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Dynamic analysis of Bernoulli-Euler beams with interval uncertainties under moving loads

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Abstract

This paper deals with the dynamic analysis of Bernoulli-Euler beams with uncertain parameters crossed by moving loads. Uncertainties associated with mass density, Young's modulus and load velocity are modeled as interval variables with given lower bounds and upper bounds. In order to evaluate the bounds of the interval displacement of the beam, an efficient procedure based on the use of the classical modal superposition method in conjunction with the *improved interval analysis via extra unitary interval* is developed. The key idea is to seek the bounds of the response by exploring just a few combinations of the endpoints of the uncertain parameters appropriately selected based on those yielding the minimum and maximum values of the interval natural frequencies.

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Keywords: Bernoulli-Euler beam; loving load; interval uncertainty; lower bound and upper bound

1. Introduction

The dynamic analysis of continuous structures, like beams, plates, and shells under moving loads is a topic of great interest in several fields of engineering. Traditional approaches assume deterministic values of all input parameters. However, it is widely recognized that any design process involves uncertainties in parameters which may adversely affect the performance of engineering systems. Besides the widely used probabilistic model of

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uncertainty, recently much research effort has focused on non-probabilistic approaches, such as the interval model [1] which represents the uncertain parameters as interval variables with given lower bound (LB) and upper bound (UB). Such a model is very useful when only range information on uncertainties is available, as it happens in early design stages. One of the main drawbacks of the interval model is the overestimation of the interval solution due to the *dependency phenomenon* which often leads to useless results for design purposes. To limit the overestimation, recently the so-called *improved interval analysis via extra unitary interval* (*IIA via EUI*) [2] as well as the *parameterized interval analysis* [3,4] have been proposed. In the literature, several approaches have been developed with the aim of obtaining sharp bounds of the structural response under both static and dynamic excitations. To the best of the authors' knowledge, few studies have been devoted to incorporate interval uncertainties into typical moving-load problems (see e.g. [5]).

The present paper addresses the dynamic analysis of Bernoulli-Euler beams with interval uncertainties crossed by moving loads. Uncertainties affecting Young's modulus, mass density and the velocity of the moving load are taken into account. The proposed approach for the evaluation of response bounds relies on the use of the classical modal superposition method in conjunction with the *IIA via EUI*. The key idea is to reduce the number of combinations of the endpoints of the uncertain parameters compared to those considered by the classical combinatorial procedure. To this aim, just the combinations of the bounds of Young's modulus and mass density yielding the LB and UB of the natural frequencies of the beam are taken into account.

Numerical results presented in the paper demonstrate the accuracy of the proposed procedure through appropriate comparisons with the exact bounds provided by the classical combinatorial procedure.

2. Equations of motion of a Bernoulli-Euler beam crossed by a moving load

Let the dynamic system depicted in Figure 1 be a simplified model of a bridge structure which is treated as a simply supported uniform homogeneous Bernoulli-Euler beam of length l_{b}. The beam is crossed by a force, F, moving from left to right at constant speed, v. It is assumed that at the time instant of force arrival, $t = 0$, the beam is at rest. Under these assumptions, the equation of motion of the bridge reads:

$$\rho_{\mathrm{b}} A_{\mathrm{b}} \frac{\partial^2 u_{\mathrm{b}}(z,t)}{\partial t^2} + E_{\mathrm{b}} J_{\mathrm{b}} \frac{\partial^4 u_{\mathrm{b}}(z,t)}{\partial z^4} + D_{\mathrm{b}}(z,t) = F\,\delta(z - vt)\chi(t) \tag{1}$$

where t and z denote the time and the coordinate measured along the axis of the beam, respectively; $u_{\mathrm{b}}(z,t)$ is the transverse displacement field of the beam; ρ_{b}, A_{b}, E_{b} and J_{b} are the beam mass density, cross-sectional area, modulus of elasticity and moment of inertia, respectively; $D_{\mathrm{b}}(z,t)$ is the internal damping force and $\chi(t)$ is the so-called window function, defined in such a way that $\chi(t) = 1$ if $0 \le vt \le l_{\mathrm{b}}$, $\chi(t) = 0$ if $vt < 0$ or $vt > l_{\mathrm{b}}$.

