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## Influence of the displacement profile on the performance and mechanical stresses of an axial piston compressor for refrigeration applications

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### Abstract

Axial piston compressors are commonly equipped with rotating disk plates that make the pistons following a sinusoidal displacement. The variation of the plate angle leads to stroke increments without changing the displacement profile. The axial piston architecture allows one to make piston displacement profiles that are different from a sinusoidal one by using rotating disk with a shaped circumferential profile. In this work, a detailed analysis on the thermodynamic cycle of compressors with different disk geometries was carried out. A lumped parameter numerical model of a compressor for refrigeration application was developed. The compressor performance (i.e. indicated power, compressed mass of gas and specific power) was estimated by imposing piston displacement profiles that are different from the sinusoidal one. The influence on the cycle COP in which the compressor runs was evaluated for each analysis. For each profile, the study of the forces acting on the rotating plate was also investigated. A sensitivity analysis allowed the definition of a profile design that guarantees the optimization of both the thermodynamic cycle and the mechanical stresses.

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**Nomenclature***Greek letters*

$\gamma$	Specific heats ratio
$\theta$	Angular position [°]
$\rho$	Density [kg/m <sup>3</sup> ]

*Latin letters*

$A$	Area [m <sup>2</sup> ]
$a$	Acceleration [m/s <sup>2</sup> ]
$COP$	Coefficient of Performance
$F$	Force [N]
$h$	Displacement [m]
$K$	Flow coefficient
$M$	Mass [kg]
$\dot{m}$	Mass flow rate [kg/s]
$p$	Pressure [Pa]
$W$	Power [W]

*Subscripts*

$01$	Total upstream
$1$	Upstream
$2$	Downstream
$ax$	Axial
$AXP_i$	Piston profile
$cyl$	Cylinder
$iner$	Inertia
$lat$	Lateral
$p$	Piston
$pres$	Pressure
$sin$	Sinusoidal
$v$	Valve

**1. Introduction**

In the vapor compression refrigeration cycles, the reciprocating compressor efficiency influences directly the COP of the cycle by increasing or reducing the full cycle absorbed power for a fixed refrigerating effect [1]. It follows that it is important to optimize the work amount spent to raise the pressure of the gas in order to obtain advantages in terms of both absorbed energy and operating costs. For this reason, the performance optimization (i.e. reduction of the absorbed specific work) of reciprocating compressors is fundamental, especially for machines operating continuously and for long time as in refrigeration applications.

The geometry is one of the main aspects that affects the compressor performance and thus the thermodynamic efficiency of the refrigeration cycle. Therefore, many sensitivity analyses on the influence of the geometric parameters on the compressor performance can be found in the literature. Baek et al. [2] studied how the cylinder volume ratio affects the performance of a twin rotary compressor working in a refrigeration cycle. Jang et al. [3] developed a selection method for the optimum volume ratio for a two-stage compressor operating in an air source heat pump. Farzaneh-Gord et al. [4] performed a theoretical analysis to simulate a natural gas reciprocating compressor and they investigated the influence of the dead volume on the performance. The results showed that an increase of the dead volume leads to a reduction of both the mass flow rate and the compressor work, and to an enhancement of the work per mass unit. The influence of the dead volume was investigated also by Castaing-Lasvignottes and Gibout [5], who analyzed a reciprocating compressor working in a refrigeration system with R134a. The lower is the dead volume, the greater is the mass flow rate and the volumetric efficiency tends to one, whereas the isentropic efficiency is relatively insensitive. Rigola et al. [6] evaluated the influence of stroke-to-bore ratio on the performance of a hermetic reciprocating compressor in a refrigeration cycle. When stroke-to-bore ratio increases, although maintaining the chamber volume, the volumetric efficiency tends to an asymptotic value and the power consumption is maintained almost constant with a small increase. On the other hand, the COP of the cycle presents a maximum value and then begins to decrease. In addition, Chaudhary and Gupta [7] analyzed the influence of the stroke-to-bore ratio on the reciprocating compressor for domestic refrigeration showing that the COP of the cycle is constant and the volumetric efficiency slightly increases. These studies carried out sensitive analyses that lead to modify the geometry of the compressor chamber.

