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# Numerical studies on capillary discharges as focusing elements for electron beams

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ABSTRACT

Active plasma lenses are promising technologies for the focusing of high brightness electron beams due to their radially symmetric focusing and their high field gradients (up to several kT/m). However, in a number of experimental situations, the transverse non-uniformity of the current density flowing in the lens causes beam emittance growth and increases the minimum achievable spot size. To study the physics of the capillary discharge processes employed as active plasma lenses, we developed a 2-D hydrodynamic computational model. Here, we present preliminary simulation results and we compare the computed magnetic field profile with one from literature, which has been experimentally inferred. The result of the comparison is discussed.

## 1. Introduction

The application of Active Plasma Lenses (APLs) to electron beam focusing has recently gained new interest [1–4], because of the possibility of combining active plasma focusing with laser/particle-driven plasma acceleration techniques. In fact, an APL can reach magnetic field gradients up to several kT/m, even higher than those achievable in permanent magnet quadrupoles, is capable of focusing a beam in both transverse planes at the same time, and its focusing strength, $K$, scales like $1/\gamma$, which makes it suitable also at high energies. This makes APLs attractive for use in configurations of plasma wakefield acceleration which require to reach small spot sizes or to capture highly diverging beams.

The working principle of an APL [5] is the focusing of a charged particle beam by means of the azimuthal magnetic field generated by a current flowing through a plasma channel in the same direction as the beam current. In the most recent applications [2,3], the plasma is generated by a capillary discharge. The lines of the magnetic field lay on the transverse plane and wrap around the axis of the capillary, giving rise to a focusing Lorentz force on the beam particles. In this device the spatial distribution of the current density is crucial, as it is the only responsible for the magnetic field inside the capillary, given that the system is in quasi-static magnetic conditions. For instance, in order to have a perfectly linear lens one would require a current density distributed uniformly in the transverse plane. At the present status of the technology, a major problem for an optimal operation of an APL for electron beams appears to be transverse non-uniformity of the current density [2,3], which induces transverse non-linearities of the magnetic field; these are in turn responsible for an increase of both the emittance and the minimum achievable spot size. The behavior of the magnetic field can be explained with the relation between plasma temperature $T$ and electrical conductivity $\sigma$. The electrical conductivity of a plasma, under the experimental conditions of our interest (temperature: 1–5 eV, density: $10^{16}$–$10^{18}$ cm$^{-3}$), scales as $\sigma \propto T^{3/2}$. Due to the cooling effect of the capillary walls, the plasma is hotter on axis, therefore one expects that the current concentrates at that location [2,3].

Furthermore, the effect commonly named as *passive plasma lensing* can play a non-negligible role. The plasma electrons always tend to reorganize to conserve overall neutrality during the beam passage, thus partially shielding the beam space charge. This corresponds to a focusing of the beam, with an associated focusing strength which depends on the beam shape and density compared to the background electron density. Recent studies on beam degradation effects due to passive lensing in a discharge capillary show that significant emittance growth may occur due to gas jets leaking from the capillary extremities [6]. Thus, the

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development of plasma plums from the capillary edges also needs to be carefully studied.

In order to develop an APL with satisfying optical properties, the discharge process has to be simulated with a model that allows for studying the effect of the changes in the most relevant settings, e.g. the initial gas pressure, the shape of the current pulse, the geometry of the capillary and of the electrodes. Accurate discharge simulations are also a key element in order to boost the development of novel acceleration technologies involving plasmas. The computed quantities (e.g. electron and ion density, as well as the magnetic field) can be used as input to plasma wakefield codes [7–9] to perform reliable start-to-end studies on relevant acceleration schemes.

To take into account in a realistic way the geometry of the capillaries, and especially the gas outflow during the discharge process, a 2-D or 3-D model is necessary. Since we are currently interested in studying capillaries with circular cross-section, a two-dimensional axially symmetric approximation is considered a good compromise between accuracy and runtime.

