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Decentralized Control Performances of an Experimental Web Handling System

Regular Paper

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Abstract Robust and good tracking control of the speed and the tension in web handling systems in spite of changes of set point is surely one of the important challenges in the web transport systems future development. In this paper, the authors experimentally demonstrate the real applicability of a decentralized robust control to a multi-span web transport system, which is composed of twelve guide rollers, four main sections mutually interconnected with each other. The overlapping methodology has been applied for the system decomposition.

The experimental results carried out using the robust decentralized control show an excellent velocity and tension tracking in each controlled section of the system.

Keywords Web transport system, Large-scale system, Decentralized control, H∞ control, Overlapping decomposition

1. Introduction

Some consumer products are processed in the form of web and the handling of the web material is an important factor in the final quality of the products. It becomes very important in web handling systems to control the tension between consecutive pair of rolls at a certain value so that it does not crease or break the web while it is moving at a certain transfer speed. The industrial web handling systems are usually composed of a large number of rolls and may be considered as large-scale systems that are decomposed in many subsystems for web tension control and web speed control; until now, most industrial web transport systems use decentralized PID controllers. However, all those subsystems have strong interactions with each other and the control of each subsystem is heavily influenced by the interactions with the neighboring subsystems. In the large scale systems [2] the control decentralization is necessary because the systems to be controlled are too large and the problems to be solved are too complex. A good solution to the necessity of the decomposition was introduced since 1998 [22-23] with a methodology based on a decomposition of the system into subsystems overlapped to take into account the mutual effects of the neighboring subsystems. The simulation results [22-23] demonstrated that the controller design became easy and that the control performances were better than the conventional decentralized controllers based on a disjoint decomposition.

The experimental performances of the decentralized controllers on an experimental web platform were recently shown in [19], which investigated the possibility of a gain scheduling technique determined by the speed and roll radius.

Another quite important aspect in the industrial web handling systems is that there exist many sources of disturbances (roller non-circularity, web slipping, vibrations, variation of the radius and of the inertia of the winder and unwinder rolls etc.), which can be a dangerous damaging to the web. Then, various robust control strategies have been investigated [4,5,12,14,16]. The results [14] demonstrated the performance improvement of a multivariable H_∞ robust centralized controller compared to PID controllers. Moreover, some simulations demonstrated that H_∞ decentralized controllers, with one [4,22] or two degree of freedom [5] or with the model based feed forward [13] could improve the system performances by reducing the coupling effects between two consecutive subsystems especially when the decomposition was carried out by introducing the overlapping control strategy [23].

The research about the applicability of complex and robust control for the web handling systems is an actual topic with some recent contributes that try to test controllers on models based on the classical mathematical equation of the web handling systems: e.g., the H infinity controller with linear parameter varying [28], an hybrid structure based on PID controllers [6], a model reference adaptive PI controller [27], a back stepping controller optimized by the genetic algorithm [7]. Moreover, interesting experimental tests for investigating the relation between the tension distribution and the lateral movement of the web are shown in [17], a method for measuring the web tension from vibration measurements using a laser is shown in [26].

This work is aimed to demonstrate and to discuss the real applicability of a decentralized robust control to an experimental multi-span web transport system, which was demonstrated in the past by means of simulation [22-23]. The experimental system has been designed and realized at Kyushu Institute of Technology for creating a situation similar to a large-scale system with several sections (four main sections) mutually interconnected to each other's. In particular, robust controllers (based on H infinity formulation) have been designed for controlling the web tension forces and speed of the four sections of the system. The intrinsic performance deterioration of the decentralized control that neglects mutual interactions may be considered as an uncertainty that is controlled by the robust controller together with all the other uncertainties and nonlinearities that characterize the experimental system. Moreover, the overlapping decentralized control methodology [22] facilities the controller design, and decrease the order of each controller with many advantages in terms of computational effort and of system quickness; in this case each of the four decentralized controller is only of the fourth order.

All the steps of this research are shown in this paper: Section 2 introduces the experimental system, Section 3 describes an accurate dynamic model validated by open loop tests and used for designing the controllers, Section 4 describes the decentralized robust controllers by using the overlapping decomposition and Section 5 describes and discusses the most significant experimental results.

2. Description of the realized web handling system

The first experimental tests with the new system [8-9] were characterized by a web tension longitudinal speed of about 0.3-0.4 m/s that is quite a low transfer speed for the systems used in industrial applications. Currently the system has been considerably changed by means of some improvements applied to the mechanical part and electrical part of the web system [11].