In this work, an axial piston compressor is studied in order to determine the influence of the piston displacement profile on the compressor performance without changing the geometry of the compressor chamber. Axial piston compressors are commonly equipped with a rotating disk that moves the pistons. The disk is a circular and flat plate whose circumferential profile leads to a sinusoidal piston displacement. By modifying the rotating disk angle, the

stroke is varied without changing the sinusoidal profile of the piston displacement. This leads to the introduction of regulation opportunities by varying the compression rate [8].

With the aim of improving the compressor performance by changing the piston displacement profile without varying the compression rate, the authors focused their attention on the axial piston compressors, the only ones that allow changes in the piston displacement profile because of the presence of the rotating disk that moves the piston. A compressor equipped with automatic valves, whose openings are regulated by the difference between the pressures inside and outside the cylinder, is examined. For these type of valves, different piston displacement profiles lead to different valve opening timing on the crank-angle cycle. Once the valves are open, the suction and discharge mass flow rates are strongly dependent from the piston displacement profile, due to the cylinder pressure that changes more or less consistently depending on the displacement profile slope. The hypothesis at the basis of this study is that the displacement profile combined with automatic valve openings influences the suction and discharge phase timing, thus the pressure inside the cylinder. It follows that the absorbed work of the cycle is modified by varying the displacement profile. Furthermore, as reported in the literature on the axial piston pump [9], the checking of the mechanical stresses on the rotating disk is an important issue in the analysis of this type of machine.

A sensitivity analysis of the compressor thermodynamic cycle was carried out by varying the piston displacement profile. For this work, the authors used the lumped parameter model named Re.Co.A. (Reciprocating Compressor Analysis), that was developed and used in previous works [10,11] with good results. A wide set of harmonic piston profiles [12] were simulated in order to optimize the compressor performance in terms of indicated power, mass of gas per cycle and specific power. Finally, the influence of the displacement profile on the COP of the refrigeration cycle was evaluated. A mechanical analysis of the system was also carried out by considering the forces exerted by the gas pressure on the piston surface and the inertia forces of the moving parts (i.e. piston and connecting rod). The axial and circumferential forces acting on the rotating disk that moves the compressor piston were analyzed in detail, with the aim of identifying the one that is the best compromise between the performance and mechanical stresses.

## 2. Numerical model

The influence of the displacement profile on the compressor performance was evaluated by means of the numerical model Re.Co.A. developed in MATLAB®. This analysis represents the preliminary study of the compressor design; consequently, a very flexible and fast tool was used. The compressor model Re.Co.A. is based on a 0D quasi-steady numerical approach. In order to evaluate the variation of the thermodynamic parameters in the whole 360° of the cycle, the energy and mass continuity equations are solved in the fluid domain inside the cylinder by time-step advances. For each simulation time-step, which corresponds to a crank angle position, the displacement of the piston is computed by the proper kinematic equation implemented in each analysis. The simulations of the reciprocating compressor follow a two-step numerical computation. At the first computational step, the mass continuity and energy equations are solved to calculate the preliminary thermodynamic conditions for the new in-cylinder volume value. The computed density and specific internal energy allow calculating the in-cylinder pressure and temperature from the real gas property tables. Afterwards, the in-cylinder pressure determines the phase of the cycle (suction, discharge, compression or expansion) and computes the mass flow through the valves. By solving the mass continuity and the energy equations, the in-cylinder density and the internal energy are calculated. A scheme of an axial piston compressor with a generic circumferential profile of the rotating disk and its fluid domain is depicted in Fig. 1a. During the suction and discharge phases, the valve opening is instantaneous. The mass flow is computed using the nozzle isentropic flow formulation (Eq.1).

