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On the Fundamental Periods of Vibration of Flat-Bottom Ground-Supported Circular Silos containing Gran-like Material

"Luca Pieraccini, Michele Palermo, Silvestri Stefano, Tomaso Trombetti" *

*"*Department DICAM, University of Bologna"*

Abstract

Despite the significant amount of research effort devoted to understanding the structural behavior of grain-silos, each year a large number of silos still fails due to bad design, poor construction, with a frequency much larger than other civil structures. In particular, silos frequently fails during large earthquakes, as occurred during the 1999 Chi-Chi, Taiwan earthquake when almost all the silos located in Taichung Port, 70 km far from the epicenter, collapsed. The EQE report stated that "*the seismic design of practice that is used for the design and construction of such facilities clearly requires a major revision*". The fact indicates that actual design procedures have limits and therefore significant advancements in the knowledge of the structural behavior of silo structures are still necessary. The present work presents an analytical formulation for the assessment of the natural periods of grain silos. The predictions of the novel formulation are compared with experimental findings.

Keywords: flat-bottom on-ground circular grain-silo, seismic response, fundamental period, code-like formula

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1. Introduction

The structural design of grain-silos requires accounting for the effect of the ensiled grain on the wall both under static and under dynamic conditions. Grain-silos are considered different to many other civil structures [1] and are usually classified as "non-building structures" [2, 3].

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* Corresponding author. Tel.:+39 051 209 3232; fax: + 39 051 209 3236.
E-mail address: luca.pieraccini5@unibo.it

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As widely known, the identification of the natural periods is the basic step for any seismic design. Unfortunately, for flat-bottom grain-silos no reliable formulas are available for the evaluation of the natural periods to date.

Experimental tests show that grain-silos present a marked non-linear dynamic behavior. Therefore, the common methods adopted for the dynamic analysis of common civil structures cannot be straightly applied to grain-silos.

The present study aims at providing an analytical formulation for the estimation of the fundamental period of vibration of flat-bottom circular grain-silos referable to the class of silo with isotropic continuous wall (such as rolled steel plate silos). Starting from the analytical framework proposed by [4] and [5] and adopting the same idealized model, the dynamic behavior of grain-silos is re-conducted to that of an equivalent linear-elastic system.

In the first part of the present paper, a brief review of the experimental and theoretical research works conducted by many Authors related to the dynamic behavior of grain-silos is briefly presented. In the second part of the paper, the theoretical framework adopted, the assumptions, the closed-form expressions for the analytical evaluation of the fundamental period of vibration are presented. Finally, the theoretical estimation is compared with the experimental data gathered via shaking-table tests on a silo specimen containing Ballottini-glass and data available from the scientific literature.

Nomenclature

$t_{w,i}$	Constant thickness of the *i*-th wall portion
Δz_i	Length of the *i*-th wall portion
r	Total number of the wall portions
H_{beam}	Vertical length between the silo bottom and the highest solid-wall contact
d_c	Diameter of the silo
R	Radius of the silo
$\bar{t}$	Uniform equivalent thickness of the equivalent beam
z	Vertical distance between the generic grain layer and the grain free-surface
$m_b(z)$	Mass per unit length corresponding to the *effective mass* of the grain (or bulk solid)
$p_{wf}(z)$	Wall frictional traction at a distance *z* under static condition
μ_{GW}	Grain-wall friction coefficient
γ_b	Unit weight of the ensiled bulk material
γ_w	Unit weight of the wall material
g	Gravity acceleration
λ	Pressure ratio
$p_0(z)$	Horizontal pressure according to Janssen’s formulation
$m_w(z)$	Mass per unit length of the silo wall
$t_w(z)$	Thickness of the silo wall
$\overline{m}$	Uniform mass per unit length of the equivalent beam
$\overline{t}_w$	The uniform thickness of the equivalent beam leading to the same wall mass of the silo
m_{eff}	*Effective mass*
Δ	Slenderness ratio
χ	Shear coefficient
$f_{n,sh+flex}$	Fundamental frequency accounting for both shear and flexural deformations