The system (photo in Fig.1 and scheme in Fig.2) is composed of an unwinder section followed by a couple of tension sensors (one for each side of the web), a lead section (master speed control) followed by a couple of guide rolls where the web is wrapped to maximize the contact area between the web and the drive roll, a draw roll section controlled by means of a couple of tension sensors and, finally, the winder section. Four main sections strongly interlaced each other and 12 rollers constitute the system. The inputs to the system are the torque reference signals (u1, u2, u3, u4) of the four servomotors (Fig.2). The servomotors of the unwinder and winder section (750W, 2.39Nm maximum torque) are much more powerful than the servomotors of the lead section and draw-roll section (100W, 0.318Nm maximum torque); all servomotors are set in torque control mode. The outputs of the systems are the tension forces measured by tension sensors after the unwinder section and the draw-roll section (called T_1 and T_3, respectively, in Fig.2, calculated as the average value of a couple of tension sensors mounted at right and left side for each section), and the speed measured by encoders mounted on the servomotors for the lead section and the winder section (called v_2 and v_4 in Fig.2). The guide rollers positioned before and after the tension sensors are necessary for having a definite slope of the web. The inputs signals u_i, i=1….4 are sent to the motors by using a 4 channels D/A board, the tension signals feed a 4 channels A/D board, and the 4 motor encoder signals (including the speed signals of unwinder section and lead section) feed a digital counter. The controller's CPU receives signals by A/D boards and counters, performs the control algorithm (C language and Linux operating system), and outputs the command signals in real time to the motor driver through D/A boards.

The whole system is accurately mounted on a mechanical frame, designed for supporting the components of the system, and each of the 4 motors is connected to the respective roller using connection joints. A particular attention was dedicated to cover the lead and draw rolls with rugged material in such a way to guarantee maximum friction between those rolls and the web; a wider web has been considered (width of 30 cm) for increasing the contact area. Moreover, the power cables were placed far away with respect to the signal cables (Fig.1), and all the cables were covered with insulating tape.

3. Modeling of the system

The mathematical model of the web transport systems is usually based on three laws (see details in [8,24]) applied at each section between two consecutive drive rolls. The expressions in the Laplace domain are described by Eq. 1-3:

$$\varepsilon(s) = \frac{1}{L_k \cdot s} \cdot [v_b(s) - v_a(s)] \qquad (1)$$

$$s \cdot J_k(s) \cdot \omega_k(s) = r_k \cdot [F_{k+1}(s) - F_k(s)] + U_k(s) - C_k - K_{fk} \cdot \omega_k(s) \qquad (2)$$

$$F(s) = A \cdot \eta \cdot \left(\frac{1 + T_v(s)}{T_v(s)}\right) \cdot s \cdot \varepsilon(s) \qquad (3)$$

where (1) is the law of conservation of mass for the web section for evaluating the relation between the speeds (named v_a and v_b) of two adjacent drive rolls and the strain ε in the web, L_k is the length of the span (considered constant between two drive rolls); (2) is a torque balance equation of the tension forces F_{k+1} and F_k applied to the sides of the k_{th} roll having radius r_k, inertia J_k and angular velocity ω_k. U_k is the motor torque applied to the k_{th} roll, C_k is the dry friction torque, and K_{fk} is the viscous friction coefficient. Eq. (3) expresses the linear viscoelasticity of the web-material [1,25], where F is the force applied to the web, A is the cross-sectional area of the web, η is the viscosity modulus, $T_v = \eta/E$, with E the elastic modulus.

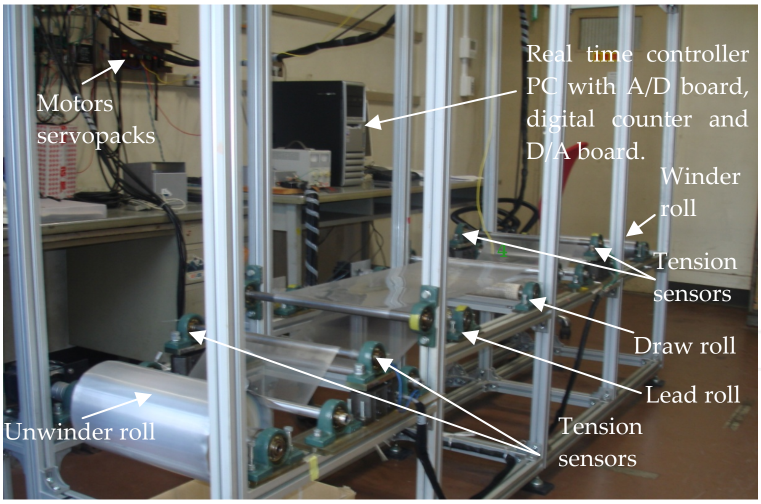

Figure 1. Experimental web transport system.