$$\dot{m} = K \cdot A_v \cdot \sqrt{\left[ \frac{2\gamma}{\gamma-1} \right]} \left\{ \rho_1 \cdot p_{01} \left[ \left( \frac{p_2}{p_{01}} \right)^{2/\gamma} \cdot \left( \frac{p_2}{p_{01}} \right)^{1+\frac{1}{\gamma}} \right] \right\} \quad (1)$$

The effective in-cylinder pressure and temperature values at a crank angle position are computed from the real gas property tables by using the values of density and specific internal energy. This procedure is repeated for several

times until two subsequent thermodynamic cycles have a negligible relative difference in terms of in-cylinder pressure.

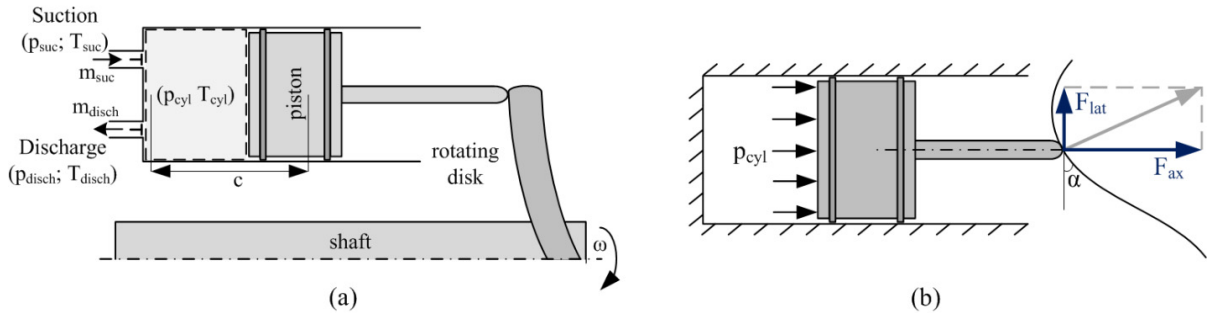


Fig. 1. (a) Re.Co.A computational domain; (b) mechanical forces on the rotating disk.

### 2.1. Force analysis

The thermodynamic cycle shows the variations of the in-cylinder pressure that exerts the force on the piston and, consequently, on the rotating disk. During the rotation of the compressor, the forces that act axially from the piston to the disk are the result of two contributions: the in-cylinder pressure ( $F_{pres}$ ) and the piston inertia ( $F_{iner}$ ) (Eq.2).

$$F_{ax} = F_{pres} + F_{iner} = p_{cyl} \cdot A_p - M_p \cdot a_p \cdot \text{sign}(a_p) \quad (2)$$

It is important to notice that the force due to the pressure has the same versus at different crank angle position (i.e. from the piston to the disk), whereas the versus of the force of inertia depends on the direction of the piston acceleration. Therefore, the contribution of the piston inertia can increase or decrease the contribution of the pressure depending on the rotating disk profile and the crank angle position.

Due to the shape of the rotating disk, also a perpendicular force  $F_{lat}$ , which causes a torque along the axis of the rotating disk, acts between the piston rod and the disk. In reference to Fig. 1b, by considering a contact between the components in a single point, the lateral force can be calculated by the trigonometric laws, where  $\alpha$  is the inclination of the profile in respect to the axial direction.

### 3. Test case

The analysis was carried out on a cylinder of a DORIN reciprocating compressor for refrigeration applications with R134a. In Fig. 1a, the functioning of an axial piston compressor with a generic profile of a rotating disk is depicted. The movement of the piston inside the cylinder is caused by the rotating disk that is linked to the shaft. As a first approach, it is hypothesized that the piston rod is in contact on the disk in a single point such that the piston displacement profile is identical to the rotating disk profile. The compressor data are summarized in Table 1.

