



Evidence for partial rotation alignment in proton emitting ^{121}Pr

M.C. Lopes^a, E. Maglione^{b,c,*}, L.S. Ferreira^a

^a Centro de Física das Interações Fundamentais, and Departamento de Física, Instituto Superior Técnico, Avenida Rovisco Pais, P1049-001 Lisbon, Portugal

^b Dipartimento di Fisica “G. Galilei”, Via Marzolo 8, I-35131 Padova, Italy

^c Istituto Nazionale di Fisica Nucleare, Padova, Italy

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ABSTRACT

Using nonadiabatic quasiparticle calculations we reproduce the experimental half-life for proton radioactivity in ^{121}Pr assuming that the decaying state has angular momentum $J^\pi = 7/2^-$, thus showing for the first time clear evidence for partial rotation alignment in a proton emitting nucleus. The treatment of the pairing interaction in the BCS approach produces profound changes in the ordering of energy levels, and at high deformation, the state $7/2^-$ coming from the $h_{11/2}$ spherical shell becomes the bandhead.

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Proton emission is a tunneling process through the Coulomb and centrifugal barrier. The half-lives are thus quite sensitive to the angular momentum of the decaying nucleus. The energy of the outgoing proton is quite low, and the process has been interpreted as decay from a resonance very low in the continuum, of the proton in the field of the core nucleus. In the case of deformed nuclei, the simple adiabatic model [1–3] that considers the wave function of the proton as a single-particle Nilsson resonance, has been very successful in determining uniquely deformation and angular momentum of the parent nucleus assumed to be a rotor, with infinite moment of inertia. ^{131}Eu was the only nucleus that presented some ambiguity in the assignment of the quantum numbers of the decaying state. This ambiguity was solved by a later measurement of the fine structure. The simultaneous interpretation of the experimental branching ratio and half-lives for decay to ground and excited 2^+ state with a consistent deformation, selected unambiguously the decaying state [4].

A more sophisticated model, the nonadiabatic quasiparticle model [5], that provides a complete and consistent description of proton emission by taking into account the finite moment of in-

ertia of the core and the pairing residual interaction, has later confirmed the spin assignment of the adiabatic approximation.

In the mass region where proton radioactivity occurs, the negative single particle parity states are mainly based on the $h_{11/2}$ spherical state. It is well known [6,7] that in this case, for oblate and moderate prolate deformations, the strong coupling model should not work, since the Coriolis interaction is very strong. One thus expects a rotational alignment, i.e. the spectrum should be characterized by a rotational band based on a $11/2^-$ bandhead with the same energy spacings as in the rotational band of the daughter even-even nucleus, independently on the K value of the Nilsson level close to the Fermi surface.

Recently, ground state proton radioactivity from ^{121}Pr [8] has been measured at Argonne National Laboratory. According to the mass model of Möller and Nix [9], this new proton emitter should have a prolate deformation with $\beta_2 = 0.318$ and $\beta_4 = 0.075$. The adiabatic calculations performed by Robinson and collaborators [8] also confirmed that the emission occurred from a highly prolate deformed state, but were not able to discriminate between a $3/2^+$ and $3/2^-$ as possible decaying Nilsson states. In the present work, we try to solve this ambiguity by performing a nonadiabatic quasiparticle calculation, and show that it is possible to give a firm assignment to the angular momentum of ^{121}Pr .

We will start by studying the decay process within the adiabatic approach as the authors of Ref. [8] in order to allow a clear

* Corresponding author at: Dipartimento di Fisica “G. Galilei”, Via Marzolo 8, I-35131 Padova, Italy.

E-mail address: maglione@pd.infn.it (E. Maglione).

