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Research Letter

Wiener's Loop Filter for PLL-Based Carrier Recovery of OQPSK and MSK-Type Modulations

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This letter considers carrier recovery for offset quadrature phase shift keying (OQPSK) and minimum shift keying-type (MSK-type) modulations based on phase-lock loop (PLL). The concern of the letter is the optimization of the loop filter of the PLL. The optimization is worked out in the light of Wiener's theory taking into account the phase noise affecting the incoming carrier, the additive white Gaussian noise that is present on the channel, and the self-noise produced by the phase detector. Delay in the loop, which may affect the numerical implementation of the PLL, is also considered. Closed-form expressions for the loop filter and for the mean-square error are given for the case where the phase noise is characterized as a first-order process.

1. Introduction

Offset quadrature phase-shift keying (OQPSK) and minimum shift keying-type (MSK-type) modulations play a prominent role in optical and radio transmission systems. The renewed interest in such modulations for application to optical and 60 GHz radio transmissions is demonstrated by the recent contributions [1, 2]. As is known, MSK-type signals can be regarded as approximated forms of OQPSK when the first term of the Laurent's decomposition dominates the series [3]. This suggests the extension of synchronization algorithms usually employed for OQPSK to MSK-type signals [4].

In carrier recovery for OQPSK and MSK-type formats, it is worth taking into account the self-noise that affects the phase detector, a noise that is not present in quadrature amplitude modulation (QAM) and phase-shift keying (PSK) formats. The design of a prefilter that mitigates the effects of the self-noise in an open-loop synchronizer is considered in [5]. In this letter, we focus on the design of the loop filter of closed-loop carrier recovery based on the phase-lock loop (PLL). The Wiener's approach, studied in [6] for QAM modulation formats, is extended here to the case where self-noise is present, enabling application to OQPSK and MSK-type formats.

2. System Model

The complex envelope of the continuous-time received signal is

$$r(t) = e^{j\theta(t)}\sum_i a_i u(t - iT) + w(t), \tag{1}$$

where j is the imaginary unit, $a_k = \pm 1$ for k even and $a_k = \pm j$ for k odd, $\theta(t)$ is the phase noise, $u(t)$ is the impulse response of the shaping filter, T is the bit repetition interval, and $w(t)$ is the complex additive white Gaussian noise with power spectral density N_0. The model given by (1) holds exactly for OQPSK and MSK, while it provides a close approximation for many MSK-type signals of practical interest, as for instance the popular GMSK with $BT = 0.3$, when the impulse response $u(t)$ is the first term of the Laurent decomposition.

The received signal is filtered through the matched filter $u^*(-t)$, sampled at the time instants kT, and filtered through the zero forcing (ZF) prefilter with frequency response

$$C(e^{j2\pi fT}) = \frac{2T}{\sum_k \left| U(f - k/2T) \right|^2}, \tag{2}$$

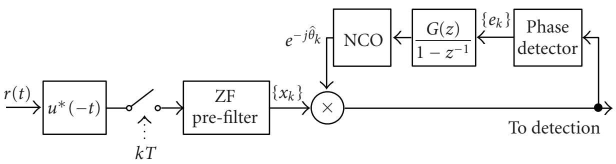

FIGURE 1: Block diagram of the system.

where $U(f)$ is the Fourier transform of $u(t)$. The kth sample of the signal at the output of the prefilter has the form

$$x_k = e^{j\theta_k} \sum_i a_i v_{k-i} + n_k, \quad (3)$$

where the samples of the impulse response $\{v_k\}$ satisfy the Nyquist condition in the form $v_0 = 1$ and $v_{2k} = 0$, for $k \neq 0$. The sequence $\{n_k\}$ is the complex additive Gaussian noise with power spectral density

$$\Psi_n(e^{j2\pi fT}) = \frac{N_0}{T} \left| C(e^{j2\pi fT}) \right|^2 \left| \sum_k U^* \left(f - \frac{k}{T} \right) \right|^2. \quad (4)$$