## 2. Computational model for two-dimensional axially symmetric capillary discharges

The gas discharge process has to be simulated with a fluid model, as a particle in cell approach would be computationally impractical, due to the time scales involved (>1 μs) and to the spatial resolution required (<100 μm). In particular, according to previous studies [10–12] a single-temperature fluid seems reasonable. A local thermodynamic equilibrium approximation is taken, which allows for computing the ionization degree at each time step only as a function of the local plasma temperature and mass density. In particular, Saha ionization model is employed.

Also, we decided to initially neglect the magnetic skin effect and the influence of Lorentz force on the plasma column. These simplifications allow us to skip the treatment of a self consistent magnetic field. The plasma is thus modeled with a hydrodynamic system, with the addition of an ohmic heating term in the energy conservation equation; the current density is computed in a static current flow approximation. In the next future it is foreseen to remove these limitations by including an equation for the evolution of the magnetic field.

The relevant equations in the model are then the following:

$$\frac{\partial}{\partial t}\rho + \nabla \cdot (\rho \mathbf{v}) = 0, \tag{1}$$

$$\frac{\partial}{\partial t}(\rho \mathbf{v}) + \nabla \cdot (\rho \mathbf{v}\mathbf{v}) = -\nabla p, \tag{2}$$

$$\frac{\partial}{\partial t}E + \nabla \cdot (E\mathbf{v}) = -\nabla \cdot (p\mathbf{v}) + \nabla \cdot (\kappa \nabla T) + \eta \|\mathbf{j}\|^2, \tag{3}$$

$$\mathbf{j} = -\frac{1}{\eta}\nabla\phi, \tag{4}$$

$$\nabla \cdot \mathbf{j} = 0. \tag{5}$$

Here $\rho$ is the mass density, $\mathbf{v}$ is the plasma fluid velocity, $p$ is the thermal pressure, $E$ is the total energy density, $\kappa$ is the thermal conductivity, $T$ is the plasma temperature, $\mathbf{j}$ is the current density, $\phi$ is the electric potential and $\eta$ is the electrical resistivity of the plasma. We expect that the gas in our studies is only partially ionized, therefore energy and pressure as function of $\rho$ and $T$ are provided by and equation of state which considers also the neutral hydrogen atom ionization energy, whereas $\eta$ and $\kappa$ are computed including the scattering of the electrons with neutrals.

The DUED code [13] has been suitably modified in order to implement the above model. DUED solves the presented equations with a finite differences approach in a Lagrangian frame, which brings, with respect to an Eulerian treatment, the advantage that only the plasma region needs to be meshed. A rezoning scheme is however required to avoid mesh pathologies.

