_{R,k}$ – at time instant $t = j\Delta t$ according to the basic dynamic linear model equations, herein reported:

$$X_{j+1} = X_j + \xi_j \quad (13)$$

$$Z_j = X_j + \eta_j$$

being the SPs $\{\xi_j; j = 1,2, \dots\}$ and $\{\eta_j; j = 1,2, \dots\}$ white Gaussian noise sequences. The first relationship of (13) refers to the “load” evolution from the state at j to $j + 1$. The latter is a simple linear model linking the observation or measurement Z_j to the true state X_j . The above assumptions lead to a straightforward application of the Kalman filter approach for the estimation of the power system inertia.

However, this kind of estimation procedure is valid only under the assumption of low values of the parameter ρ . In practical cases, the impact of the renewable could be very massive and creating abnormal areas characterized by high values of ρ .

IV. A GENERAL DYNAMIC MODEL OF POWER SYSTEM INERTIA

A dynamic modeling of power system inertia in case of large penetration of RES is proposed in this section on the basis of the classical time series theory and on the analysis of real data of power system inertia. The aim is to specify a probabilistic model of which the observed data is a realization. The probabilistic model is developed on the basis of numerical tests performed with respect to a suitable area of the Italian power system.

A. ANALYSIS OF ACTUAL DATA

The data used in this paper refer to information made available by the Italian TSO, Terna S.p.A. [29] and are related to an area which has been considered isolated at the aim of verifying the goodness of the dynamic modeling. The available data refer to the:

- total demand;
- production per type of energy source;
- production from RESs; and
- data of the inertia constant for each type of the energy source.

These data refer to a time horizon of one month sampled at 15 minutes. In Fig. 1, the values assumed by the parameter ρ in an area of the Italian transmission system is reported.

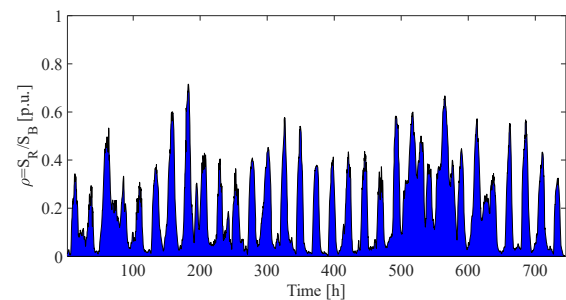


FIGURE 1. Actual data of the values that ρ assumes in the selected area of the Italian transmission network.

Fig. 1 clearly puts in evidence that the values that ρ assumes are very variable, by ranging from values slightly than zero up to values higher than 0.7, while its mean value assumes the value 0.2. In this scenario, the inertia estimation performed by exploiting the linear model (12) and the Kalman filter (13) – can involve local unacceptable errors in cases when ρ is large. Fig. 2 reports the profiles of both kinetic energy and the parameter ρ . Fig. 3, reports the values of kinetic energy corresponding to the assumed values of parameter ρ .

It is worth to note that during most of the time (larger than 85%) $\rho \leq 0.4$ and only in a few percentages of cases (lower than 2%) $\rho \geq 0.6$. This means, that the assumption of linear model can be considered still valid at the purpose of explaining the correlation between the involved variables in qualitative way. The more general estimation method, however, which overcomes the assumption of linearity, is described in the next sub-section.

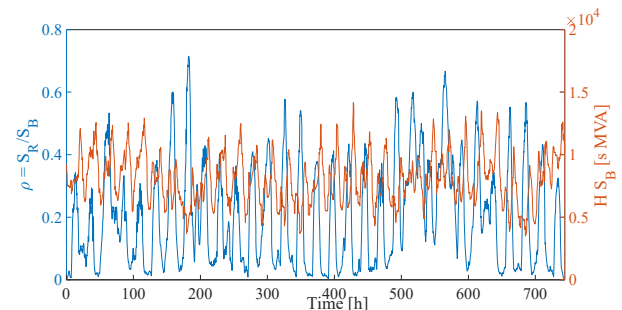


FIGURE 2. Actual values of ρ and of kinetic energy for the selected area network.