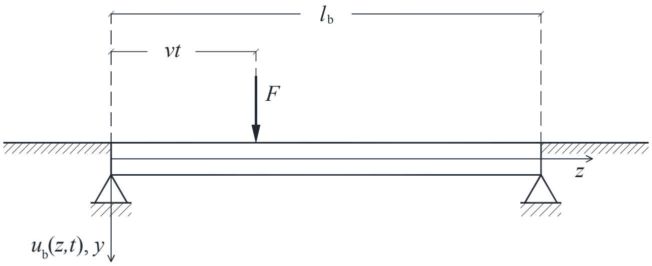

Fig. 1. Simply supported beam crossed by moving load.

Following the traditional modal superposition method, the transverse displacement field of the beam can be approximated as:

$$u_{\mathrm{b}}(z,t)=\sum_{i=1}^{n_{\mathrm{b}}}\phi_{\mathrm{b},i}(z)q_{\mathrm{b},i}(t)=\boldsymbol{\phi}_{\mathrm{b}}^{\mathrm{T}}(z)\mathbf{q}_{\mathrm{b}}(t) \tag{2}$$

where

$$\boldsymbol{\phi}_{\mathrm{b}}(z)=\left[\phi_{\mathrm{b},1}(z),\quad \phi_{\mathrm{b},2}(z),\quad \ldots,\quad \phi_{\mathrm{b},n_{\mathrm{b}}}(z)\right]^{\mathrm{T}};\quad \mathbf{q}_{\mathrm{b}}(t)=\left[q_{\mathrm{b},1}(t),\quad q_{\mathrm{b},2}(t),\quad \ldots,\quad q_{\mathrm{b},n_{\mathrm{b}}}(t)\right]^{\mathrm{T}} \tag{3a,b}$$

are the vectors collecting the first n_{b} modal shapes of the continuum, $\phi_{\mathrm{b},i}(z)$, and the associated modal coordinates, $q_{\mathrm{b},i}(t)$, respectively. As well-known, for simply supported beams, the i-th circular frequency, $\omega_{\mathrm{b},i}$, and the corresponding mass-normalized modal shape, $\phi_{\mathrm{b},i}(z)$, can be evaluated in closed-form as follows:

$$\omega_{\mathrm{b},i}=\left(i\,\pi\right)^{2}\sqrt{\frac{E_{\mathrm{b}}J_{\mathrm{b}}}{\rho_{\mathrm{b}}A_{\mathrm{b}}\,l_{\mathrm{b}}^{4}}};\quad \phi_{\mathrm{b},i}(z)=\sin\left(\frac{i\,\pi}{l_{\mathrm{b}}}z\right)\sqrt{\frac{2}{\rho_{\mathrm{b}}A_{\mathrm{b}}\,l_{\mathrm{b}}}}. \tag{4a,b}$$

Upon substituting Equation (2) into Equation (1), pre-multiplying both sides by $\boldsymbol{\phi}_{\mathrm{b}}(z)$ and integrating with respect to z between 0 and l_{b}, the following set of decoupled n_{b} ordinary differential equations is obtained:

$$\ddot{\mathbf{q}}_{\mathrm{b}}(t)+\boldsymbol{\Xi}_{\mathrm{b}}\,\dot{\mathbf{q}}_{\mathrm{b}}(t)+\boldsymbol{\Omega}_{\mathrm{b}}^{2}\,\mathbf{q}_{\mathrm{b}}(t)=\boldsymbol{\phi}_{\mathrm{b}}(vt)F\,\chi(t) \tag{5}$$

where

$$\boldsymbol{\Omega}_{\mathrm{b}}=\mathrm{Diag}\left[\omega_{\mathrm{b},1},\quad \omega_{\mathrm{b},2},\quad \ldots,\quad \omega_{\mathrm{b},n_{\mathrm{b}}}\right];\quad \boldsymbol{\Xi}_{\mathrm{b}}=2\zeta_{\mathrm{b}}\boldsymbol{\Omega}_{\mathrm{b}} \tag{6a,b}$$

with ζ_{b} being the modal damping ratio herein assumed constant in all vibration modes of the beam.