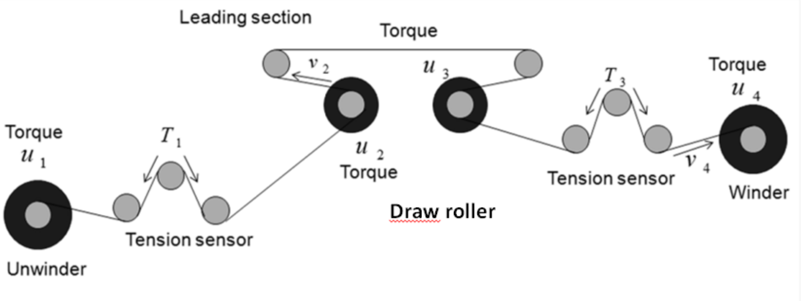

Figure 2. Scheme of the experimental web transport system.

For increasing the dynamic accuracy of the model taking into account the radius variation of the unwinder and winder rolls during the system running, Eq. (4) and (5) have been introduced in the model:

$$r_{uw}(t_k) = r_{uw}^0 - \frac{\omega_{uw}(t_k) \cdot Th \cdot \Delta t}{2\pi} \qquad (4)$$

$$r_w(t_k) = r_w^0 + \frac{\omega_{uw}(t_k) \cdot Th \cdot \Delta t}{2\pi} \qquad (5)$$

where $r_{uw}(t_k)$ and $r_w(t_k)$ are the roll radius of the unwinder and winder, respectively at time t_k, r_{uw}^0 and r_w^0 the initial unwinder and winder radius, $\omega_{uw}(t_k)$ and $\omega_w(t_k)$ the unwinder and winder angular speed (in rad/s) at time t_k, Th is the web thickness and Δt is $(t_k\text{-}t_k\text{-}1)$. On the contrary, the inertia variation has been considered negligible.

3.1 Model validation

The theoretical model contains many parameters, and some of them are strictly related to the geometry of the system (roll radius, section length, roll inertia, elastic modulus, web thickness, etc.). The other parameters are quite difficult to estimate (such as dry friction C_k, viscous friction K_{fk}), and often they are estimated iteratively by changing their values and testing the model behavior. The authors proposed a validation strategy [8] based on a multivariable optimization algorithm. The essence of this new strategy lies in the demonstration that the Laplace domain interlaced blocks system model obtained by (1)-(3) is equivalent to a classical state variable form of (6) when the inputs u_k are step inputs.

$$A\dot{Y} = BY + C \qquad (6)$$

In (6) $Y=[T_1, v_2, T_3, v_4, dT_1/dt, dv_2/dt, dT_3/dt, d^2T_3/dt^2, dv_4/dt, d^2v_4/dt^2]$ is the vector containing all the system outputs $[T_1, v_2, T_3, v_4]$, with first and second derivatives (for T_3 and v_4). The coefficients A and B are square matrices and C is a vector containing the input values. The possibility of solving (6) in a numerical way (i.e. using a classical fourth order Runge-Kutta method) permits iterative using of the solutions of the system model y_i^{th} (i= 1…4 for the considered case) in a minimum research problem expressed in (7), where $y_i^{\exp}$ is the experimental value of the system outputs

$$\min\left(\frac{y_i^{\exp} - y_i^{th}}{y_i^{\exp}}\right) \qquad (7)$$

The minimum research problem tries iteratively to solve (7) estimating a certain number of model parameters included in the square matrices A and B in such a way to validate the model. The minimum problem was solved using a sequential quadratic programming (SQP) method in the time interval [0–30 s] with sampling time 10 ms.

The model validation was carried out with the same procedure shown in [8,10], giving step-wise constant values to the 4 motors (inputs u_1, u_2, u_3, u_4) and measuring the system outputs (T_1, v_2, T_3, v_4). An input combination (shown in Table 1) was carried out for moving the web at a speed of around 0.7 m/s starting from the null position.

The preliminary tests exhibited a considerable amount of noise of the output signals even when the web is motionless; in Fig. 3 the signals T_1 and T_3 acquired when a constant force of 5.95 and 5.55 N was applied respectively to the first and second couple of tension sensors. In order to reduce the noise, a classical Kalman filter has been designed and realized for decreasing the experimental noise in the validation procedure. The filter design was carried out in an iterative way, analyzing the experimental data with and without filter in such a way not to create any phase distortion; the covariance matrices used are indicated in Table 2. The geometrical data directly measured on the experimental system and used in the model are summarized in Table 3 and 4; for the moment of inertia evaluation referred to the motor shaft for the four motors, the classical formulas of the inertia of a cylinder of know mass and radius with respect to its axis was used.

The other parameters (for k_{th} section, the dry friction torque C_k and the viscous friction coefficient K_{fk}) have been considered as unknown parameters to be estimated with the model validation procedure; so totally 8 unknown parameters have been estimated.