The kth sample at the output of the phase detector is

$$e_k = \Im\{a_k^* x_k e^{-j\hat{\theta}_k}\} = \sin(\theta_k - \hat{\theta}_k) + \lambda_k \cos(\theta_k - \hat{\theta}_k) + \xi_k, \quad (5)$$

where the superscript $*$ denotes complex conjugation, $\Im\{x\}$ extracts the imaginary part of x, and $\hat{\theta}_k$ is the kth phase sample produced by the carrier recovery loop. The sequence $\{e_k\}$ is filtered and used to produce the carrier estimate by a number controlled oscillator (NCO). The block diagram of the system is reported in Figure 1. The even and the odd samples of the Gaussian noise $\{\xi_k\}$ come from the imaginary and the real part of $\{n_k\}$, respectively, and therefore are uncorrelated. The power spectral density of $\{\xi_k\}$ is

$$\Psi_\xi(e^{j2\pi fT}) = \frac{1}{4}\Psi_n(e^{j2\pi fT}) + \frac{1}{4}\Psi_n(e^{j2\pi(f-1/2T)T}), \quad (6)$$

where the factor 1/4 comes from the folding (factor 1/2) and from the projection of the complex noise over one dimension (factor 1/2).

The kth sample of the self-noise is

$$\lambda_k = \sum_i \Im\{a_k^* a_{k-2i-1}\} v_{2i+1}. \quad (7)$$

In what follows we use the z-transform to represent sequences, where z^{-1} is the unit delay, that is, T. The z-spectrum of $\{\lambda_k\}$ is

$$\Psi_\lambda(z) = \sum_i v_{2i+1}^2 - \sum_k v_{2k+1}^2 z^{-2k-1}. \quad (8)$$

Note that the power spectrum of self-noise depends on the odd samples of $\{v_k\}$. When $|\sum_k U^*(f - k/T)|^2$ satisfies the condition of not having spectral zeros, as it happens for instance with GMSK, it is possible to design the prefilter in such a way that the condition $v_k = \delta_k$ is satisfied (δ_i denotes the Kronecker delta). This design of the prefilter induces noise enhancement. However, at high signal-to-noise ratio, noise enhancement can be accepted, enabling the approach of [5]. Also note that, when $U(f)$ is the square root of a Nyquist filter, as it often happens with OQPSK, we obtain

$$\Psi_\xi(z) = \frac{N_0}{2E_b}, \quad (9)$$

where $E_b = \int_{-\infty}^{\infty} |u(t)|^2 dt$ is the energy per bit.

3. Derivation of The Optimal Transfer Function

For small phase error, the phase detector can be linearized, $\sin(\theta - \hat{\theta}) \approx \theta - \hat{\theta}$, $\cos(\theta - \hat{\theta}) \approx 1$, leading to the error polynomial

$$E(z) = \Theta(z) - \hat{\Theta}(z) + \Lambda(z) + \Xi(z). \quad (10)$$

The *open-loop transfer function* is

$$G(z) = \frac{\hat{\Theta}(z)}{E(z)} = \sum_{k=K}^{\infty} g_k z^{-k}, \quad (11)$$

where $K > 0$ is the delay of the loop. The number of poles of $G(z)$ is the *order* of the loop, while the number of poles of $G(z)$ at $z = 1$ is the *type* of the loop. In the analysis of the loop, it is useful to introduce the polynomial

$$Y(z) = \Theta(z) + \Lambda(z) + \Xi(z). \quad (12)$$

We assume that $\{\theta_k\}$, $\{\lambda_k\}$, and $\{\xi_k\}$ are independent random sequences, therefore the z -spectrum of $\{y_k\}$ is

$$\Psi_y(z) = \Psi_\theta(z) + \Psi_\lambda(z) + \Psi_\xi(z). \quad (13)$$

The *closed-loop transfer function* is

$$H(z) = \frac{G(z)}{1 + G(z)} = \frac{\hat{\Theta}(z)}{Y(z)}. \quad (14)$$

The optimal $H(z)$ is obtained from the Wiener-Hopf equations. Following the steps of [6], we get

$$H(z) = \alpha^2 (1 - P(z)) \left[(1 - P(z^{-1})) \Psi_\theta(z) \right]_K^\infty, \quad (15)$$