3. Uncertain input parameters modeled as interval variables

According to the *Classical Interval Analysis* (*CIA*) [1], denoting by $\mathbb{IR}$ the set of all closed real interval numbers, a real interval variable $\alpha_i^I \triangleq \left[\underline{\alpha}_i,\overline{\alpha}_i\right]\in\mathbb{IR}$ is defined in such a way that $\underline{\alpha}_i \le \alpha_i \le \overline{\alpha}_i$, with $\underline{\alpha}_i$ and $\overline{\alpha}_i$ representing the lower bound (LB) and upper bound (UB) of α_i^I, respectively; the apex I denotes interval variables. Alternatively, the interval variable α_i^I is characterized by the midpoint value, $\alpha_{0,i}$, and the deviation amplitude, $\Delta\alpha_i$, i.e.:

$$\alpha_{0,i}=\frac{\overline{\alpha}_i+\underline{\alpha}_i}{2};\quad \Delta\alpha_i=\frac{\overline{\alpha}_i-\underline{\alpha}_i}{2}>0. \tag{7a,b}$$

According to the so-called *improved interval analysis via extra unitary interval* (*IIA via EUI*) [2], the interval variable α_i^I can be expressed in the following form:

$$\alpha_i^I=\alpha_{0,i}+\Delta\alpha_i\hat{e}_i^I \tag{8}$$

where $\hat{e}_i^I \triangleq \left[-1,1\right]$ is a particular unitary interval, called *EUI* which does not follow the rules of the *CIA* [6].

Let us assume that the elastic modulus and mass density of the beam, E_{b} and ρ_{b}, and the speed of the moving load, v, are uncertain-but-bounded parameters, defined following the *IIA via EUI*, i.e.:

$$E_{\mathrm{b}}^I=E_{\mathrm{b},0}(1+\Delta\alpha_E\hat{e}_E^I);\quad \rho_{\mathrm{b}}^I=\rho_{\mathrm{b},0}(1+\Delta\alpha_\rho\hat{e}_\rho^I);\quad v^I=v_0(1+\Delta\alpha_v\hat{e}_v^I) \tag{9a-c}$$

where $E_{b,0}$, $\rho_{b,0}$ and v_0 are the nominal values, while $\hat{e}_E^I$, $\hat{e}_\rho^I$ and $\hat{e}_v^I$ are the associated *EUI*s. The interval parameters are assumed independent and are collected into the interval vector $\boldsymbol{\alpha}^I$:

$$\boldsymbol{\alpha}^I = \begin{bmatrix} E_b^I & \rho_b^I & v^I \end{bmatrix}^T. \tag{10}$$

Due to uncertainties affecting the input parameters, the transverse displacement field of the beam is interval in nature and is ruled by the following interval partial differential equation:

$$\rho_b^I A_b \frac{\partial^2 u_b^I(z,t)}{\partial t^2} + E_b^I J_b \frac{\partial^4 u_b^I(z,t)}{\partial z^4} + D_b(z,t) = F\,\delta(z - v^I t)\chi(t). \tag{11}$$

By applying the modal superposition method, the interval transverse displacement can be approximated as:

$$u_b^I(z,t) = \sum_{i=1}^{n_b} \phi_{b,i}^I(z) q_{b,i}^I(t) = \boldsymbol{\phi}_b^{I^T}(z)\mathbf{q}_b^I(t) \tag{12}$$

where $\boldsymbol{\phi}_b^I(z)$ and $\mathbf{q}_b^I(t)$ are the vectors collecting the interval modal shapes and modal coordinates, defined as interval extensions of those in Equations (3a,b). In particular, the interval modal coordinates are ruled by the interval extension of Equation (5):

$$\ddot{\mathbf{q}}_b(\boldsymbol{\alpha}^I;t) + \boldsymbol{\Xi}_b(E_b^I,\rho_b^I)\dot{\mathbf{q}}_b(\boldsymbol{\alpha}^I;t) + \boldsymbol{\Omega}_b^2(E_b^I,\rho_b^I)\,\mathbf{q}_b(\boldsymbol{\alpha}^I;t) = \boldsymbol{\phi}_b(\rho_b^I, v^I t)F\chi(t) \tag{13}$$

whose $j-$th equation reads:

$$\ddot{q}_{b,j}(\boldsymbol{\alpha}^I;t) + 2\zeta_b\omega_{b,j}(E_b^I,\rho_b^I)\dot{q}_{b,j}(\boldsymbol{\alpha}^I;t) + \omega_{b,j}^2(E_b^I,\rho_b^I)\,q_{b,j}(\boldsymbol{\alpha}^I;t) = \sqrt{\frac{2}{\rho_b^I A_b\, l_b}}\sin\left(\frac{j\pi}{l_b}v^I t\right)F\chi(t). \tag{14}$$