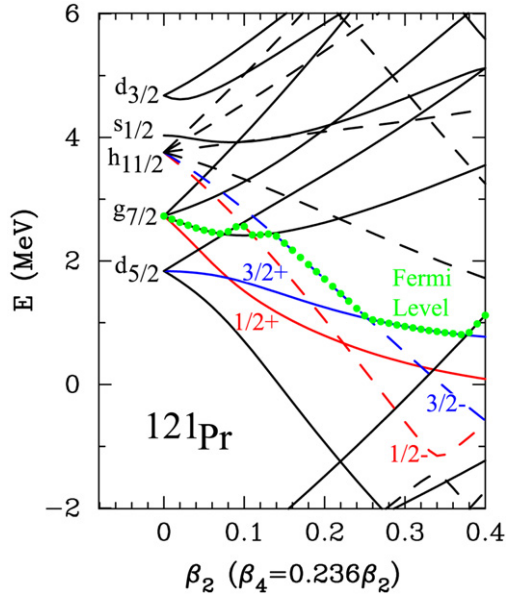


Fig. 1. Nilsson proton levels of ^{121}Pr as a function of quadrupole deformation (β_2). The hexadecapole deformation was scaled as $\beta_4 = 0.236\beta_2$ in order to maintain the ratio suggested by Möller and Nix [9]. The levels closest to the Fermi level (dotted line) are $K = 1/2$ and $K = 3/2$ with both positive and negative parities (full and dashed lines, respectively). The positive parity states are coming from the spherical $g_{7/2}$ and $d_{5/2}$ shells, while the negative parity ones are both coming from the spherical $h_{11/2}$ shell.

comparison between both calculations and single out the effect of the rotational spectrum of the daughter nucleus and the pairing residual interaction.

The single particle Nilsson energies are reported in Fig. 1. In order to compare our results with the ones obtained in Ref. [8], we have used the same parameterization of the mean field. As has been shown in Ref. [10] for strongly deformed nuclei, the decay widths are practically independent of the single particle potential used. The $K^\pi = 3/2^+$ and $K^\pi = 3/2^-$, assigned as decaying state in Ref. [8], lie close to the Fermi surface for the predicted [9] large deformation, and are the most probable candidates as decaying state. According to Ref. [1] the decay width in the adiabatic approach is given by

$$\Gamma_{l_p j_p}^{K_i} (r) = \frac{\hbar^2 k}{\mu(j_p + 1/2)} \frac{|\alpha_{l_p j_p}^{K_i}(r)|^2}{|G_{l_p}(kr) + iF_{l_p}(kr)|^2} u_{K_i}^2, \quad (1)$$

where F and G are the regular and irregular Coulomb functions respectively, and $\alpha_{l_p j_p}^{K_i}$ the component with angular momentum j_p , equal to the spin of the decaying nucleus, of the Nilsson wave function. The quantity $u_{K_i}^2$ is the spectroscopic factor, corresponding to the probability that the single particle level in the daughter nucleus is empty, evaluated in the BCS approach. Since usually the emitting nucleus is in the band-head of the rotational band, $j_p = K_i$.

The half-lives corresponding to the widths given by Eq. (1), are shown in Fig. 2. Our results for both $3/2$ states are similar to the ones presented by Robinson [8]. Their results, however, are not able to reproduce the experimental half-life in the region of deformations predicted by Möller and Nix [9]. In our case, assuming decay from the $3/2^-$ state, the half-life reproduces the experimental value. This is not only due to the different (0.26) β_4/β_2 relation used in Ref. [8]. The main reason being that in our calculations, in contrast to Ref. [8] where it is kept constant, the spectroscopic factor u^2 , that corresponds to the probability that the single-particle level in the daughter nucleus is empty, comes from a BCS calculation and changes with deformation, leading to a smaller value

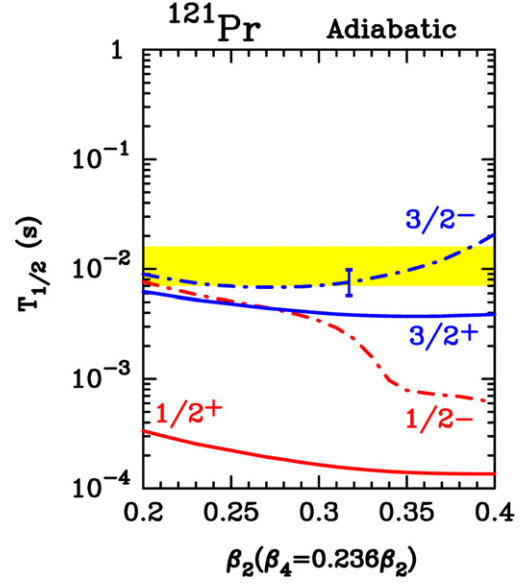


Fig. 2. Half-life for the $K = 1/2$ and $K = 3/2$ Nilsson states of Fig. 1 as function of β_2 , obtained with adiabatic calculations. The shaded (yellow) area represents the experimental half-life, taking into account the error bar. The error bar shown on the theoretical calculation for the $j^\pi = 3/2^-$ state at the deformation suggested by Möller and Nix is representative of the uncertainty in the calculation due to the experimental error in the Q_p -value of 900(10) keV [8]. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this Letter.)

at higher deformations. This effect is particularly pronounced for the state $3/2^-$, which becomes lower than the Fermi level (Fig. 1), making it less probable to find this state empty. As a consequence, the half-life of this state becomes longer and closer to the experimental value.