where the notation $[X(z)]_i^j = \sum_{k=i}^j x_k z^{-k}$ is used. The real number α^2 is computed by Szego's formula, and $1 - P(z)$ is the causal monic and minimum phase transfer function which results from the spectral factorization

$$(1 - P(z)) \alpha^2 (1 - P(z^{-1})) = \Psi_y^{-1}(z). \quad (16)$$

4. Approximate Solution

We observe that in many cases of practical interest, the bandwidth of the loop filter is small compared to the bit frequency, therefore we aim to approximate the spectrum of self-noise and the spectrum of channel noise in the low-frequency region. The spectrum of self-noise is well approximated in the low-frequency region by

$$\Psi_\lambda(z) = (1 - z^{-1})\gamma_2(1 - z), \quad (17)$$

with $\gamma_2 = \sum_{k=0}^{\infty}(2k+1)^2 v_{2k+1}^2$. The spectrum of the additive channel noise $\{\xi_k\}$ is approximated in the low-frequency region by (9). Using (9) and (17) in (13) and performing the spectral factorization, we obtain

$$H(z) = \frac{[Q(z) + \alpha^2\gamma_2 z^{-1}]_K^\infty}{1 + Q(z)}, \quad (18)$$

and the mean square error (MSE)

$$E\{(\theta_i - \hat{\theta}_i)^2\} = \frac{1}{\alpha^2} + \sum_{k=1}^{K-1}\frac{q_k^2}{\alpha^2} - 2\gamma_2 - \frac{N_0}{2E_b} - (\alpha^2\gamma_2^2 + 2\gamma_2 q_1)\delta_{K-1}, \quad (19)$$

where $1 + Q(z) = (1 - P(z))^{-1}$.

5. Case Study

Assume that the phase noise is the sum of random phase walk plus white phase. The phase noise spectrum is

$$\Psi_\theta(z) = \frac{\gamma_{-2}}{(1 - z^{-1})(1 - z)} + \gamma_0. \quad (20)$$

The spectral factorization is performed by root finding. The result is

$$1 - P(z) = \frac{1 - z^{-1}}{(1 - z_1 z^{-1})(1 - z_2 z^{-1})}, \quad (21)$$

$$Q(z) = \frac{(1 - z_1 - z_2)z^{-1}}{1 - z^{-1}}\left(1 + \frac{z_1 z_2}{1 - z_1 - z_2}z^{-1}\right), \quad (22)$$

where the two roots are

$$z_i = \frac{x_i - \sqrt{x_i^2 - 4}}{2}, \quad i = 1, 2, \quad (23)$$

with

$$x_{1,2} = 2 + \frac{\gamma_0 + N_0/2E_b}{2\gamma_2} \pm \sqrt{\left(\frac{\gamma_0 + N_0/2E_b}{2\gamma_2}\right)^2 - \frac{\gamma_{-2}}{\gamma_2}}. \quad (24)$$

Putting (22) in (18) and using (14), one gets

$$H(z) = \frac{(1 - z_1 - z_2 + z_1 z_2)z^{-K}}{(1 - z_1 z^{-1})(1 - z_2 z^{-1})}, \quad (25)$$

$$G(z) = \frac{(1 - z_1 - z_2 + z_1 z_2)z^{-K}}{(1 - z^{-1})\left(1 - z_1 z_2 z^{-1} + \sum_{k=1}^{K-1}(1 - z_1 - z_2 + z_1 z_2)z^{-k}\right)}. \quad (26)$$

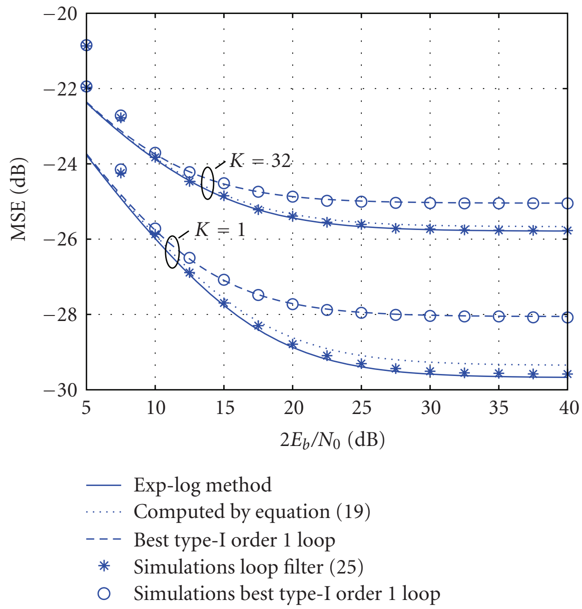

Figure 2: MSE versus $2E_b/N_0$ for OQPSK with roll-off 0.1.