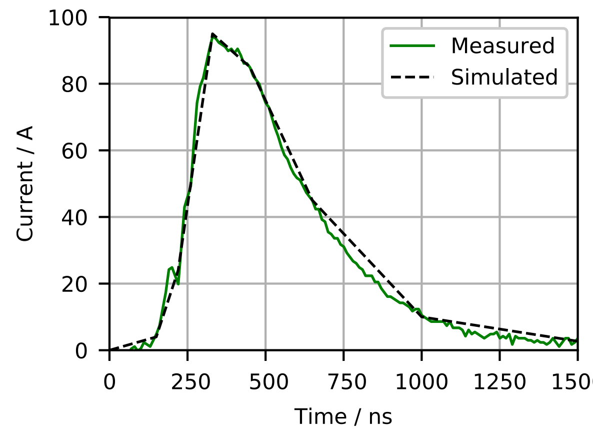


**Fig. 1.** Simulated and measured time profile of the current in the capillary discharge studied.

## 3. Preliminary studies

Some preliminary results have been produced by simulating the same capillary discharge as the one employed in Ref. [2] for focusing a high-brightness electron beam. The capillary under study is a 3 cm-long sapphire tube with a circular cross section of diameter 1 mm and open ends. It is pre-filled with hydrogen at a backing pressure of approximately 40 mbar. Its extremities are open and two flat electrodes are stuck at the ends, each with a hole as wide as the capillary aperture. In the simulation the gas has been initialized at rest and perfectly confined inside the capillary, with an initial mass density of $2.5\times10^{-6}$ g cm$^{-3}$ (corresponding to a density of atomic hydrogen of $1.5\times10^{18}$ cm$^{-3}$). The initial temperature was set to 0.34 eV, which corresponds to 4000 K, for providing a minimal fictitious ionization degree to let the discharge process start. The temperature at the capillary boundary is kept at the fixed value of $T_\mathrm{b} = 0.09$ eV. Simulations show that results are independent of $T_\mathrm{b}$ for $0.04\,\mathrm{eV} < T_\mathrm{b} < 0.17\,\mathrm{eV}$.

The current flowing in the capillary was imposed at each time step to mimic the measured profile of the discharge under consideration, which is shown in Fig. 1.

## 4. Results

Fig. 2 shows the computed mass and electron density map after 750 ns from the start of the discharge, together with the velocity field of the gas exiting from the capillary at that moment. We note that the order of magnitude of the bulk velocity near the electrodes (approximately $3\times10^6$ cm s$^{-1}$) is in accordance with the ion acoustic velocity, which is the velocity one would expect in a free-expansion case:

$$c_\mathrm{s} = \sqrt{\frac{\gamma T}{m_\mathrm{p}}} \approx 10^6\ \mathrm{cm\ s^{-1}}, \quad (T = 1\ \mathrm{eV}), \tag{6}$$

where $\gamma$ is the adiabatic index of the gas, $m_\mathrm{p}$ is the proton mass, and we choose 1 eV as characteristic temperature of the plasma near the capillary exit, in accordance with the simulation results. The results of the simulation enforce the idea that employing a 2-D model is crucial to take accurately into account the outflow of gas from the capillary during the discharge process. From Fig. 2 we can also notice that the plasma electron density ramps up from a value of less than $10^{17}$ cm$^{-3}$ near the capillary extremity, to approximately $5\times10^{17}$ cm$^{-3}$ at the center of the capillary; a 1-D radial approximation would prevent from observing this effect.

We now briefly focus on the optical properties of the device studied. In the thin lens approximation, the bending of the trajectory of a charged particle due to a transverse magnetic field depends on the integral of the field performed along the longitudinal direction. In order to obtain



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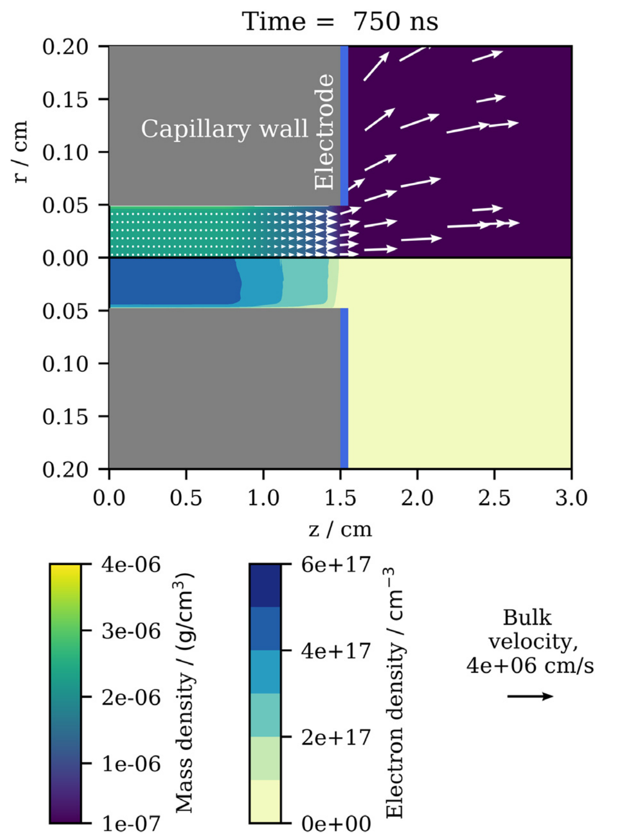


**Fig. 2.** Mass density (top) and electron density (bottom) maps of the gas flowing from the capillary at 750 ns from the start of the discharge. The arrows superimposed to the mass density plot represent the velocity field of the plasma.