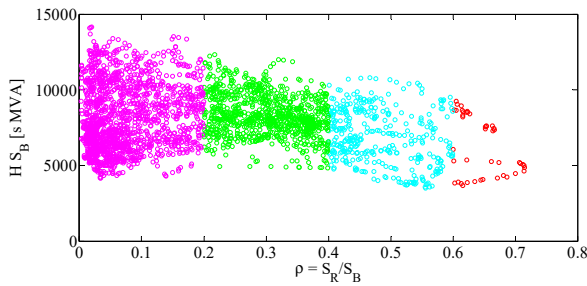


FIGURE 3. Measured values of ρ and corresponding values of the kinetic energy of the selected area network.

At the aim of determining the total inertia of the selected area, the kinetic energy is primarily evaluated by the knowledge of both the inertia time constant of the synchronous generators installed in the selected area network and the individual generator breaker position, available by an existing SCADA system. Further information inherent to the dynamic evolution of the renewable amount allows for determining the so-called “measured” values of inertia, shown in the Fig. 4. In other word, the measured values of power system inertia are built from (8), where the right-hand member is effectively measured [28]. In Fig. 4, high correlation is clearly exhibited between H_{sys} and the parameter ρ : the valleys of the system inertia H_{sys} correspond to the peak values of the parameter ρ , that is the peak values of the power produced by the renewable energy resources.

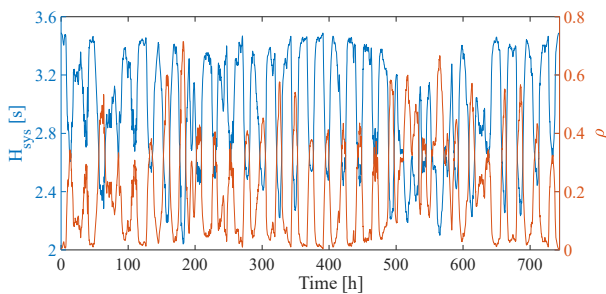


FIGURE 4. Time varying inertia and ρ of the selected area network.

A large correlation between the system inertia and the renewable power production S_R is also registered, while a slight level of correlation can be measured between the total inertia and the rotating power, S_B . This can be motivated by the formula (12), $H_{sys} \cong H(1 - S_R/S_B)$. Since the correlation coefficient is a measure of the linear dependence between variables, indeed, it is obvious the linear dependence from S_R while this circumstance does not hold for H_{sys} and S_B ¹. This is confirmed in Fig. 5, where the combined plot of H_{sys} with S_R (Fig. 5.a) and S_B (Fig. 5.b) are reported.

¹ It is opportune to remark that, with reference to the relation $H_{sys} \cong H(1 - S_R/S_B)$, that is to say $Z = k(1 - X/Y)$, Z may be weakly correlated (linearly) to Y while, obviously, even if heavily dependent on Y . A simpler example of the above is the following: let (Q, W) a pair of random variables, such that $W = Q^2$, then they are heavily dependent. Nonetheless if, f.i. Q is

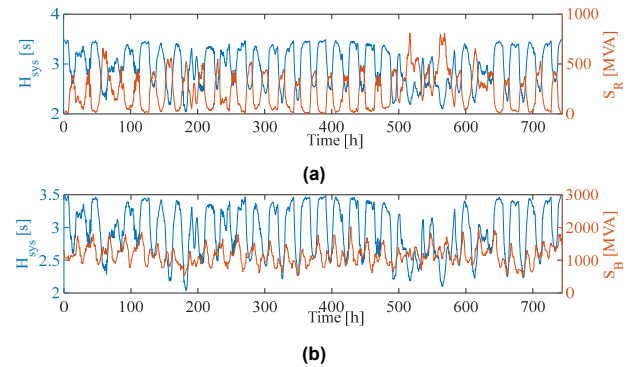


FIGURE 5. Time varying inertia and S_R (a) and S_B (b) of the selected area network.