The reciprocating compressors for refrigeration applications are able to rotate both clockwise and counter-clockwise. Therefore, in order to obtain the same compressor cycle in both the rotation directions, the circumferential profiles of the rotating plate have to be symmetrical with respect to  $180^\circ$  of the crank angle. By considering the first  $180^\circ$ , a set of test profiles is generated. It is worth pointing out that the harmonic profile is obtained in the common axial piston compressors equipped with a planar rotating disk. In this study, different profiles were constructed by coupling segments of the harmonic curves in such a way that lead to continuous displacement, velocity and acceleration curves. Since these curves have a zero acceleration at the mid-point, a discontinuity in the acceleration is avoided by coupling two different curves at this point. Therefore, it is necessary to assure the matching between the velocities at the junction in order to have a smooth and continuous piston displacement [12]. Fig. 2a shows the profile obtained with the segment AB and the segment BC of two different harmonic curves by imposing the maximum displacement equal to  $2h_1$  and  $2h_2$  and the angular position corresponding to the mid-point equal to  $\theta_1$  and  $\theta_2$ . By equating the velocities, the following equation is obtained:

$$\frac{h_1}{h_2} = \frac{\theta_1}{\theta_2} \quad (3)$$

Moreover, by considering that these parameters have some constraints (i.e. the sums of the angles and of the heights have to be fixed), the profiles can be obtained with the variation of only one parameter. In this work, the tested profiles are obtained by varying the value of  $h_1$  and are depicted in Fig. 2b that shows the displacement profile as a function of the crank angle. As can be seen, five curves were tested: one is the sinusoidal one (namely “SIN”), two have an initial slope profile steeper and the other two less steep than the sinusoidal one.

The numerical evaluations were carried out with typical conditions at the boundaries: evaporator and condenser temperatures equal to 5°C and 50°C were considered respectively. Values of 10°C and 3.5 bar was considered for the temperature and the pressure at the suction side, whereas a pressure of 13.18 bar for the discharge side.

Table 1. Compressor data.

Parameter		Value
Rotating speed	[rpm]	2950
Bore	[mm]	46
Stroke	[mm]	30
Piston mass	[g]	95

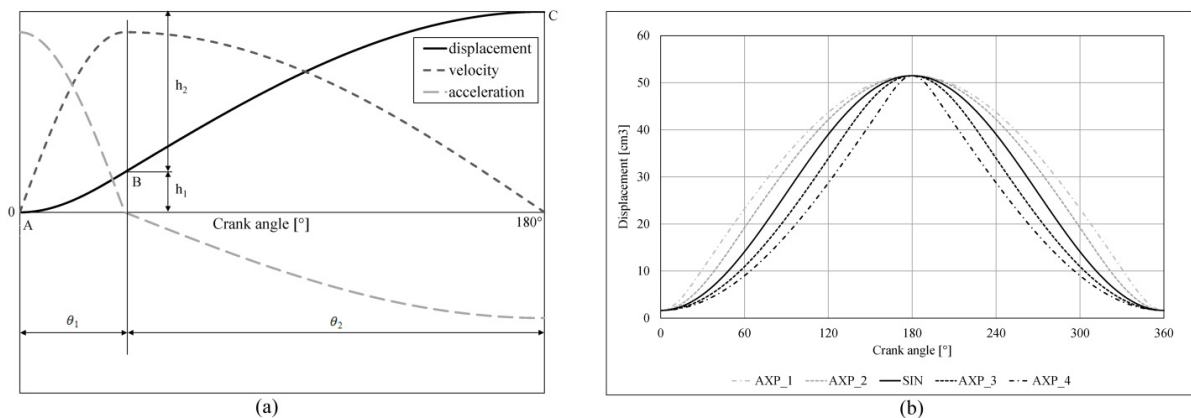


Fig. 2. (a) Generic harmonic curves with velocity and acceleration; (b) tested profiles.