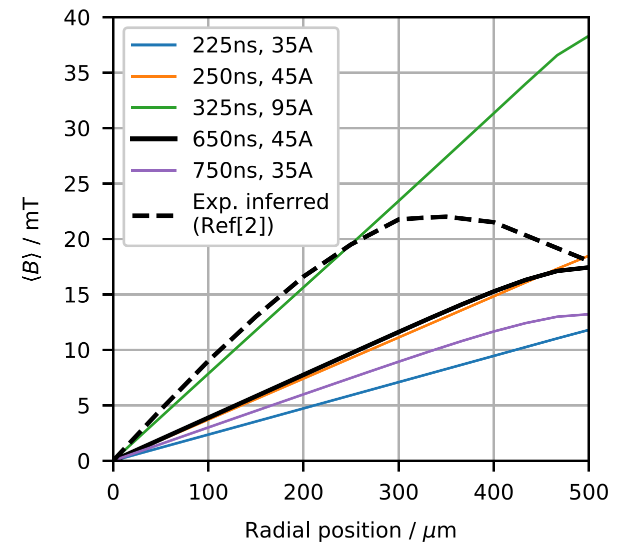


**Fig. 3.** Radial profiles of the longitudinally averaged magnetic field computed at different times from the start of the capillary discharge (continuous lines), and magnetic field profile experimentally inferred in Ref. [2] (dashed line).

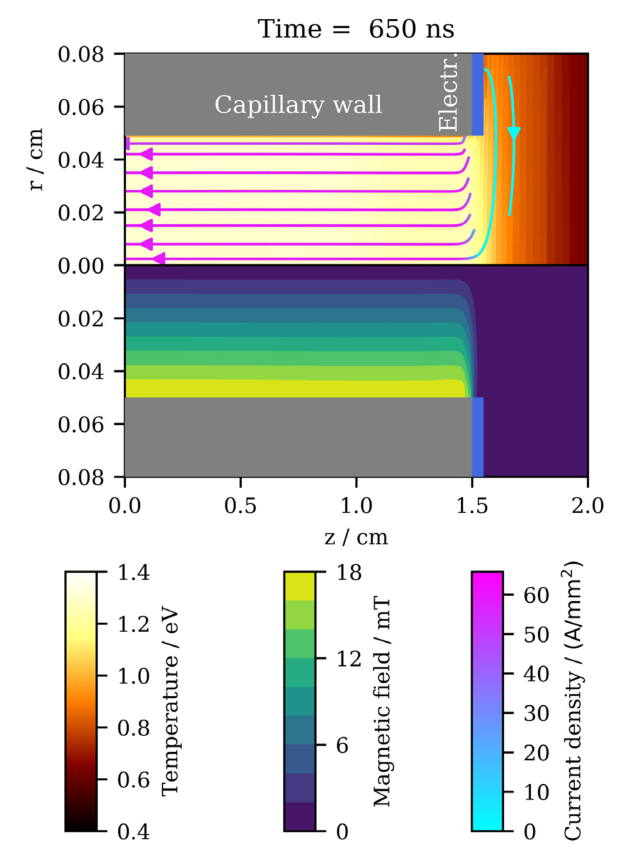


**Fig. 4.** Plasma temperature (top) and azimuthal magnetic field (bottom) maps in the capillary and near the right electrode at 650 ns from the start of the discharge. The current density, represented by the streamlines superimposed to the temperature plot, intensifies where the plasma is hotter, due to the approximate proportionality between the plasma electrical conductivity and its temperature to the power of 3/2.