B. GENERAL ESTIMATION MODEL

The spectral analysis performed with respect to the data referring to Fig. 4 puts in evidence the presence of a particular case of time-varying harmonics, characterized by changes of only amplitudes and phases (the fundamental frequency and, consequently, the harmonics frequencies are fixed) [30]. The inertia process can be described by the simple stochastic model:

$$H_{sys}(t) = s(t) + Z(t) \quad (14)$$

where $s(t)$ is the sum of time-varying harmonics and $Z(t)$ is the non-periodic stochastic component of the process Inertia. Hence, the function $s(t)$ can be defined as:

$$s(t) = a_0 + \sum_{h=1}^n a_h(t) \cos(\omega_h t) + b_h(t) \sin(\omega_h t) \quad (15)$$

Due the low number of significant harmonics, the Goertzel technique exhibits recognized more efficient performances with respect to the classical Fast Fourier Transform. As well known, it is based upon the employment of a second-order infinite impulse response filter for estimating the parameters of the generic harmonic [31].

The discretized version of (14) becomes, with the obvious meaning of the symbols:

$$H_{sys,j} = s_j + Z_j \quad (16)$$

where it was assumed the time $t = j\Delta t$. At the aim of analyzing the statistical properties of the above process, based upon an adequate statistical data analysis, it is worth to note that Z_j is weakly stationary since its mean function $\mu_{z,j}$ is independent of time and the covariance function $\gamma_{z,j+h,j}$ is independent of time for each lag $h = 1, 2, \dots$

uniformly distributed in the real interval $[-1, 1]$, so that Q has the expected value and all its odd moments equal to zero, then W (being $W = Q^2$) and Q are uncorrelated although no statistically independent. Indeed, $E(QW) = E(Q^3) = 0$, which is also the product of the means. This implies that the correlation coefficient is zero, i.e. the two random variables are uncorrelated.

A deep analysis of the available data puts also in evidence that Z_j is an SP non-normal distributed. At the aim of a feasible and accurate description of Z_j , the authors propose an autoregressive model whose innovations have non-normal underlying distribution. More specifically, the Logistic distribution is adopted as underlying distribution, as it results the most suitable from classical statistical tests.

Under the assumption of a Logistic underlying distribution, the following first-order autoregressive model is proposed:

$$Z_j - \hat{\phi}_z Z_{j-1} = \hat{\mu}_z + \hat{a}_j \quad (17)$$

where $\hat{\phi}_z$ is a proper parameter, $\hat{\mu}_z$ is the estimate of the mean value of Z_j and \hat{a}_j is distributed according to the symmetric Logistic distribution.

As it is well-known [31], the estimation of the parameters $\hat{\phi}_z$ and $\hat{\mu}_z$ can be obtained by exploiting the following relationship:

$$\begin{bmatrix} \hat{\mu}_z \\ \hat{\phi}_z \end{bmatrix} = \begin{bmatrix} n_s & \sum y_{j-1} \\ \sum y_{j-1} & \sum y_j^2 \end{bmatrix}^{-1} \begin{bmatrix} \sum y_j \\ \sum y_j y_{j-1} \end{bmatrix} \quad (18)$$

where n_s is the number of samples and y_j is given by the difference of the inertia and the periodic function s_j . In the case we assume the symmetric logistic distribution, an estimate of the parameters $\hat{\phi}_z$ and $\hat{\mu}_z$ can be performed by starting from the well-known likelihood function, $L(\phi_z, \mu_z, \beta_z)$ [32]:

$$L(\phi_z, \mu_z, \beta_z) = (1/\beta_z)^{n_s} \prod_{j=1}^{n_s} \frac{e^{-x_j}}{(1 + e^{-x_j})^2} \quad (19)$$