#### 4. Results

The behavior of the piston displacement profiles was analyzed in terms of performance and mechanical stresses. At a first time, the mass of compressed gas, the absorbed power and the specific work were evaluated for each case. In Fig. 3, the results are reported in comparison to the values obtained with the sinusoidal profile. Because of the simulation of a volumetric machine with the same thermodynamic conditions, almost identical values of the mass flow rate were obtained: despite of the different displacement profile, the volume swept by the piston is the same. Conversely, the power calculated from the indicated cycle depends on the case. By considering the piston at the TDC (top dead center) at 0° of the crank shaft, the absorbed power is higher for the profile in which the displacement variation is faster in the first crank angles and it decreases as the slope in the first part of the profile is reduced. The specific work, expressed by the ratio between the power and the mass flow rate, follows the same trend of the power due to negligible variations of the mass flow rate. Under a thermodynamic point of view, the performance are evaluated by minimizing the specific work. Therefore, the results show that better performance are achieved by using a profile with an initial slope that is less steep than the sinusoidal one.

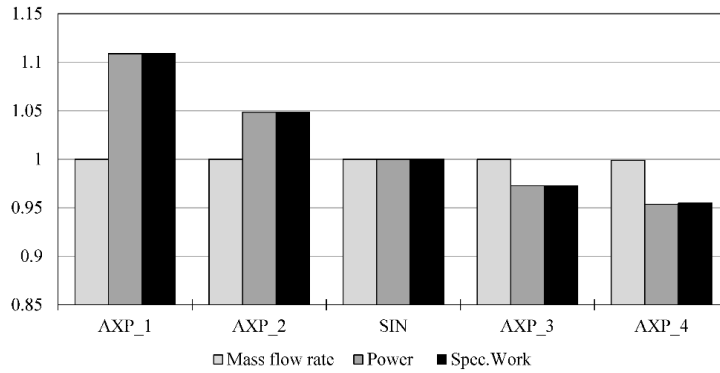


Fig. 3. Mass flow rate, produced power and specific work for different displacement profile.

The trend of the produced power can be explained by the P-V cycle (Fig. 4). The higher is the area of the cycle, the greater is the absorbed power and the specific work. For the tested profiles, the values of the in-cylinder mass with closed valves are almost identical. For this reason, the trends of the in-cylinder pressure profiles during compression and expansion phases are the same. Moreover, the suction and discharge phases start at the same piston position in the whole of the cases. During these phases, the in-cylinder pressure is not constant like it is in the ideal process and it follows different trends due to the piston velocity. In the suction phase, the in-cylinder pressure variation from the suction pressure is very small. On the other hand, during the discharge phase, the in-cylinder pressure exceeds consistently the discharge pressure and the variation depends on the displacement profile. It is important to consider that the discharge phase occurs in the final part of the stroke from the bottom dead center (BDC) to the top dead center (TDC). In Fig. 2b it can be seen that the slope of the profile, and so the piston velocity, during the discharge phase is higher in the ΔXP\_1 profile than in the other ones, for which it decreases gradually. Consequently, the rise of the in-cylinder pressure respect to the discharge pressure is higher for the ΔXP\_1 that leads to a greater area of the cycle. The high piston speed has the effect of increasing the pressure loss through the valve, thus increasing absorbed power.