2. A review of the research on the seismic behavior of grain-silos

The knowledge of the dynamic properties (at least natural periods of vibration, equivalent damping ratios) of a structural system is the basic step toward its reliable seismic design. The dynamic behavior of common linear-elastic system, such as frame structures, is well established in structural dynamics and their design methods are nowadays well consolidated in the design codes. Nonetheless, for other structural systems, such as grain-silos, the prediction of the seismic behavior is still a relevant challenge, due to the strong non-linear behavior under seismic excitation and complex mass-structure interaction phenomena. Consequently, the lack of a general and universally accepted theoretical framework for the dynamic behavior of grain-silos reflects in important shortcomings in actual seismic design provisions [6, 7].

From a scientific point of view one of the main challenge to be faced deals with the evaluation of the period of vibration, which involves both the assessment of the *effective mass* involved in the dynamic response and the contribution of the silo wall in terms of stiffness. Clearly, a scientific advancement in such direction would also benefit the development of more consistent design rules. As matter of fact, even though many current design codes deal with the design of elevated and on-ground circular grain-silos, their provisions do not explicitly give formulas for the evaluation of the fundamental period of vibration of grain-silos [1], or suggest to simply consider a rigid-body response [3] (see §15.7.9.2).

Extensive experimental tests have been conducted during the last decades aimed to the study of the dynamics of flat-bottom ground-supported silos and to fully understand the complex interaction between cylindrical shell and ensiled content under earthquake excitation. Experimental dynamic tests were conducted by [9, 10, 11, 12, 13, 14, 15, 16, 6, 17, 18, 31]. In general, from the analysis of the aforementioned experimental works, it appears that:

- The dynamic response of grain silos is significantly affected by the nature of the dynamic input and by the properties of the ensiled material;
- The values of the *effective mass* seems to be significantly influenced by the excitation: large values (around 0.8) are obtained when the silo in excited close to its natural frequency; lower values are obtained under Withe Noise (WN), seismic excitation (EQK), and also under harmonic excitation far from the resonance;
- The natural frequencies are largely influenced by the excitation type WN or Harmonic Signal (HS) and acceleration amplitude (values larger or smaller than the critical acceleration a_{crit} at which horizontal grain sliding occurs, around 0.30 g);

3. An analytical formulation for the fundamental period of grain-silos

3.1. The basic assumptions

The formulation that is here proposed for the evaluation of the fundamental period of flat-bottom on-ground circular grain-silos is grounded on the Silvestri-Pieraccini theory assumptions [4, 5], and make use of the approach by [19], which is specialized for the case of grain-silo systems. The approach by [19] consists in modeling the cylindrical shell with its content as a uniform linear-elastic shear-flexural cantilever beam. Additional assumptions are here necessary to extend the [19] approach to grain-silos: (1) horizontal input is applied only; (2) the *effective mass* is independent on the profile and amplitude of the horizontal accelerations; (3) in the deformed configuration, plain section remain plain and no section ovalizations occur; (4) the stiffness of the system is provided by the silo wall only; (5) the overall mass of the equivalent beam consists of 2 contributions: the grain mass corresponding to the *effective mass* and the mass of the silo structures, and is considered as uniformly distributed along the height. The analytical estimation of the fundamental period of vibration of grain-silos

Based on the aforementioned assumptions, the fundamental period of the realistic flat-bottom on-ground circular grain-silo of Fig. 1a is evaluated with reference to the idealized equivalent uniform shear-flexural cantilever beam model, as represented in Fig. 1b. The silo of Fig. 1a has smooth wall with stepwise variable thickness $t_{w,i}$ (i is the i-th wall portion characterized by constant thickness $t_{w,i}$ and length Δz_i, r is the total number of wall portions).

All the other relevant geometrical properties of the silo are indicated in Fig. 1a. The equivalent cantilever beam of Fig. 1b, has a height H_{beam}, an hollow uniform circular cross-section of diameter d_c and thickness $\bar{t}$, and is clamped at the base. The value $\bar{t}$ varies with respect to the homogenization criteria: equal mass, equal shear frequency, equal flexural frequency. The three criteria will be specified in the following sections.