4. Bounds of the interval displacement field

Since at each time instant the response is a monotonic function of the uncertain parameters, the exact bounds of the interval displacement field can be computed by applying the classical combinatorial procedure which, in the present case, requires 2^3 deterministic dynamic analyses, one for each combination of the endpoints of the three uncertain parameters E_b^I, ρ_b^I and v^I . In order to reduce the computational effort, according to the proposed approach, first the bounds of the natural frequencies and associated modal shapes of the beam are evaluated. Then, two different modal analyses are performed by employing the computed LB and UB of the natural frequencies. According to interval arithmetic, the LB and UB of the interval natural frequencies can be readily evaluated as [7]:

$$\underline{\omega}_{b,j} = (j\pi)^2\sqrt{\frac{\underline{E}_b J_b}{\overline{\rho}_b A_b\, l_b^4}};\quad \overline{\omega}_{b,j} = (j\pi)^2\sqrt{\frac{\overline{E}_b J_b}{\underline{\rho}_b A_b\, l_b^4}}. \tag{15a,b}$$

By inspection of the previous equations, it is observed that the natural frequencies reach their LB when Young's modulus and mass density are set at the LB and UB, respectively. Conversely, the maximum value of the natural circular frequencies is achieved when Young's modulus and mass density are set at the UB and LB, respectively. Furthermore, the modal shapes associated to the LB and UB of the natural frequencies are obtained from Equation (4b) setting the mass density at its UB and LB, respectively. Indeed, only physically consistent combinations of the bounds of the interval parameters need to be considered, that is the mass density must assume the same value when evaluating the natural frequencies and the associated modal shapes. To this aim, the *IIA via EUI* associates an *EUI* to each interval variable. Taking into account that the interval velocity of the moving load can arbitrarily range between its bounds, the LB and UB of the interval displacement field can be evaluated as:

$$\begin{aligned} \underline{u}_{\mathrm{b}}(\boldsymbol{\alpha};z,t) &= \min\left\{u_{\mathrm{b}}^{(1)}(z,t);\ u_{\mathrm{b}}^{(2)}(z,t);\ u_{\mathrm{b}}^{(3)}(z,t);\ u_{\mathrm{b}}^{(4)}(z,t)\right\}; \\ \overline{u}_{\mathrm{b}}(\boldsymbol{\alpha};z,t) &= \max\left\{u_{\mathrm{b}}^{(1)}(z,t);\ u_{\mathrm{b}}^{(2)}(z,t);\ u_{\mathrm{b}}^{(3)}(z,t);\ u_{\mathrm{b}}^{(4)}(z,t)\right\}. \end{aligned} \tag{16a,b}$$

where the following deterministic solutions pertaining to four physically consistent combinations of the endpoints of the uncertain parameters are considered:

$$\begin{aligned} u_{\mathrm{b}}^{(1)}(z,t) &= \sum_{i=1}^{n_{\mathrm{b}}}\phi_{\mathrm{b},i}(\underline{\rho}_{\mathrm{b}};z)\,q_{\mathrm{b},j}\left(\overline{E}_{\mathrm{b}},\underline{\rho}_{\mathrm{b}},\underline{v};t\right);\quad u_{\mathrm{b}}^{(2)}(z,t) = \sum_{i=1}^{n_{\mathrm{b}}}\phi_{\mathrm{b},i}(\underline{\rho}_{\mathrm{b}};z)\,q_{\mathrm{b},j}\left(\overline{E}_{\mathrm{b}},\underline{\rho}_{\mathrm{b}},\overline{v};t\right); \\ u_{\mathrm{b}}^{(3)}(z,t) &= \sum_{i=1}^{n_{\mathrm{b}}}\phi_{\mathrm{b},i}(\overline{\rho}_{\mathrm{b}};z)\,q_{\mathrm{b},j}\left(\underline{E}_{\mathrm{b}},\overline{\rho}_{\mathrm{b}},\underline{v};t\right);\quad u_{\mathrm{b}}^{(4)}(z,t) = \sum_{i=1}^{n_{\mathrm{b}}}\phi_{\mathrm{b},i}(\overline{\rho}_{\mathrm{b}};z)\,q_{\mathrm{b},j}\left(\underline{E}_{\mathrm{b}},\overline{\rho}_{\mathrm{b}},\overline{v};t\right). \end{aligned} \tag{17a-d}$$