The estimated parameters are shown in Table 5 and the comparison between the updated model and the experimental data (both the data filtered with the same Kalman filter) is shown in Fig.4. The model, after the transitory, matches quite well the experimental data for both tension forces and speeds when a step variation is applied to the system motors.

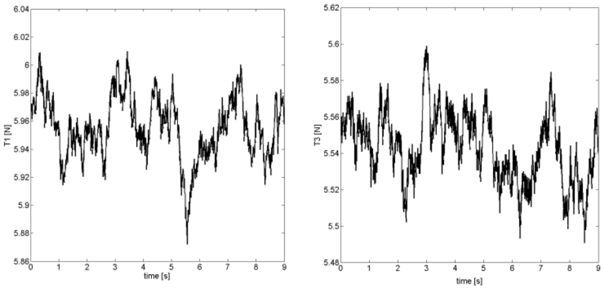

Figure 3. Preliminary analysis in static conditions.

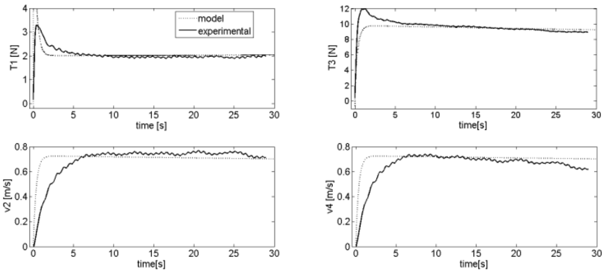

Figure 4. Comparison of the model with the web system.

4. Decentralized robust controller design

In the modern manufacturing companies, there are many reasons for the decentralization of the controllers [2] such as the high dimensionality of the system or subsystems assigned to different authorities. For this reason, it is very important to test the efficiency of the decentralized control on a web transport system.

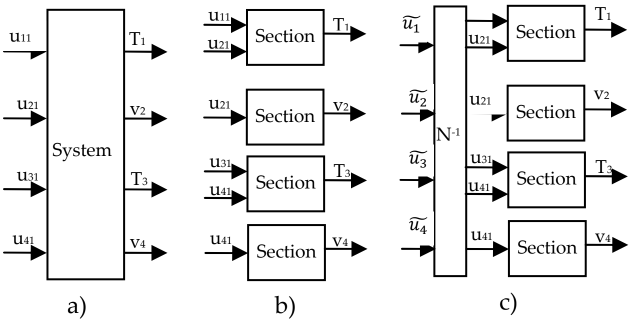

Figure 5. Overlapped subsystems decomposition.

Since 1998 it was demonstrated [22] that a decentralized controller based on overlapping decomposition permits considering some mutual interactions between subsystems. The resultant control system has better control performance compared with a decentralized controller based on disjoint decomposition, and yet it makes the controller design simpler. Following this approach, the realized web handling system (Fig 5a) has been divided in 4 overlapped sections (Fig.5b), and the new input controls (Fig 5c) are calculated through the matrix N^{-1} (8).

$$N^{-1} = \begin{bmatrix} \frac{r_1}{J_1} & \frac{r_2}{J_2} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{r_3}{J_3} & \frac{r_4}{J_4} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}$$

Moreover, in order to increase the control robustness when many sources of disturbance appear, a robust H infinity control has been designed for each subsystem making use of the mixed sensitivity approach [15] called S/KS/T scheme. In Fig. 6, *w* is the subsystem input, z the controlled signal, W_P, W_u, W_t the frequency weighting functions, and K(s) the controller to design.

The closed loop transfer matrix T_{zw} of this control scheme is given by (9) where $S = (I+GK)^{-1}$ is the sensitivity function and $T=(I-S)$ is the complementary sensitivity function.

$$T_{zw} = \begin{bmatrix} W_p S \\ W_u K S \\ W_t T \end{bmatrix} \tag{9}$$

H-infinity controller K(s) is calculated using the "loop-shifting formulae" [21] in order to minimize the $H\infty$ norm of the transfer function T_{zw}.

About the selection of the weighting functions for the 4 subsystems, as described in [3] there are no objective set of criteria, and the relationship between frequency dependent weighting functions and the closed-loop time response is difficult to obtain, although only simple guidelines exist. The weighting functions for the four subsystems were selected considering the guidelines given in [3] and verifying the performance of the controlled system on the validated model. The designed system decomposition (Figure 5) is almost symmetrical: the sections 1 and 2 control the motors input u_1 and u_2, while the sections 3 and 4 control the motors input u_3 and u_4. Moreover, the sections 2 and 4 have a similar equation linking the input u_2 to the output v_2 (section 2) and the input u_4 to the output v_4 (section 4). Starting from these considerations, the choice of the weighting functions shown in Figure 6 was carried out considering similar functions for the subsystems 1 and 3 and similar functions for the subsystems 2 and 4. For the subsystems 1 and 3 the general structure of W_P defined by (10) [14]