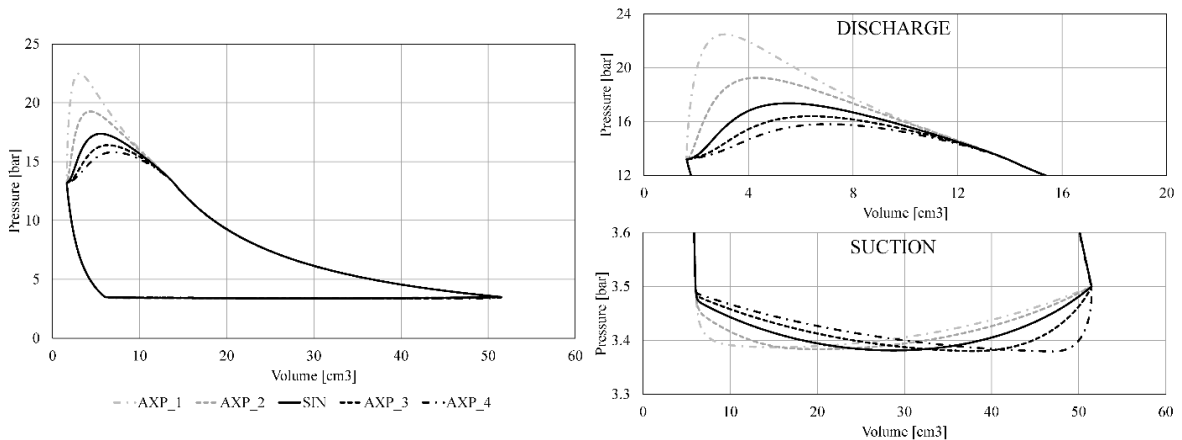


Fig. 4. P-V cycles for the tested profiles and the in-cylinder pressure trend during the suction and discharge phases.

The compressor performance influences strongly the COP of the refrigeration cycle. The COP is calculated as the ratio of the cooling capacity to the compressor power. In Fig. 5, the COP percentage differences of each analysis compared to the sinusoidal profile (Eq. 4) are shown. Due to the negligible variation of the mass flow rate, for the

different cases the cooling capacity is almost the same and the variation of COP depends only on the compressor power.

$$\Delta COP\% = \frac{COP_{AXP\_i} - COP_{sin}}{COP_{sin}} = \frac{W_{sin}}{W_{AXP\_i}} - 1 \quad (4)$$

The COP variation follows an opposite trend in comparison to that of the power. Therefore, the higher value of COP is obtained by using a circumferential profile of the rotating disk with a less steep slope in the first crank angles of the cycle. In this work, a COP improvement of about 5% can be obtained with the profile AXP\_4 in comparison to the sinusoidal profile. In comparison to [6,7], the influence of the displacement law on the mass flow rate, as well as the variation of the stroke-to-bore ratio, is negligible due to the unchanged displacement volume. On the other hand, the variation of the displacement profile has a strong influence on the COP of the refrigeration cycle: in this work, the COP variation could be about 15% (i.e. variation from AXP\_1 to AXP\_4), conversely to the results obtained by varying the stroke-to-bore ratio.

Finally, the circumferential profile of the rotating disk influences also the mechanical stresses between the piston and the disk. Due to the main influence of the in-cylinder pressure on the forces in the axial direction, the peak of the axial force occurs during the discharge phase. Consequently, as shown in Fig. 6a, the higher value of the axial force is achieved by using a rotating disk profile with the highest initial slope (i.e. AXP\_1). However, the piston inertia, which depends on the piston acceleration, influences the mechanical stresses as well. The combination of two curves to obtain the harmonic profile leads to a trend of the acceleration that changes suddenly in the coupling point, as shown in Fig. 2a. Similarly, in the axial force trend there is a sudden change that is stronger for a displacement profile with a lower initial slope (AXP\_4). It is worth noticing that the axial force is always positive in each angular position so that the piston is maintained in contact with the rotating disk.

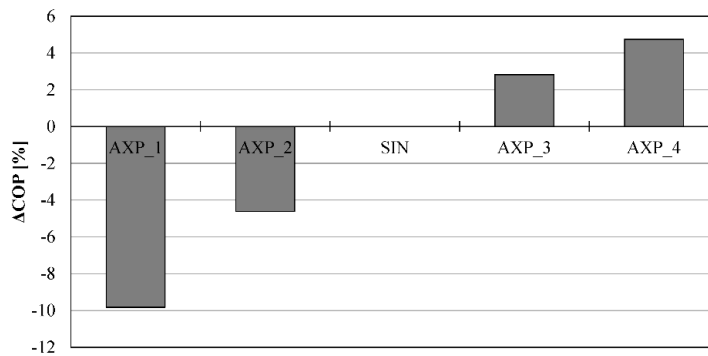


Fig. 5. COP variation for each case compared to the sinusoidal profile.