Note that with $K = 1$ and without self-noise, one would obtain an optimal type-I order 1 loop [6]. The presence of self-noise has increased the order of the optimal loop inducing a pole at $z = z_1 z_2$. For $K > 1$, a type-I order K loop is obtained. Using (22) and substituting $\alpha^2 = z_1 z_2/\gamma_2$ in (19), we get for the MSE

$$E\{(\theta_i - \hat{\theta}_i)^2\} = \gamma_2\left(\frac{1}{z_1 z_2} + 2z_1 + 2z_2 - z_1 z_2 - 4\right) + \frac{\gamma_2}{z_1 z_2}(K-1)(1 - z_1 - z_2 + z_1 z_2)^2 - \frac{N_0}{2E_b}. \quad (27)$$

6. Numerical Results

We derive numerical results for OQPSK with raised-cosine shaping and roll-off 0.1 and GMSK with $BT = 0.3$. The spectrum of phase noise that affects the free-running oscillator used for the analysis is characterized by $\gamma_{-2} = 5\cdot 10^{-5}$ and $\gamma_0 = 10^{-4}$. Figure 2 shows analytical results and simulation results for the MSE versus $2E_b/N_0$ for $K = 1$ and $K = 32$ with OQPSK. Note that, while analytical results have been obtained assuming that no decision errors affect the phase detector (5), in the simulations the phase detector makes use of decisions in place of true data. At low SNR simulation results deviate from the theory due to decision errors. The best MSE is given by the optimum $H(z)$ obtained without approximations using the numeric exp − log method [7]. From the figure we see that the use of the loop filter we propose gives a benefit of about 2 dB on the floor compared to the classical order-I type 1 loop when $K = 1$. When $K > 1$, the benefit diminishes. Also observe that the performance of the approximated transfer function derived in Section 5 is close to the optimal one. Figure 3

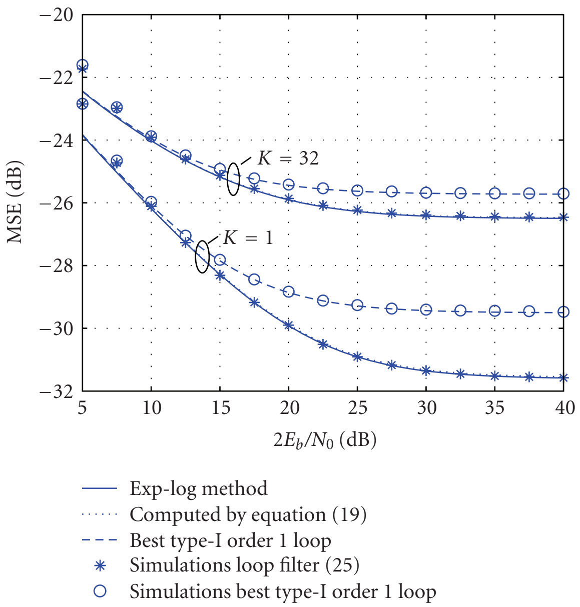

FIGURE 3: MSE versus $2E_b/N_0$ for GMSK with $BT = 0.3$.

reports the performance of GMSK. The benefits obtained by our method are similar to those shown in Figure 2. In this case, the performance of the approximated transfer function is virtually the same as that of the optimal one.

7. Conclusions

The main novel results of the letter are the optimal loop filter and the MSE for carrier recovery based on discrete-time PLL for OQPSK and MSK-type modulation formats. Compared to previous literature, self-noise that affects the phase detector and delay in the loop are taken into account in the design of the loop filter. Closed-form expressions for the loop filter and for the MSE are given for the case where the phase noise is modelled as a first-order process.

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