In Fig. 2 the half-lives for the $1/2^+$ and $1/2^-$ are also reported, since, changing slightly the spin-orbit interaction and consequently the relative positions of the positive and negative parity states, they could become the Fermi level. However, the half-lives are quite far from the experimental value and therefore are ruled out as candidates for ground state.

Improvements to the strong-coupling approach, taking into account the Coriolis coupling, have been presented in Refs. [11,12]. One has to consider the total Hamiltonian:

$$\begin{aligned} H &= H_{\text{int}} + H_{\text{col}} \\ &= H_{\text{int}} + \frac{\hbar^2}{2\mathcal{I}} (J^2 + j^2 - 2\vec{J} \cdot \vec{j}), \end{aligned} \quad (2)$$

which includes in H_{int} the Nilsson Hamiltonian and the pairing residual interaction, and thus has to be diagonalized between quasiparticle states [5]. The vector \vec{J} and \vec{j} are the total angular momentum of the nucleus and the angular momentum of the odd proton respectively.

The moment of inertia of the rotor \mathcal{I} appearing in Eq. (2), is usually determined from the energy of the 2^+ state of the daughter nucleus. Unfortunately, there is no experimental information concerning the 2^+ energy in the case of ^{120}Ce . Therefore, we estimate this value by using the Grodzins formula [7,13] which provides an empirical relation between the E^{2^+} and the quadrupole deformation (β_2) of the nucleus in the following way

$$E^{2^+} \approx \frac{1225}{A^{(7/3)} \beta_2^2} \text{ MeV} \quad (3)$$

obtaining a value of 0.167 MeV for the E^{2^+} in ^{120}Ce using a $\beta_2 = 0.318$ as suggested by Möller and Nix [9].

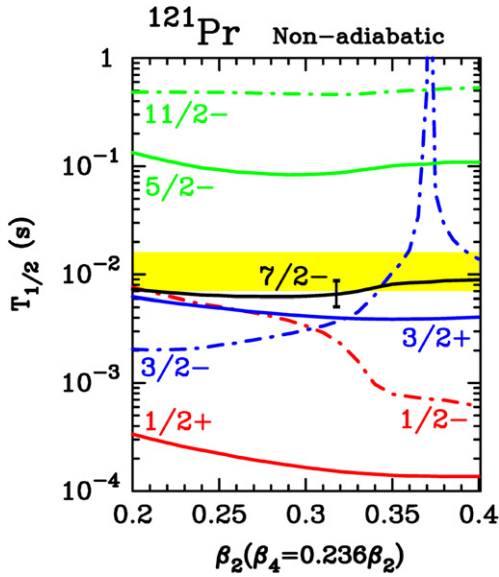


Fig. 3. Half-life of $^{121}_{59}\text{Pr}$ as a function of deformation β_2 for different I^π states. The calculations were performed within the nonadiabatic approach with residual pairing interaction. The experimental value, and the theoretical and experimental uncertainties are reproduced as in Fig. 2.

In the nonadiabatic quasiparticle model the partial decay widths are given by [5],

$$\Gamma_{I_p j_p}^R(r) = \frac{\hbar^2 k}{\mu} \frac{2(2R+1)}{(2J+1)} \times \left| \sum_{K>0} u_K^f \langle j_p K R 0 | J K \rangle a_K^J \frac{\phi_{I_p j_p}^K(r)}{G_{I_p}(kr) + iF_{I_p}(kr)} \right|^2, \quad (4)$$

where F , G and u_K^f are defined as in Eq. (1). The quantity K is the projection of the angular momentum on the symmetry axis of the rotor and a_K^J , the components of the wave function after the diagonalization on the basis of states $|JKM\rangle$. R is the angular momentum of the daughter nucleus. In the case of decay to the ground state of the daughter nucleus $R=0$ and Eq. (4) reduces to:

$$\Gamma_{I_p j_p}^0(r) = \frac{\hbar^2 k}{\mu} \frac{2}{(2J+1)} \left| \sum_{K>0} u_K^f a_K^J \frac{\phi_{I_p j_p}^K(r)}{G_{I_p}(kr) + iF_{I_p}(kr)} \right|^2 \delta_{J, j_p}. \quad (5)$$

The residual pairing interaction is treated within the BCS approach using a constant gap $\Delta \approx 12/\sqrt{A}$ [14].