$$W_p(s) = \frac{\frac{s}{M} + \omega_B}{s + \omega_B \cdot \varepsilon_0} \tag{10}$$

where M is the maximum peak amplitude of S, $\|S\|_\infty \le M$, ω_B is the required bandwidth frequency and ε_0 is the steady-state error. The choice for the selection of W_P was to guarantee a steady state error equal to 0, fixing $\varepsilon_0 = 0$. In addition the main control interest was related to low frequencies, so the bandwidth ω_B was chosen equal to 1 and the maximum peak amplitude M=10. The polynomials W_u is used to avoid large control signal equal to 1, while the weighting function W_t was defined as first order polynomials with one zero and one pole (the pole is 10 times the zero). The same guidelines were used for the subsystems 2 and 4 but the transfer function of these subsystems have one pole on the *jw* axis; in this case a loop transformation technique is necessary [18,20] for designing the controllers. The weighting functions designed and used in all the experimental tests are shown in Table 6. The frequency curves of the four close loop subsystems with the designed controllers are shown in Fig.7.

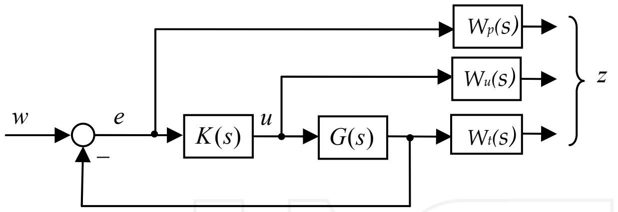

Figure 6. Scheme of the H∞ control.

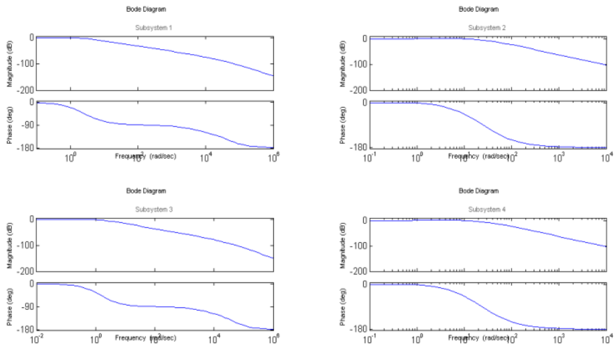

Figure 7. Bode diagrams of the 4 subsystems with the designed controllers.

The system decomposition technique and the good choice of the weighting functions considerably decrease the resulting controller order to obtain only the 4th order filter for each subsystem; this aspect guarantees a very small computational effort of the real time controller and an agile control action in spite of the complexity of the experimental system.

The decentralized robust controller design was realized with the objective of having a very short rise time and a good tracking of all the reference variables. Finally, each designed controller is discretized by a bilinear Tustin approximation at a 10 ms sampling time and used for the experimental tests. The discrete transfer functions of the designed controllers for the experimental tests are shown in Table 7.

5. Experimental results and discussions

Several experiments were carried out on the system that is depicted in Fig.1 for testing the decentralized controllers performances. At this aim, the designed controller was used for moving the web starting from a standstill. Different values of T_1 and T_3 and of the web velocity setpoint v_2 and v_4 were selected: in Fig. 8 the results with setpoint values of 4 and 6 N respectively for T_1 and T_3 and of 0.5 m/s for v_2 and v_4. In Fig.9 the controller outputs u_1, u_2, u_3, u_4 for the test shown in Fig.8 demonstrate the effectiveness of the controller design; for all the test duration, all the controller outputs (expressed in Nm) are limited in the motors operative ranges and they don't saturate the motors.

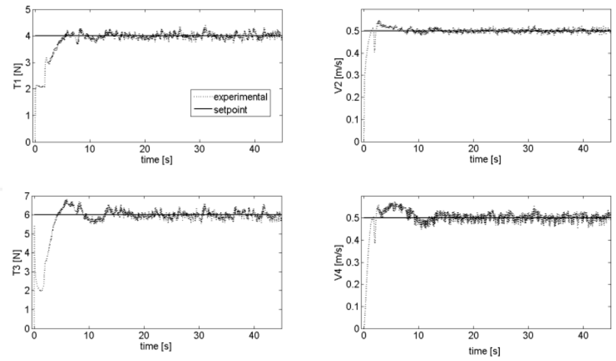

Figure 8. Experimental results starting from a standstill position.