with:

$$X_j = (1/\beta_z)(Z_j - \phi_z Z_{j-1}) - \mu_z \quad (20)$$

where β_z is the parameter of the symmetric Logistic distribution which is related to the variance of \hat{a}_j :

$$\text{Var}(\hat{a}_j) = \frac{\pi^2}{3} \beta_z^2 \quad (21)$$

By equating the partial derivatives of the log-likelihood function with respect to the unknowns μ_z , ϕ_z and β_z to zero:

$$\begin{aligned} \frac{\partial \ln L(\phi_z, \mu_z, \beta_z)}{\partial \mu_z} &= 0 \\ \frac{\partial \ln L(\phi_z, \mu_z, \beta_z)}{\partial \phi_z} &= 0 \\ \frac{\partial \ln L(\phi_z, \mu_z, \beta_z)}{\partial \beta_z} &= 0 \end{aligned} \quad (22)$$

the following equations system is obtained:

$$\begin{aligned} n_s - 2 \sum_{j=1}^{n_s} \frac{1}{1 + e^{x_j}} &= 0 \\ \sum_{j=1}^{n_s} Z_{j-1} - 2 \sum_{j=1}^{n_s} Z_{j-1} \frac{1}{1 + e^{x_j}} &= 0 \\ n_s - \sum_{j=1}^{n_s} X_j + 2 \sum_{j=1}^{n_s} X_j \frac{1}{1 + e^{x_j}} &= 0 \end{aligned} \quad (23)$$

whose solution provides the estimate of the parameters $\hat{\mu}_z$, $\hat{\phi}_z$ and $\hat{\beta}_z$.

Eventually, the proposed on-line estimation method can be summarized as follows:

- based on the set on the available data of inertia, their periodic function s_j is derived;
- the set of data y_j are derived as the difference of the inertia data and the periodic function s_j ;
- evaluation of the parameters of the Z_j SP according to the proposed auto-regressive model with innovations having a symmetric Logistic underlying distribution;
- estimate of inertia time variation is obtained through (14).

V. APPLICATION TO THE REAL CASE

In this section the application of the proposed method for the on-line estimation of the power system inertia is reported. The data used to test the time series approach proposed in Section IV is that reported in Fig. 4.

The selected area under investigation is an islanded system connected via 400 kV undersea cables; the reason of this choice is the relatively small extension of the electrical system and the high penetration level of the installed inverter-based generation. Large share of power production derives from renewable and dispersed generation, which mainly includes photovoltaic (23%) and wind (16%) systems.

With reference to the data under study (Fig. 4), 2880 samples (in the following they are referred to as ‘measured samples’) were used to analyze the inertia behavior and to derive the inertia estimate. The last 96 samples – which correspond to the last samples of the available day – were used to test the estimate at the aim of verifying the proposed method (in the following they are referred to as ‘test samples’). First of all, the periodic function s_j ($j=1, \dots, 2880$) was derived on the basis of a different number of samples. In order to analyze the features of the periodic components of the signal, Fig. 6 reports the spectrum of the whole test sample set. In the figure it is shown that the inertia of the system clearly shows four main periodic components, corresponding to 24 hours, 12 hours, 6 hours, and 4 hours.

The frequency of the four most relevant periodic component and their amplitude which corresponds to a difference number of samples are reported in Fig. 7. In particular, the four main harmonic components are reported corresponding to sample’s number ranging in the interval [1500, 2880].

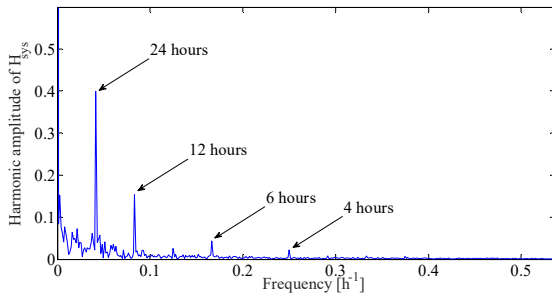


FIGURE 6. Amplitude of the harmonics of the time varying inertia of the selected area network.