5. Numerical application

To assess the accuracy of the presented method a simply supported beam with the following geometrical and mechanical properties is analyzed: length $l_{\mathrm{b}} = 28\ \mathrm{m}$, cross-section $A_{\mathrm{b}} = 0.954\ \mathrm{m}^2$, moment of inertia $J_{\mathrm{b}} = 0.355\ \mathrm{m}^4$, nominal Young' modulus $E_{\mathrm{b},0} = 35\times 10^6\ \mathrm{kN/m}^2$, nominal mass density $\rho_{\mathrm{b},0} = 2500\ \mathrm{kg/m}^3$ and modal damping ratio $\zeta_{\mathrm{b}} = 0.020$. The beam is crossed by a moving load of intensity $F = 96530.40\ \mathrm{N}$. Without loss of generality, the uncertain elastic modulus, mass density and velocity of the moving load are modeled as interval variables with the same deviation amplitude, that is $\Delta\alpha_E = \Delta\alpha_\rho = \Delta\alpha_v = \Delta\alpha$. Different values of the nominal load velocity, v_0 , and of the deviation amplitude of the uncertain parameters, $\Delta\alpha$, are considered. The accuracy of the proposed method is demonstrated by appropriate comparisons with the exact solution provided by the classical combinatorial procedure.

Figure 2 shows a very good agreement between the proposed and exact time-histories of the LB and UB of the midspan displacement of the beam crossed by a moving load with nominal velocity $v_0 = 20\ \mathrm{m/s}$, for two different degrees of uncertainty, say $\Delta\alpha = 0.05$ and $\Delta\alpha = 0.10$. Similar results are reported in Figure 3 for a larger value of the nominal velocity, namely $v_0 = 40\ \mathrm{m/s}$.

In order to quantify the dynamic effects induced by the moving load, the so-called dynamic magnification factor for the interval midspan deflection is evaluated. Within the interval framework, such a coefficient is defined for the LB and UB of the displacement at the abscissa z as follows:

$$D_{\underline{u}_b}(z) = \frac{\underline{u}_{b,\max}(z)}{\underline{u}_{b,\max}^{(s)}(z)};\quad D_{\overline{u}_b}(z) = \frac{\overline{u}_{b,\max}(z)}{\overline{u}_{b,\max}^{(s)}(z)} \tag{18a,b}$$

where $\underline{u}_{b,\max}(z)$ and $\overline{u}_{b,\max}(z)$ denote the maximum dynamic LB and UB of the response; $\underline{u}_{b,\max}^{(s)}(z)$ and $\overline{u}_{b,\max}^{(s)}(z)$ are the maximum static LB and UB of deflection. In Figure 4, the proposed and exact dynamic magnification factors for the LB and UB of midspan displacement versus the nominal load velocity v_0 are plotted for $\Delta\alpha = 0.05$ and $\Delta\alpha = 0.10$. Notice that the proposed method is able to accurately predict the dynamic magnification induced by the moving load on the UB and LB of the response. Furthermore, it is observed that uncertainties in the input parameters may significantly increase the dynamic amplification due to the moving load.