According to assumptions 1, 2, 5 and with reference to the silo configuration represented in Fig. 1, the mass per unit length to be used for the estimation of the fundamental period of vibration is made of the following contributions: (i) the *effective mass* of the grain; (ii) the mass of the silo wall.

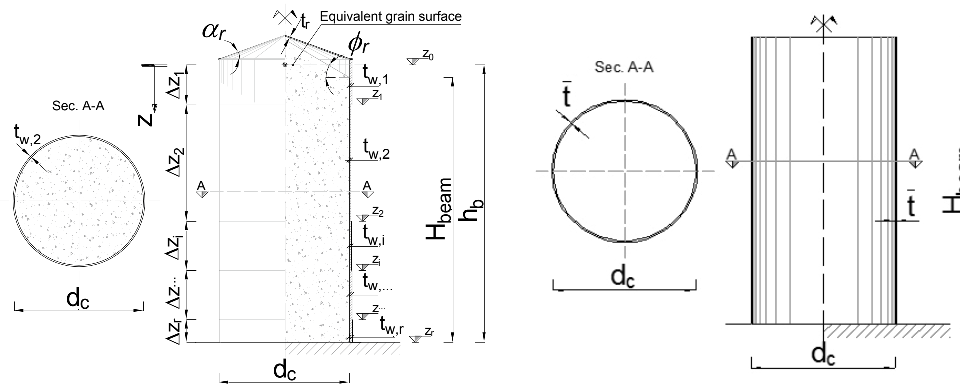

Fig. 1 – (a) Geometry of a realistic flat-bottom ground-supported circular grain-silo;(b) Geometry of the corresponding equivalent beam

The mass per unit length corresponding to the *effective mass* of the grain (or bulk solid) $m_b(z)$ is given by:

$$m_b(z)=2\pi R\cdot\left[\mu_{GW}\cdot p_0(z)\right]/g=(\gamma_b/g)\cdot\pi R^2\cdot\left[1-e^{-(2\cdot\mu_{GW}\cdot\lambda\cdot z)/R}\right] \tag{1}$$

The mass per unit length of the silo wall $m_w(z)$ can be expressed as:

$$m_w(z)=2\pi R\cdot t_w(z)\cdot(\gamma_w/g) \tag{2}$$

The equivalent uniform mass per unit length $\overline{m}$ of the Equivalent Beam Model (EBM) results equal to:

$$\overline{m}=(\gamma_b/g)\cdot\pi R^2\cdot m_{eff}+2\pi R\cdot\overline{t_w}\cdot(\gamma_w/g) \tag{3}$$

where $\overline{t_w}=\left[\sum_{i=1}^{r}\Delta z_i\cdot t_{w,i}\right]\Big/H_{beam}$ and the analytical expression of m_{eff} results:

$$m_{eff}=1+\left(1-e^{\omega}\right)/\omega \tag{4}$$

where $\omega=-4\cdot\mu_{GW}\cdot\lambda\cdot\Delta$.

The main elastic properties of the EBM as represented in Fig. 1b can be explicated as follows:

$$A_w^{'}=2\pi R\cdot\overline{t_{w,sh}}/\chi \tag{5}$$

$$I_w=\pi R^3\cdot\overline{t_{w,flex}} \tag{6}$$

where χ is the shear coefficient; $\overline{t_{w,sh}}$ and $\overline{t_{w,flex}}$ are the thickness of the uniform shear and flexural beam:

Equal shear frequency $\overline{t_{w,sh}}=H_{beam}{}^2\Big/\sum_{i=1}^{r}\left[\left(z_i^2-z_{i-1}{}^2\right)/t_{w,i}\right]$ (7)

Equal flexural frequency $\overline{t_{w,flex}}=H_{beam}{}^4\Big/\sum_{i=1}^{r}\left[\left(z_i^4-z_{i-1}{}^4\right)/t_{w,i}\right]$ (8)