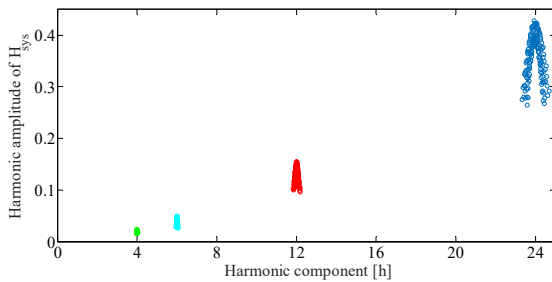


FIGURE 7. Harmonic components corresponding to different number of samples.

As clearly shown in Fig. 7, the four harmonic components always correspond to the four components highlighted in the Fig. 6. Obviously, some slight differences are highlighted due to the continuous variation of the system inertia. It is interesting to note that the number of harmonics to be calculate is small, thus allowing the estimation of the harmonics' parameters through the Goertzel technique [31]. Once the harmonic components have been found, the periodic function of the system s_j ($j=1, \dots, 2880$) is derived. The comparison of the periodic function and the measured inertia values is reported in Fig. 8. The difference between the measured data and the periodic function – that is Z_j ($j=1, \dots, 2880$) – is reported in Fig. 9.

As already put in evidence in the previous section, it appears convenient to describe Z_j as an auto-regressive model with innovations with non-normal underlying distribution. At this purpose, the comparison between the classical Gaussian distribution and the Logistic distribution, reported in Fig. 10, at a glance put in evidence the better ability in describing the innovations.

The maximum Likelihood approach presented in the section IV allows deriving the parameters $\hat{\mu}$, $\hat{\phi}$ and $\hat{\beta}$ to estimate the SP of Z_j . It derives $\hat{\mu} = -0.0191$, $\hat{\phi} = 0.6308$, and $\hat{\beta} = 0.0111$.

Based on these values, the estimated values of the system inertia have been derived for the last day of the available data ($j=2880, \dots, 2976$). Both estimated and measured data are reported in Fig. 11. Fig. 12 reports the absolute percentage error of the estimated inertia compared to the actual inertia.

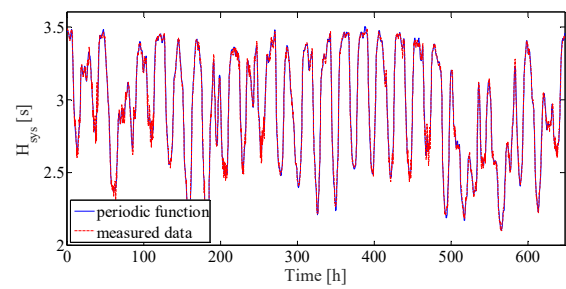


FIGURE 8. Periodic function of the system inertia and measured data.

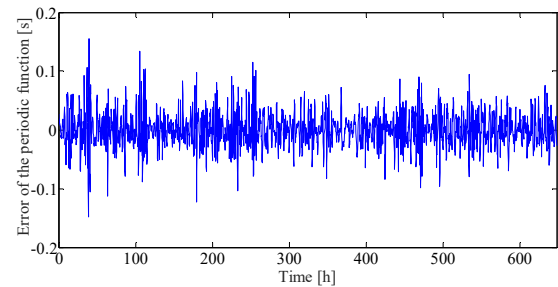


FIGURE 9. Difference between the measured data and the periodic function.