We performed the nonadiabatic calculations, diagonalizing the Coriolis interaction in the correlated field of quasiparticles, considering different J values for both positive and negative parity states.

As one can see from Fig. 3, the half-lives for both parities $J=1/2$ states remain the same as in the adiabatic calculation, meaning that the Coriolis interaction does not mix appreciably the different basis states. The main reasons for this behaviour are the quite large separation in energy of the $K=1/2$ basis states, and magnitude of the Coriolis matrix elements that are weak for positive parity states, since they have small angular momentum, and for negative parity states since they are coming from different major shells. The states with $J^\pi=3/2^+$ and $J^\pi=3/2^-$ are both modified due to the Coriolis interaction and the good agreement observed earlier in Fig. 2 for the state $3/2^-$ disappears, although, due to the large theoretical uncertainty, they cannot be discarded as possible candidates for the emitting state. All the other half-lives for states coming from negative parity appear much higher than the experimental value, with the exception of $J^\pi=7/2^-$ for which it reproduces exactly the experimental value.

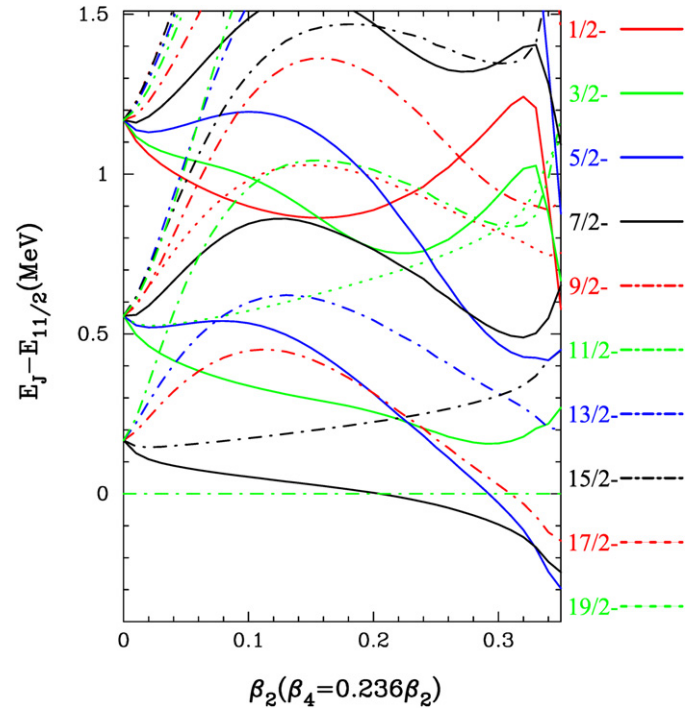


Fig. 4. Eigenvalues with negative parity of the Hamiltonian of Eq. (2) with respect to the $J^\pi=11/2^-$ state. In these calculations the E^{2+} energy was assumed to be constant.

It might look surprising that the $7/2^-$ and $3/2^-$ have similar half-lives, since they have quite different centrifugal barriers. This is due to the fact that the size of the $f_{7/2}$ component is much larger than the $p_{3/2}$ component as can be seen easily using perturbation expansions arguments. The $f_{7/2}$ is coupled at first order through the quadrupole deformation, while the $p_{3/2}$ comes from a second order coupling of the quadrupole or first order of the hexadecapole fields, therefore are smaller.

As it was shown, the calculations for the half-lives of the decay from ^{121}Pr with the adiabatic and the nonadiabatic quasiparticle approaches suggest a different value for the angular momentum of the decaying state. We interpret this result as a consequence of the large strength of the Coriolis interaction that mixes considerably the Nilsson states coming from the spherical $h_{11/2}$ level. However, since proton emission is a slow process compared to gamma decay, it can occur only from either the ground or an isomeric state. Therefore to identify the emitting state it is not enough to reproduce the experimental half-life, but one has to show that it is also a bandhead.