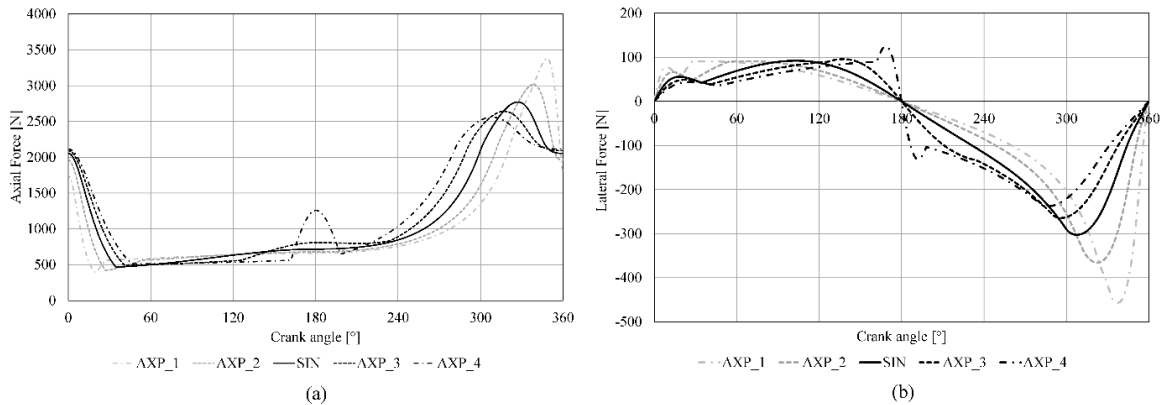


Fig. 6. (a) Axial forces vs crank angle; (b) lateral forces vs crank angle.

The variation of the lateral force on the rotating disk with different displacement profiles (Fig. 6b) has a similar trend of the axial one. By using a profile with a lower slope in proximity of the TDC, the lateral force reaches a lower peak value, but it has a more marked change of the trend at the BDC. Therefore, the use of a profile that increases the compressor performance in terms of specific work, leads to the advantage of lowering the peak of the mechanical stresses on the rotating disk.

## 5. Conclusions

The COP of compression refrigeration cycles depends on the compressor efficiency that increases or reduces the full cycle absorbed power while the refrigeration effect is fixed. Dealing with an axial piston compressor, the authors focused their attention on the effects of the piston displacement profile on the compressor performance. As well known, axial piston architecture provides a rotating disk plate that moves the piston following a sinusoidal profile. The piston displacement profile could be modified by using a rotating disk with a shaped circumferential profile.

By taking advantage of the 0D numerical model Re.Co.A. developed for the analysis of the reciprocating compressor cycle, the authors carried out a study on the performance of an axial piston compressor by making the piston following a set of different harmonic profiles. The analysis was focused on both thermodynamic and mechanical aspects: the numerical model allowed to compute the compressor performance and the forces (i.e. sum of gas pressure and piston inertia) acting on the compressor rotating disk.

The results were compared with the ones obtained with the sinusoidal piston profile. As a general statement, it was demonstrated that both the compressor performance and the mechanical stresses acting on the rotating disk are strongly influenced by the piston displacement profile. Then, the COP of the refrigeration cycle where the compressor works changes too. It was observed that a more planar slope of the piston displacement profile in the suction and discharge phases leads to lower pressure losses, by reducing the in-cylinder pressure thus the indicated absorbed power of the cycle. In particular, a 5% increase of COP respect to the sinusoidal profile was obtained. The results showed that, conversely to other works in the literature that analyze the impact of the stroke-to-bore to the cycle performance, the displacement profile has a relatively strong influence on the COP. Moreover, a reduction of both the axial and lateral forces acting on the rotating disk are consistently reduced in proximity of the TDC. On the other hand, by using these profiles the mechanical stresses have a more marked change in proximity of the TDC. In conclusion, the proper profile has to be chosen as a compromise between the performance, the peak and the trend of the mechanical stresses.

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