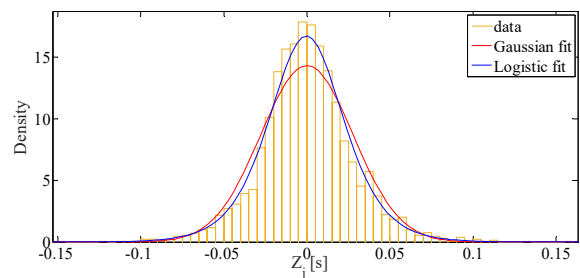


FIGURE 10. Gaussian and Logistic fitting distributions for innovations.

The accuracy of the proposed approach is demonstrated in Fig. 11, which shows that the estimation of the inertia quite agrees with the actual profile of the test samples. It is interesting to note that the proposed method allows catching the variation of the inertia in both decreasing (between hours 725 and 730) and in the increasing (between hours 735 and 740) behaviors of the inertia. This is thanks to the ability of the proposed approach to find the periodic nature of the inertia.

The error reported in Fig. 12 refers to the absolute percentage difference between the estimated and measured inertia. The figure shows that the error always is lower than 8%. With reference to the whole day, the mean value of this error is 2.5%. This error is aligned with the errors typically occurring in the on-line estimation methods [9]. Also, it can be noted that the highest errors occur in the period of the time when inertia changes.

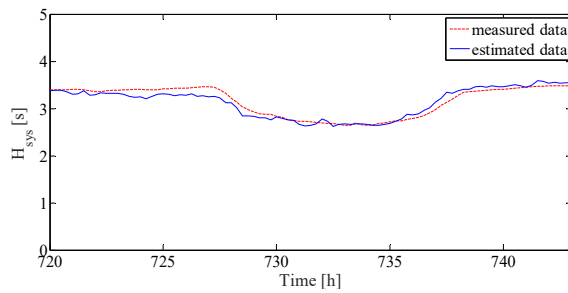


FIGURE 11. Measured and estimated inertia.

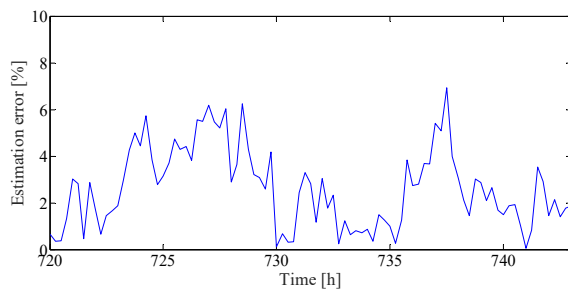


FIGURE 12. Absolute percentage error of the estimated inertia.

VI. CONCLUSIONS

On the basis of a deep insight based upon adequate statistical analyses of available real data, the authors realized that a simple dynamic model could be adopted for a feasible and adaptive description of the inertia stochastic process, as a function of renewable source contribution to the total power generation, i.e. on the share of synchronous and renewable – based generators. More specifically, it has been shown in the paper that the inertia dynamics may be regarded as the sum of a periodic component and a noise stochastic process distributed according a non-Gaussian model, and an autoregressive model with innovations with Logistic underlying distribution has been adopted. Statistical inference of such model has been performed by means of the classical Maximum Likelihood approach, and its performances, in terms of a comparison between measured and estimated inertia has been successfully illustrated. It is expected that such kind of studies can have a significant impact on the correct real time operation of power systems in the framework of the recent switch towards higher and higher penetration of renewables. Future works will be devoted to more extensive applications of the method combined with all the measurements needed for effective analyses. Also, different scenarios including contingency and post-contingency events will be analyzed on the basis of statistical data related to longer time periods.