The energies of the negative parity states with respect to the $J=11/2^-$ are represented in Fig. 4. The bunching of levels at zero deformations corresponds to the coupling of the $h_{11/2}$ spherical proton level, to the rotational spectrum of the core, since the calculation has been performed using a fixed value of the moment of inertia obtained from the energy of the 2^+ given by Eq. (3). The $3/2^-$ states is never the ground state and, although in some way it reproduces the experimental half-life, can be definitely eliminated as a possibility. One can also observe that, for low deformation, the lowest state in energy is the $11/2^-$, while, as deformation increases, the $7/2^-$ lowers its energy, until it becomes the ground state and consequently the decaying state for deformation larger than ≈ 0.2 .

In fact, looking at the systematics of odd- A praseodymium isotopes shown in Fig. 5, one can see that the isotopes with higher neutron number and lower deformation, display a rotational band based on a $11/2^-$ state [15–17], and the energy spacings between levels have a great correspondence to the ones of the ground state

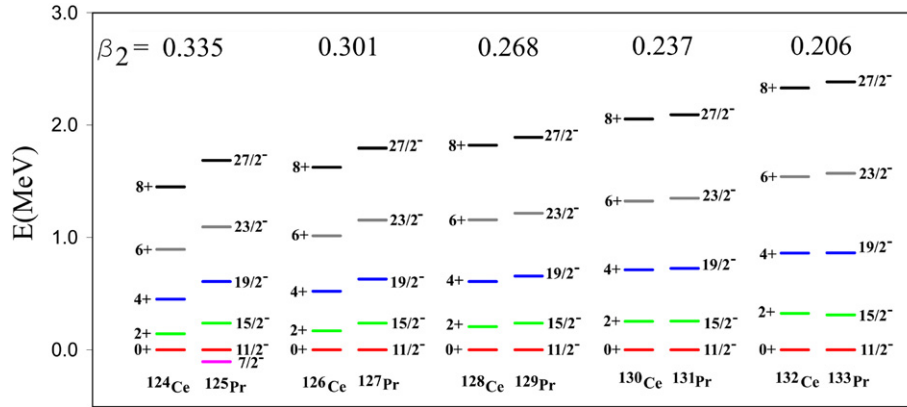


Fig. 5. Ground state rotational bands of different Ce isotopes compared to the negative parity bands in the neighbour Pr nucleus. The quadrupole deformation assigned to the different nuclei has been calculated using the Grodzins formula [7,13]. Experimental data are taken from Refs. [15–20].

rotational band in the Ce daughter nuclei [18–20]. This effect is also present, for low deformation, in the results of our model presented in Fig. 4 and gives support to our calculation. In the past, similarities like these were the main inspiration for the development of the so-called rotation aligned scheme [6]. This scheme relies on the notion that it is energetically easier to achieve a given spin by combining a particular angular momentum aligned nearly along the rotation axis with a smaller core rotation than coupling a particular angular momentum aligned elsewhere with a large core rotation.

The importance of the rotation alignment scheme was first realized in 1975 by Stephens [6]. If we analyze the total Hamiltonian of Eq. (2), minimization of the total energy shows that for a fixed total angular momentum J and a more or less fixed proton angular momentum j , the $\vec{J} \cdot \vec{j}$ term of the rotor Hamiltonian tries to align the intrinsic spin j with the total spin J .

In lighter praseodymium isotopes, the energy of the $15/2^-$ states increases with respect to the 2^+ of the daughter nucleus as the deformation increases, an effect also present in Fig. 4. Finally, the lightest and most deformed praseodymium isotope for which the experimental spectrum is known, ^{125}Pr , has a $7/2^-$ bandhead showing that approaching the proton drip-line, there is a crossing between the $7/2^-$ and $11/2^-$ levels.

This experimental evidence gives further support to our calculation and assignment of the angular momentum of the proton emitting state.

For these reasons our analysis indicates a strong evidence for a partial rotation alignment in ^{121}Pr , with the spin and parity of the proton emitting state as $J^\pi = 7/2^-$, although we cannot completely exclude the possibility of being a $3/2^+$.

In conclusion, ^{121}Pr is the first proton emitting nucleus for which partial rotation alignment has been clearly established, di-

rectly from the interpretation of the half-life, showing how powerful the observation of proton radioactivity can be for the knowledge of nuclear structure.

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