a quantity which is comparable to transverse field profiles belonging to 1-D models, such integral can be normalized by the capillary length:

$$\langle B(r)\rangle = \frac{\int_{-\infty}^{+\infty} B_\theta(r,z)\mathrm{d}z}{L_{\mathrm{cap}}}. \tag{7}$$

In Fig. 3 some radial profiles of these magnetic field averages, $\langle B(r)\rangle$, are presented, for different timings during the discharge process. We are interested in understanding whether the computed magnetic field can explain the beam degradation effects experimentally observed [2] for the very same capillary discharge of the present study. The profile which would explain the behavior of the observed beam is shown by Ref. [2] and has been also reported in Fig. 3 (dashed line). This profile has to be compared with one obtained in our work at 650 ns, since at that simulation time the current reaches the value of 45 A, with descending slope.

For the sake of completeness, the 2-D map of the magnetic field in the simulated discharge at that timing is presented in Fig. 4, together with the streamlines of the current density and the map of the plasma temperature. It is clear that the plasma temperature is not higher than 1.4 eV; it is opinion of the authors that this is due to the relatively high mass density contained inside the capillary, and therefore to the high heat capacity of the gas. In other simulations of ours, when an initial condition with a smaller amount of hydrogen gas is chosen (e.g. 2.5 g cm$^{-3}$), with no changes in the other settings, the peak temperatures values at the same location and timing are almost doubled.

In addition, for a quantitative comparison of the profiles in Fig. 3 we compute the spherical aberration parameter $\Delta K/K$, defined as the relative difference in focusing strength between the axis and the wall of the lens. We compute it for a mono-energetic beam and in the approximation of thin lens:

$$\Delta K/K \doteq \frac{K(0)-K(r_{cap})}{K(0)} = 1 - \frac{\langle B(r_{cap})\rangle / r_{cap}}{\frac{\partial}{\partial r}\langle B\rangle|_{r=0}} \tag{8}$$



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**Table 1**
Aberration parameters for some relevant discharge timings, computed from the simulated magnetic field and from the field inferred in Ref. [2].

| Current | Time | $\Delta K/K$ |
|---|---|---|
| 95 A | 325 ns | 0.025 |
| 45 A | 250 ns | 0.004 |
| 45 A | 650 ns | 0.10 |
| 35 A | 225 ns | 0.002 |
| 35 A | 750 ns | 0.12 |
| 45 A | Computed from exp. inferred field | 0.61 |

The values of the aberration parameters for the same discharge timings presented in Fig. 3 are shown in Table 1.

It is clear that the field computed in the present simulation at 650 ns cannot explain the beam degradation shown in Ref. [2] both because of the difference in the profiles shapes and in the values of the aberration parameters.

Possible explanations for this incongruence are the lack in our model of the treatment of a self consistent magnetic field, and the choice of both the initial value and (flat) profile of gas density distribution in the simulation, which might represent the experimental environment not realistically enough.

## 5. Conclusion and outlook

We presented preliminary results from a simulation of a hydrogen-filled capillary discharge with a hydrodynamic plasma model exploiting the approximation of static current flow for the computation of the current density distribution at each time step. The obtained results cannot explain the emittance growth which occurs when the real capillary is employed as focusing element for a high-brightness electron beam. The reasons for this discrepancy can be sought for both in the incomplete physical model employed and in the initial condition which has been set for the gas.

To reach the final goal of consistently simulate the capillary discharge processes, one main additional improvement to the simulation code needs to be done, which consists in the inclusion of a self consistent time-varying magnetic field.

Moreover, in the next future we will validate the simulation results by comparing the calculated electron density maps with longitudinally and time resolved measurements obtainable at SPARC_LAB [14] by means of the Stark Broadening technique [15].

After completion of the above upgrades, it will be possible to study numerically the capillary discharge with a model that captures the most relevant physical processes. This will allow to better understand APL's performance and to optimize its configuration.

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