The formula for the evaluation of the *n-th* natural frequency of a continuous uniform linear elastic cantilever shear-beam $f_{n,sh}$ are given by [28] and results:

$$f_{n,sh}=\frac{(2n-1)}{4\cdot H_{beam}}\cdot\sqrt{\frac{G_w\cdot A_w^{'}}{\overline{m}}} \tag{9}$$

The *n-th* natural frequency of a continuous uniform linear elastic cantilever flexural-beam $f_{n,flex}$ may be found in [29]. According to [30], $f_{n,sh+flex}$ of an equivalent shear-flexural beam can be computed as follows:

$$1\big/\left(f_{n,sh+flex}\right)^2 = 1\big/\left(f_{n,sh}\right)^2 + 1\big/\left(f_{n,flex}\right)^2 \qquad (10)$$

After some calculations, the following expression of $f_{n,sh+flex}$ can be obtained:

$$f_{n,sh+flex} = f_{n,sh} \cdot \sqrt{1\big/1+\left(f_{n,sh}/f_{n,flex}\right)^2} = f_{n,sh} \cdot \sqrt{1\big/1+\psi_n \cdot \left[\bar{\Delta}^2\big/\chi \cdot \left(1+\upsilon_w\right)\right] \cdot r_t} \qquad (11)$$

where $\psi_n = \left[\pi \cdot \left(2n-1\right)\big/\left(\phi \cdot H_{beam}\right)_n^2\right]^2$ is a function of *n*, $\bar{\Delta} = H_{beam}\big/d_c$ is the filling slenderness ratio and $r_t = \overline{t_{w,sh}}\big/\overline{t_{w,flex}}$ is the ratio of the thickness of the uniform shear beam on the thickness of the uniform flexural beam, E_w and υ_w is the Young’s modulus and Poisson’s ratio of the wall material, respectively, $\left(\phi \cdot H_{beam}\right)_n^2$ is the second power of the product between the *n*-th root of the secular equation and the beam length, as given by [29].

4. Experimental verification of the analytical formulation and modeling technique

4.1. Experimental results from the scientific literature

Table 1 compares the values of the first natural frequencies (*n=1*) of grain-silo models available from the scientific literature with those obtained according to Eq. (19). Almost all the tests are performed on squat and intermediate slender silo models with coal as ensiled material subjected to a harmonic signal HS at the resonance. For this reason, the analytical values are calculated by using a fixed *effective mass* value of 0.8. The only exception is represented by the WN test of [18], whose analytical frequency is estimated by using the *effective mass* value given by Eq. (9). On average, excluding one of the test by [11] (relative error 100 %), the relative error in the prediction of the first natural frequency is of the order of 15%. For the single test of [18], the relative error is of the order of 5%.

Table 1 - Comparison of the experimental fundamental frequencies of silo specimens and Eq. (19)

Reference	Specimen			Frequencies [*Hz*]				
	Wall material	Δ [-]	Ensiled material	Type	a [g]	Experimental	Analytical [Eq. 19] (*)	Relative error [%]
[10]	Acrylic resin	1.01	Coal	HS	0.05	19	22.5	-18
[11]	PVC resin	1.01	Coal	HS	0.30	13	13.5	-4
	PVC resin				0.10	20	20.5	-3
	PVC resin				0.10	22	29	-32
	Steel				0.10	23	46	-100
[13]	Acrylic plastic	1.33	Coal	HS	0.05	28.6	36	-26
					0.10	31.0	36	-16
					0.20	33.7	36	-7
					0.30	28.6	36	-26
			Slag		0.10	24.5	25	-2
[18]	Polycarbonate (roughened)	1.0	Ballottini glass	WN	0.10	15.6	14.9	4

5. Conclusions

An analytical formulation for the estimation of the natural periods of grain-silos is proposed. The formula is grounded on the Silvestri-Pieraccini theory. The silo is modelled as an equivalent shear-flexural cantilever beam with an applied mass equal to the mass of the silo structure plus the mass corresponding to the portion of the ensiled mass activated during the earthquake ground motion. Doing so, a fully analytical formula has been derived and can be easily implemented even in a simple excel spreadsheet for the prediction of the natural periods of grain-silos.

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