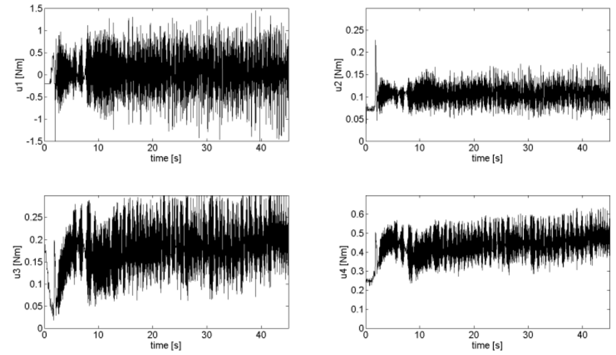

Figure 9. Experimental results: controller output for the test in Fig.8.

Other tests were carried out introducing a second step after 20 seconds for increasing the speed set point and maintaining constant the tensions T_1 and T_3; the results are good (see two examples with different speed set points in Figs 10-11) and several other tests were similarly carried out increasing the final speed until 1.5 m/s.

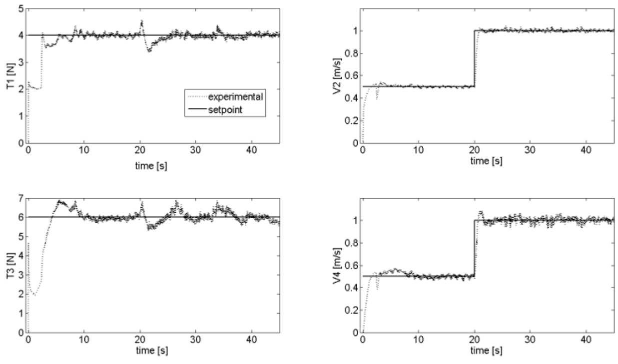

Figure 10. Experimental results increasing the web velocity after 20 seconds from 0.5 to 1 m/s.

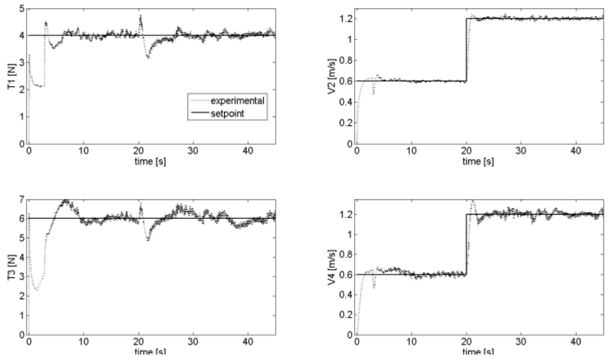

Figure 11. Experimental results increasing the web velocity after 20 seconds from 0.6 to 1.2 m/s.

Moreover tracking tests were carried out introducing an increasing and decreasing stepwise input variation around a set point for the velocity with maintaining constant tensions T_1 and T_3 (examples in Figs 12), demonstrating the excellent capacity of velocity tracking of the control system.

Also in the case of tension tracking tests introducing a second step for tension T_1 (Fig.13) and T_3 (Fig.14), the web velocity is maintained constant even for time-variant variables T_1 and T_3 (Fig.15), showing the effectiveness of the proposed decentralized controller for tension tracking.

Finally (Fig.16), in the case of a difficult situation with a stepwise input variation around a set point regarding all the variables (T_1 ,v_2, T_3, v_4) at the same time, the control performance was tested to confirm a complete controllability of all the sections.

All the tests show an excellent behavior of the controller: from Fig.8 it is possible to consider a very short rising time for the speed (around 2 seconds) and also quite short for T_1 and T_3 (around 5 seconds) starting from a standstill.

Tests in Figs. 10 and 11 were carried out with the aim of testing the web control system reaching at a relatively high speed of the web (1 or 1.2 m/s); the results show a good tracking performance of the speed for the increasing of the speed setpoint while maintaining the tension T_1 and T_3 at constant values. Only a small oscillation of the tension T_1 and T_3 appears at the time of setpoint change (20 seconds) and immediately it is damped in consequence of a very quick correction of the web speed to the desired value. The double step tests (Figs 12-16) were executed for testing the tracking performances for the control of the web speed and tension.

The oscillations that appear around the setpoint are mainly due to the high nonlinearity of the experimental system and to the coupling between consecutive subsystems nevertheless the overlapping.

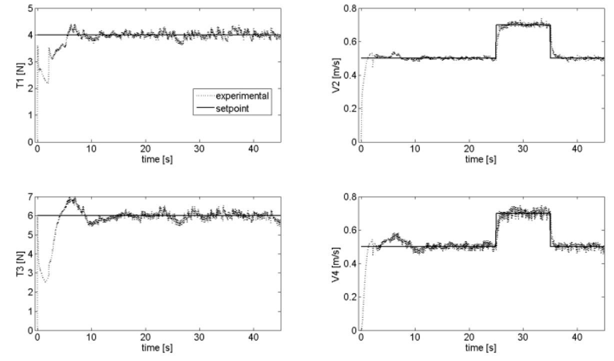

Figure 12. Experimental results tracking the web velocity.