REFERENCES

- [1] M. Pavella, P.G. Murthy. *Transient stability of power systems. Theory and Practice.* John Wiley, N.Y., 1994.
- [2] F. Milano, F. Dörfler, G. Hug, D. J. Hill, G. Verbič, “Foundations and Challenges of Low-Inertia Systems,” Proc. of Power Systems Computation Conference (PSCC), Dublin, 11-15 June 2018.
- [3] A. Ulbig, T.S. Borsche, G. Andersson, “Impact of Low Rotational Inertia on Power System Stability and Operation,” *IFAC Proceedings Volumes*, 47(3), 2014, pp. 7290-7297.
- [4] H. Shao et al., “Equivalent Modeling and Comprehensive Evaluation of Inertia Emulation Control Strategy for DFIG Wind Turbine Generator,” *IEEE Access*, vol. 7, pp. 64798-64811, 2019.
- [5] E. Chiodo, D. Lauria, F. Mottola, “On-Line Bayes Estimation of Rotational Inertia for Power Systems with High Penetration of Renewables. Part I: Theoretical Methodology,” Proc. of Int. Symposium on Power Electronics, Electrical Drives, Automation and Motion (SPEEDAM), Amalfi Coast, Italy, June 20-22, 2018.
- [6] W. Winter, K. Elkington, G. Bareux, J. Kostevc, “Pushing the limits: Europe’s new grid: innovative tools to combat transmission bottlenecks and reduced inertia,” *IEEE Power and Energy Magazine*, 13(1), 2015, pp. 60-74.
- [7] K. Shi, H. Ye, W. Song and G. Zhou, “Virtual Inertia Control Strategy in Microgrid Based on Virtual Synchronous Generator Technology,” *IEEE Access*, vol. 6, pp. 27949-27957, 2018.
- [8] A. Fernández-Guillamón, E. Gómez-Lázaro, E. Muljadi, Á. Molina-García, “Power systems with high renewable energy sources: A review of inertia and frequency control strategies over time,” *Renewable and Sustainable Energy Reviews*, Vol. 115, 2019.
- [9] O. Beltran, R. Peña, J. Segundo, A. Esparza, E. Muljadi, D. Wenzhong, “Inertia Estimation of Wind Power Plants Based on the Swing Equation and Phasor Measurement Units,” *Appl. Sci.* 2018, 8, 2413.
- [10] M. Ashton, G. A. Taylor, A. M. Carter, and W. Hung, “Application of phasor measurement units to estimate power system inertial frequency response,” in Proc. of IEEE Power Engineering Society General Meeting, Vancouver, BC, Canada, 21-25 July 2013.
- [11] T. Inoue, H. Taniguchi, Y. Ikeguchi, and K. Yoshida, “Estimation of power system inertia constant and capacity of spinning-reserve support generators using measured frequency transients,” *IEEE Trans. on Power Systems*, vol. 12, no. 1, pp. 136–143, Feb. 1997.
- [12] D. P. Chassin, Z. Huang, M. K. Donnelly, C. Hassler, E. Ramirez, and C. Ray, “Estimation of WECC system inertia using observed frequency transients,” *IEEE Trans. on Power Systems*, vol. 20, no. 2, pp. 1190–1192, May 2005.
- [13] P. M. Ashton, C. S. Saunders, G. A. Taylor, A. M. Carter and M. E. Bradley, “Inertia Estimation of the GB Power System Using Synchrophasor Measurements,” *IEEE Trans. on Power Systems*, vol. 30, no. 2, pp. 701-709, March 2015.
- [14] P.M. Ashton, G. A. Taylor, M. R. Irving, I. Pisica, A. Carter, and M. E. Bradley, “Novel application of detrended fluctuation analysis for state estimation using synchrophasor measurements,” *IEEE Trans. on Power Systems*, vol. 28, no. 2, pp. 1930–1938, May 2013.
- [15] G. Cai, B. Wang, D. Yang, Z. Sun and L. Wang, “Inertia Estimation Based on Observed Electromechanical Oscillation Response for Power Systems,” *IEEE Trans. on Power Systems*, vol. 34, no. 6, pp. 4291-4299, Nov. 2019.
- [16] D. del Giudice, S. Grillo, “Analysis of the Sensitivity of Extended Kalman Filter-Based Inertia Estimation Method to the Assumed Time of Disturbance,” *Energies* 2019, 12, 483.
- [17] P. Wall, V. Terzija, “Simultaneous Estimation of the Time of Disturbance and Inertia in Power Systems,” *IEEE Trans. on Power Delivery*, vol. 29, no. 4, pp. 2018-2031, Aug. 2014.
- [18] P. Wall, F. González-Longatt, V. Terzija, “Estimation of Generator Inertia available during a disturbance,” Proc. 2012 IEEE Power and Energy Society General Meeting, San Diego, CA, USA, 22-26 July 2012.
- [19] P. Wall, P. Regulski, Z. Rusidovic, and V. Terzija, “Inertia estimation using PMUs in a laboratory,” Proc. of Innovative Smart Grid Technologies Conference Europe (ISGT-Europe), Istanbul, Turkey, 12-15 Oct. 2014.
- [20] J. Schiffer, P. Aristidou and R. Ortega, “Online Estimation of Power System Inertia Using Dynamic Regressor Extension and Mixing,” *IEEE Trans. on Pow. Sys.*, vol. 34, no. 6, pp. 4993-5001, Nov. 2019.
- [21] X. Cao, B. Stephen, I. F. Abdulhadi, C. D. Booth and G. M. Burt, “Switching Markov Gaussian Models for Dynamic Power System Inertia Estimation,” *IEEE Trans. on Power Systems*, vol. 31, no. 5, pp. 3394-3403, Sept. 2016.
- [22] LiuJun Hu, Yong Li, WeiYu Wang, Yi Tan, Yijia Cao, Kwang Y. Lee, “Inertia Estimation of Power Grid with VSC-MTDC System,” *IFAC-PapersOnLine*, Vol. 51, no. 28, 2018, pp. 197-202