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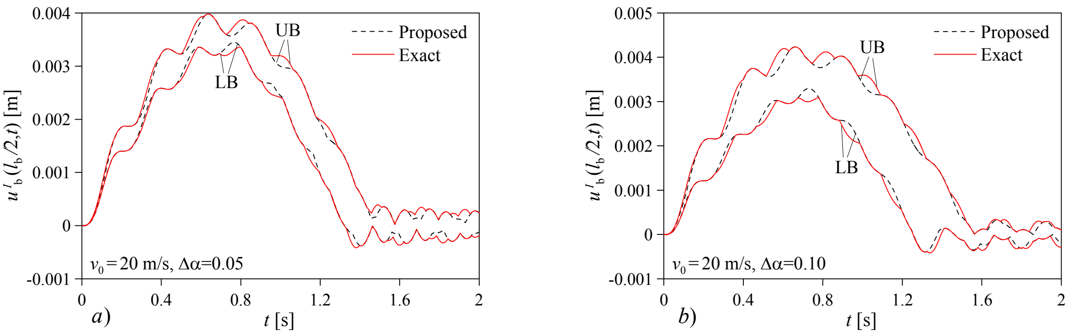

Fig. 2. Proposed and exact time-histories of the LB and UB of the midspan displacement of the beam crossed by a moving load with nominal velocity $v_0 = 20$ m/s for two different deviation amplitudes of the uncertain parameters (a) $\Delta\alpha = 0.05$; (b) $\Delta\alpha = 0.10$.

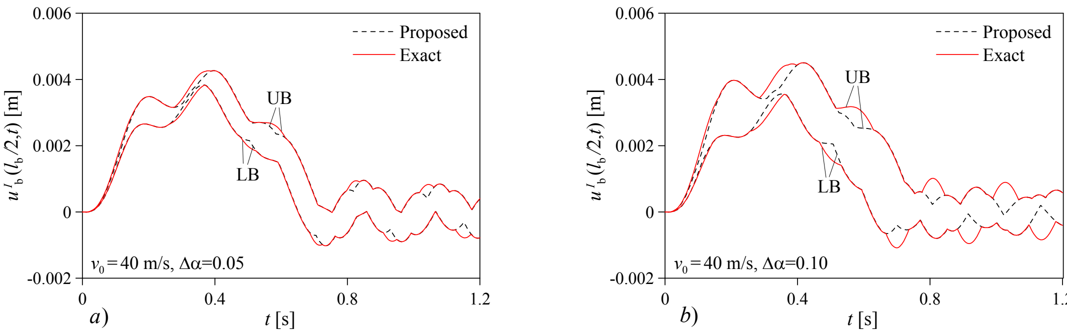

Fig. 3. Proposed and exact time-histories of the LB and UB of the midspan displacement of the beam crossed by a moving load with nominal velocity $v_0 = 40$ m/s for two different deviation amplitudes of the uncertain parameters (a) $\Delta\alpha = 0.05$; (b) $\Delta\alpha = 0.10$.

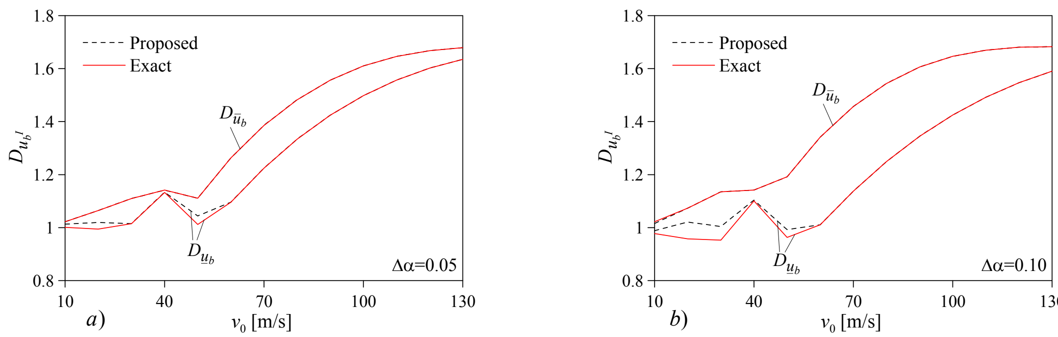

Fig. 4. Dynamic magnification factor for the LB and UB of the midspan displacement of the beam versus the nominal load velocity v_0 for two different deviation amplitudes of the uncertain parameters (a) $\Delta\alpha = 0.05$; (b) $\Delta\alpha = 0.10$.