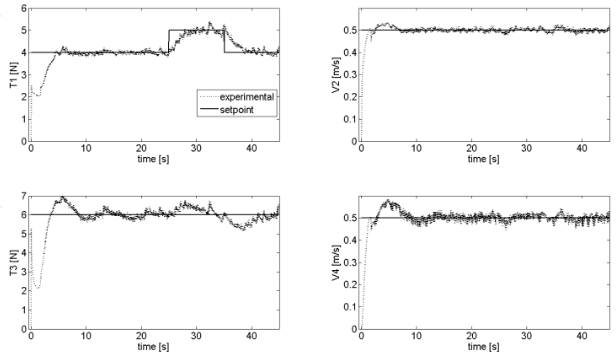

Figure 13. Experimental results tracking the tension T_1.

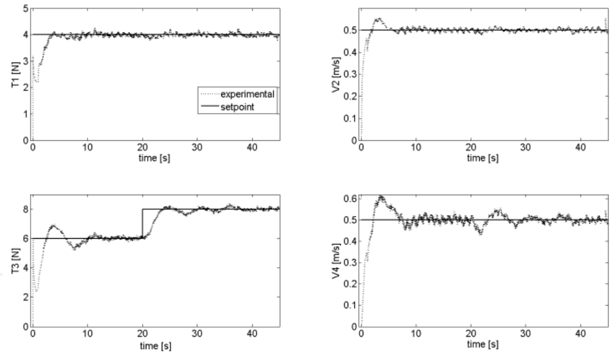

Figure 14. Experimental results increasing the tension T_3 after 20 seconds.

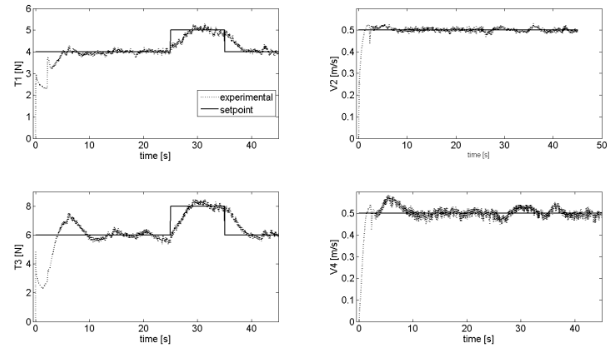

Figure 15. Experimental results tracking the tension T_1 and T_3 with a constant velocity.

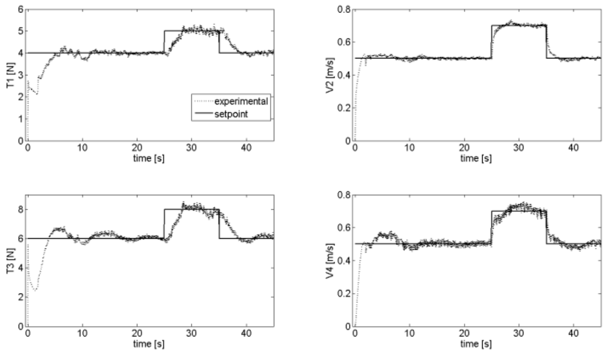

Figure 16. Experimental results tracking the tension T_1 and T_3 and the velocities .

The following results may be given from the experimental results:

- very good dynamic performances and tracking properties, with respect to all the controlled variables even when a second step or double step reference is applied;
- the tracking of web speed variations (Figs 10-12) is very quick and precise and in all the cases, and the second step reference does not provoke substantial increase of fluctuations of T_1 and T_3 , which are maintained at a constant value;
- the tracking of tension T_1 and T_3 variation (Figs 13-16) is quick, but slightly slower compared to the speed tracking. The web speed maintains a constant value without increase of fluctuations during the tension tracking control.

The experimental results here presented may be compared with other results recently shown in literature; in [14] a similar behavior was obtained for an experimental web system with a centralized H∞ control or LPV controller with similar oscillations of the web tension when velocity variations occur for a reduced-size system containing 3 motors. The same experimental system was later studied [4] by decomposing into 2 subsystems with an overlapping decentralized control; the results show oscillations of the unwinder and winder tensions similar to the oscillations found in the experiments of the present paper. Other interesting experimental results referred to the decentralized control of a 4 sections web handling system were presented in [19] using PI controllers with quite high velocity (reference velocity of 5.08 and 7.62 m/s). Significant oscillations of the controlled variables (especially for the tensions) are found also in the results shown in this study[19].