- [23] L. Lugnani, D. Dotta, C. Lackner, J. Chow, "ARMAX-based method for inertial constant estimation of generation units using synchrophasors," *Electric Power Systems Research*, vol. 180, 2020, 1060973
- [24] J. Zhang and H. Xu, "Online Identification of Power System Equivalent Inertia Constant," *IEEE Trans. on Industrial Electronics*, vol. 64, no. 10, pp. 8098-8107, Oct. 2017.
- [25] V. Sagar and S. Kumar Jain, "Estimation of Power System Inertia Using System Identification," 2019 IEEE Innovative Smart Grid Technologies - Asia (ISGT Asia), Chengdu, China, 2019, pp. 285-290.
- [26] M. Akbari and S. M. Madani, "A new method for contribution of DFIG-based wind farms in power system frequency regulation," Proc. of North American Power Symposium, Arlington, TX, 26-28 Sept. 2010.
- [27] E. Chiodo, D. Lauria, F. Mottola, "On-Line Bayes Estimation of Rotational Inertia for Power Systems with High Penetration of Renewables.Part II: Numerical applications," Proc. of IEEE International Symposium on Power Electronics, Electrical Drives, Automation and Motion (SPEEDAM), June 20-22, 2018, Amalfi Coast, Italy.
- [28] Entsoe, "Future system inertia" Technical Report. [Online] Available: https://docstore.entsoe.eu/Documents/Publications/SOC/Nordic/Nordic_report_Future_System_Inertia.pdf
- [29] TERN A S.p.A. <https://www.terna.it/it>
- [30] C. A. G. Marques, M. V. Ribeiro, C. A. Duque, E. A. B. da Silva (April 17th 2013). Parameters Estimation of Time-Varying Harmonics, Power Quality Issues, Ahmed Zobaa, IntechOpen, DOI: 10.5772/53816.
- [31] G. Goertzel, "An algorithm for the evaluation of finite trigonometric series," *The American Mathematical Monthly*, 65134351958
- [32] W.K. Wong, G. Bian, "Estimating parameters in autoregressive models with asymmetric innovations," *Stat. Probab. Lett.* 2005,71, 61-70