This paper analyses the control performances of a 4 sections web handling experimental system from a standstill position demonstrating that a robust decentralize controller may guarantee good tracking characteristics for all the controlled variables (tensions and velocity), with some shrewdness in the controller design such as the overlapping decomposition, even if not using antialiasing filters for the sensors signals.

6. Conclusion

The importance of testing control performance for web handling systems in such a way to guarantee tracking properties of web speed and tension is well appreciated by the researchers in this field, and it is probably of great interest for the industries to successfully operate these kinds of system as well.

This paper shows the results of our research that tried to realize a web handling system, which can be regarded as a large-scale system, in a laboratory. Previous studies [8,22,23] were used to realize an accurate dynamic model of the multi-span web transport system and for designing a decentralized control.

For achieving a high level of robustness of the controller with respect to parameters variations, a centralized H∞ control was chosen as controller scheme and designed with excellent dynamic characteristics for tracking step variations of speed and tension by other researchers [4].

It is important to underline that the design and the realization of such decentralized control are necessarily linked to the availability of a validated model of the system: the model is necessary for tuning the subsystem controllers and for estimating the overlapping parameters of the overlapping decomposition.

The great advantage of our approach, as discussed in this paper, lies in the absolute robustness of the designed controller that maintains some characteristics (quick dynamics and high capacity to track) for different cases of set point profiles and for all the variables controlled in different sections of the system. This aspect is interesting for a potential industrial applicability. Another interesting aspect is the possibility of decentralizing the controller for each subsystem that makes it of very low order (4th in this case) with easiness of realization, very low computational cost and absolute real time applicability (as demonstrated by the results of this paper).

u_1[Nm]	u_2[Nm]	u_3[Nm]	u_4[Nm]
0.1	0.1	0.1	0.7

Table 1. Input combination for the model validation

	Q_k	R_k	N_k
Tension sensors	10^{-5}	10^{-1}	0
Speed sensors	10^{-4}	10^{-1}	0

Table 2. Noise covariance matrices of the Kalman filter.

Parameter definition	Symbol in (1)-(3)	*k*=1 unwinder section	*k*=2 lead section	*k*=3 draw-roll section	*k*=4 winder section
Radius of drive roll [m]	r_k	0.084	0.021	0.021	0.026
Web length [m]	L_k	0.75	1.2	1.25	-
Moment of inertia [Nms^2]	J_k	0.024	0.000269	0.000269	0.00473

Table 3. Geometrical data of the system.

Parameter definition	Symbol in (1)-(3)	Value
Cross sectional area [m^2]	A	1.2 x10^{-5}
Viscosity modulus [Ns/m^3]	η	1.5 x10^9
Elastic modulus [N/m^2]	E	9.8 x10^9

Table 4. Data of the web.

Parameter definition	Symbol in (1)-(3)	*k*=1 unwinder section	*k*=2 lead section	*k*=3 draw-roll section	*k*=4 winder section
Viscous friction coefficient [Ns]	K_{fk}	0.0045	0.0045	0.0055	0.00558
Dry friction torque [Nm]	C_k	0.23	0	0	0.29

Table 5. Estimated parameters of the model.

Weighting function	Subsystem 1	Subsystem 2	Subsystem 3	Subsystem 4
W_p	$(0.1s+1)/s$	$(0.1s+1)/s$	$(0.1s+1)/s$	$(0.1s+1)/s$
W_u	1	1	1	1
W_t	$(s+1000)/$ $(s+10000)$	$(s+600)/$ $(s+1200)$	$(s+1000)/$ $(s+10000)$	$(s+600)/$ $(s+1200)$

Table 6. Weighting functions for the 4 subsystems.

Controller discrete transfer functions for the 4 subsystems	
Subsystem 1	$\frac{0.0111 z^4 + 0.0222 z^3 + 0.000143 z^2 - 0.0195 z - 0.00098}{z^4 - 0.0081 z^3 - 1.869 z^2 + 0.00672 z + 0.8705}$
Subsystem 2	$\frac{0.00812 z^4 - 0.02159 z^3 - 0.138 z^2 + 0.0216 z + 0.0568}{z^4 - 1.8318 z^3 + 0.2734 z^2 + 0.9485 z - 0.39017}$
Subsystem 3	$\frac{0.01077 z^4 + 0.0216 z^3 + 0.00139 z^2 - 0.0189 z - 0.00952}{z^4 - 0.006162 z^3 - 1.8709 z^2 + 0.00482 z + 0.8723}$
Subsystem 4	$\frac{0.1036 z^4 - 0.00275 z^3 - 0.176 z^2 + 0.00275 z + 0.00726}{z^4 - 1.84177 z^3 + 0.2863 z^2 + 0.9527 z - 0.3972}$

Table 7. Controller discrete transfer functions for the 4 subsystems.

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